QUESTION BANK WITH SOLUTIONS IN MATHEMATICS-II

FOR 2ND SEMESTER DIPLOMA COURSES FROM SUMMER 2025 ONWARDS

UNIT-I

A. 02 marks Questions & Solutions

1. Define a unit matrix . Give an example of a unit matrix of order 3 **Level-1(Remembering)**

Answer:A diagonal matrix whose diagonal elements are all unity is called as unit matrix.Example : A 3rd order unit matrix is $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$.

2.Define a singular matrix with example.**Level-1(Remembering)**

Answer: A square matrix whose determinant is zero is called a singular matrix.

$\begin{bmatrix}1 & 2\\ 3 & 6\end{bmatrix}$ is a singular matrix.

3.Evaluate $\begin{vmatrix}1 & \omega\\ -\omega & \omega\end{vmatrix}$. **Level-1(Remembering)**

Answer: $\begin{vmatrix}1 & \omega\\ -\omega & \omega\end{vmatrix}$= 1X ω - ω X(- ω) = $\omega+\omega^2$ =-1 (1+ $\omega+\omega^2=0$)

4. Find the minimum value of $\begin{vmatrix}sinx & cosx\\ -cosx & 1+sinx\end{vmatrix}$ **Level-2(understanding)**

Answer : $\begin{vmatrix}sinx & cosx\\ -cosx & 1+sinx\end{vmatrix}$ = sin x(1+sin x)- cos x (-cos x)

= sin x + $\sin^2$x+ $\cos^2$ x

= sinx+1 (as $\sin^2$ x+$\cos^2$ x=1)

We know that the minimum value of sinx is -1.

Hence minimumvalue of sin x+1=-1+1=0.

5. Find M_{23} and C_{32} of the determinant $\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 4 & 2 \end{vmatrix}$ **Level-1(Remembering)**

Answer: $M_{23}= \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}$ = 1x4 - 2x1= 2

C_{32} = $(-1)^{3+2}M_{32} = (-1)^5 M_{32} = -M_{32} = -\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -(3-2) = -1$

6. What is the order of the matrix B if $\begin{bmatrix} 3 & 4 & 2 \end{bmatrix} B = \begin{bmatrix} 2 & 1 & 0 & 3 & 6 \end{bmatrix}$ **Level-2(understanding)**

Answer: Order of matrix $\begin{bmatrix} 3 & 4 & 2 \end{bmatrix}$ is 1x3

Order of matrix $\begin{bmatrix} 2 & 1 & 0 & 3 & 6 \end{bmatrix}$ is 1x5

Hence order of B is = 3 x 5.

7. Construct a 2x3 matrix having elements a_{ij}= i+j **Level-2(understanding)**

Answer: Let the matrix be $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$.

Given that a_{ij} = i+ j

So. a_{11} = 1 + 1 = 2 $a_{12} = 1 + 2 = 3$ $a_{13} = 1 + 3 = 4$

$a_{21} = 2 + 1 = 3$ $a_{22} = 2 + 2 = 4$ $a_{23} = 2 + 3 = 5$

Hence the required matrix is $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$.

8. If A = $\begin{bmatrix} 2 & 4 \\ 3 & 13 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then find $A - \alpha I$, $\alpha \in R$. **Level-2(understanding)**

Answer: Here A= $\begin{bmatrix} 2 & 4 \\ 3 & 13 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

So, $A - \alpha I = \begin{bmatrix} 2 & 4 \\ 3 & 13 \end{bmatrix} - \alpha \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 13 \end{bmatrix} - \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} = \begin{bmatrix} 2-\alpha & 4 \\ 3 & 13-\alpha \end{bmatrix}$.

9. Solve $\begin{vmatrix} 4 & x+1 \\ 3 & x \end{vmatrix}$ = 5 .**Level-3(Apply)**

Answer : $\begin{vmatrix} 4 & x+1 \\ 3 & x \end{vmatrix}$ = 5

⇒ 4x -3(x+1) = 5

⇒4x-3x-3 = 5

$\Rightarrow$x-3 =5

$\Rightarrow$x = 8

10. Find x and y if $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ **Level-2(understanding)**

Answer : Given $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x+3y \\ 2x-y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$\Rightarrow$ $x+3y=4$ and $2x-y$ = 1

Now $x+3y=4$

3 ($2x-y$ = 1) adding the two equations we get 7x= 7 $\Rightarrow x=1$ and y= 2x-1 = 1

11. Find x and y if $\begin{bmatrix} x & -2y \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ 0 & -2 \end{bmatrix}$ **Level-2(understanding)**

Answer : Here $\begin{bmatrix} x & -2y \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ 0 & -2 \end{bmatrix}$

$\Rightarrow$ x = 1 and -2y = -8 (since two matrices are equal their corresponding elements are equal)

$\Rightarrow$ x = 1 and y = $\frac{-8}{-2}$ = 4 .

12. If A = $\begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$ then find 2A + B **Level-2(understanding)**

Answer : Given A = $\begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$

So, 2A+B = 2 $\begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 4+6 & 6+3 \\ 4+1 & 2+4 & 8+5 \end{bmatrix}$

$= \begin{bmatrix} 7 & 10 & 9 \\ 5 & 6 & 13 \end{bmatrix}$.

13. Define minor in a determinant & find the minors of the elements of $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ **Level-1(Remembering)**

Answer : The minor of any element of a determinant is the determinant obtained by eliminating the rows and columns in which the element occur.

M_{11} =4

M_{12} =3

M_{21} =2

$M_{22} = 1$

14.Define inverse of a matrix .**Level-1(Remembering)**

Answer: If A and B are two square matrices of same order such that AB=BA=I (Identity matrix) then B is called the multiplicative inverse of A . It is denoted by A^{-1} .

15. If A = $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ then prove that $A^2 = A$**Level-2(understanding)**

Answer: Given A = $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

$A^2 = A.\ A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1\times 1 - 1\times 0 & 1\times -1 - 1\times 0 \\ 0\times 1 + 0\times 0 & 0\times -1 + 0\times 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} =$ A (Proved)

16. Given $[x \quad y \quad z] - [-4 \quad 3 \quad 1] = [-5 \quad 1 \quad 0]$ determine x,y,z .**Level-2(understanding)**

Answer:Given $[x \quad y \quad z] - [-4 \quad 3 \quad 1] = [-5 \quad 1 \quad 0]$

$\Rightarrow [x+4 \quad y-3 \quad z-1] = [-5 \quad 1 \quad 0]$

$\Rightarrow x+4=-5, \quad y-3=1 \quad z-1=0$

$\Rightarrow x = -5-4=-9, y=1+3=4, z=$ 0+1 = 0

17. Define transpose of a matrix with an example .**Level-1(Remembering)**

Answer: The transpose of a matrix A is the matrix of order nxm obtained by interchanging the rows and columns of A . transpose of a matrix is denoted by A'

Example: if A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ then A' = $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

18. Define adjoint of a matrix**Level-1(Remembering)**

Answer :The adjoint of a matrix is the transpose of the cofactors matrix of the given matrix. The adjoint of a matrix A is denoted by Adj A.

19. For which value of x the determinant $\begin{vmatrix} x & 1 \\ 2x & 3 \end{vmatrix} = 0$ satisfies ? **Level-2(understanding)**

Answer :for $\begin{vmatrix} x & 1 \\ 2x & 3 \end{vmatrix} = 0 \Rightarrow 3x - 2x = 0 \Rightarrow x = 0$. So for x=0 the given determinant satisfies.

20. Find the co-factors of $\begin{vmatrix} 7 & 8 \\ 3 & 4 \end{vmatrix}$ **Level-1(Remembering)**

Answer : Here A=$\begin{vmatrix} 7 & 8 \\ 3 & 4 \end{vmatrix}$

Co-factors of an element is $C_{ij}=(-1)^{i+j}M_{ij}$

$C_{11}= (-1)^{1+1}M_{11}= 1x4=4$, $C_{12}= -1X3=-3$, $C_{21}=(-1)^{2+1}M_{21}= -1X8=-8$ and $C_{22}=(-1)^{2+2}M_{22}=1X7=7$

21.Find the value of x if $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$ **Level-3(Apply)**

Answer: Using R1=R1+R2+R3

$$\begin{vmatrix} 3+x & 3+x & 3+x \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$$

Taking 3+x as a common factor ,we get $(3+x)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$

Using $C_1=C_1-C_2, C_2=C_2-C_3$,we get , $(3+x)\begin{vmatrix} 0 & 0 & 1 \\ -x & x & 1 \\ 0 & -x & 1+x \end{vmatrix} = 0$

Expanding $(3+x)x^2 = 0$ *so the value of* $x = -3$ *or* $0.$

22. Check the system of equation 2x-y=5 and 6x-3y=15 is consistent or in consistent. **Level-2(understanding)**

Answer: $\Delta= \begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix}$ = -6+6=0 , $\Delta_1=\begin{vmatrix} 5 & -1 \\ 15 & -3 \end{vmatrix}$=-15+15=0 and $\Delta_2 = \begin{vmatrix} 2 & 5 \\ 6 & 15 \end{vmatrix}$ = 30-30=0

Since $\Delta=\Delta_1=\Delta_2=0$

The system is consistent and has infinitely many solutions.

B. 05 MARKS QUESTIONS & SOLUTIONS :

1. Solve 2x-y = 2 , 3x+y = 13 by Cramer's rule **Level-3(Apply)**

Answer:Given 2x-y = 2 , 3x+y = 13

$$\Delta = \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 2 + 3 = 5$$

$$\Delta_1 = \begin{vmatrix} 2 & -1 \\ 13 & 1 \end{vmatrix} = 2 + 13 = 15$$

$$\Delta_2 = \begin{vmatrix} 2 & 2 \\ 3 & 13 \end{vmatrix} = 26 - 6 = 20$$

By Cramer's rule $\frac{x}{\Delta_1} = \frac{y}{\Delta_2} = \frac{1}{\Delta}$

so $x = \frac{\Delta_1}{\Delta}$ and $y = \frac{\Delta_2}{\Delta} \Rightarrow x = \frac{15}{5}$ and $y = \frac{20}{5}$

$\Rightarrow x = 3$ and $y = 4$

2. Prove that $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$ = abc (a-b)(b-c)(c-a) . level-3(Analyse)

Answer: L.H.S. = $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$

= abc $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ (Taking a,b,c from R_1, R_2, R_3)

=abc $\begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$ (replacing R_1 by R_1-R_2 and replacing R_2 by R_2-R_3)

= abc (a-b)(b-c) $\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$ (Taking(a-b), (b-c) common from R_1&R_2 respectively)

=abc (a-b)(b-c) $\begin{vmatrix} 1 & a+b \\ 1 & b+c \end{vmatrix}$

= abc(a-b)(b-c)(b+c-a-b)

= abc(a-b)(b-c)(c-a) = R.H.S

3. If A= $\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ then verify that A^2-12A-I_2 =0 where I_2 is an identity matrix of order 2 . **Level-3(Apply)**

Answer: Given A = $\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

So, A^2 =AXA = $\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ = $\begin{bmatrix} 5\times5+3\times12 & 5\times3+3\times7 \\ 12\times5+7\times12 & 12\times3+7\times7 \end{bmatrix}$

= $\begin{bmatrix} 25+36 & 15+21 \\ 60+84 & 36+49 \end{bmatrix}$ = $\begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix}$

Now, 12A= 12 $\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ = $\begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix}$ and I_2 = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Thus A^2-12A-I_2 = $\begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} 61-60-1 & 36-36-0 \\ 144-144-0 & 85-84-1 \end{bmatrix}$ = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = 0 (verified)

4. Verify that $(AB)^T = B^T A^T$ where A = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$ **level-3(analyse)**

Answer :GivenA = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

A.B= $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

= $\begin{bmatrix} 1+4-3 & 2+0+3 \\ 3-4-1 & 6+0+1 \end{bmatrix}$= $\begin{bmatrix} 2 & 5 \\ -2 & 7 \end{bmatrix}$

$(AB)^T$= $\begin{bmatrix} 2 & -2 \\ 5 & 7 \end{bmatrix}$

B^T= $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ and A^T = $\begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \end{bmatrix}$

B^TA^T = $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \end{bmatrix}$

= $\begin{bmatrix} 1+4-3 & 3-4-1 \\ 2+0+3 & 6+0+1 \end{bmatrix}$= $\begin{bmatrix} 2 & -2 \\ 5 & 7 \end{bmatrix}$

Hence $(AB)^T = B^T A^T$

5. Find the inverse of the matrix $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$. **Level-3(Apply)**

Answer : Let A= $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$|A|$ = $\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 2 \times 3 - (-1) \times 1 = 6 + 1 = 7 \neq 0$

Hence A^{-1}exists .

We know $A^{-1} = \frac{adjA}{|A|}$

Minors:

M_{11}= 3 , M_{12}= 1, M_{21} = -1 and M_{22}= 2

Co-factors:

C_{11}= M_{11}= 3 , C_{12}= -M_{12}= - 1 , C_{21}= - M_{21} = 1 and C_{22}= M_{22}= 2

Adj A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$A^{-1} = \frac{adjA}{|A|}$$

$$= \frac{\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}}{7} = \begin{bmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{bmatrix}$$

6. Solve the equations x+2y =3 and 3x+y=4 by matrix method. **Level-3(Apply)**

Answer : The given system of equations is of the form AX= B

Where A= $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, B= $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and X = $\begin{bmatrix} x \\ y \end{bmatrix}$

Now , $|A|$ = $\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$ = 1-6 = -5 ≠ 0

So the inverse exists.

Now AX= B

⇒ X = $A^{-1}B$

To find A^{-1}

Minors:

M_{11}= 1 , M_{12}= 3 , M_{21}=2 and M_{22}= 1

Cofactors:

C_{11}=M_{11}= 1, C_{12}= - M_{12}= -3 , C_{21}= -M_{21}= -2 and C_{22}=M_{22}= 1

Adj A = $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ and $A^{-1} = \frac{adjA}{|A|}$ == $\frac{\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}}{-5}$ = $\begin{bmatrix} \frac{1}{-5} & \frac{-2}{-5} \\ \frac{-3}{-5} & \frac{1}{-5} \end{bmatrix}$

= $\begin{bmatrix} \frac{1}{-5} & \frac{2}{5} \\ \frac{3}{5} & \frac{1}{-5} \end{bmatrix}$

Now,X = $A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{-5} & \frac{2}{5} \\ \frac{3}{5} & \frac{1}{-5} \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix}$ = $\begin{bmatrix} \frac{-3}{5} + \frac{8}{5} \\ \frac{9}{5} - \frac{4}{5} \end{bmatrix}$ = $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Hence x=1 and y=1

7. Find A and B where

$2A + B = \begin{bmatrix} 2 & 2 & 5 \\ 5 & 4 & 3 \\ 1 & 1 & 4 \end{bmatrix}$ *and* $A - 2B = \begin{bmatrix} 1 & 6 & 5 \\ 5 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix}$ **Level-2(Understanding)**

Answer : Given $2A + B = \begin{bmatrix} 2 & 2 & 5 \\ 5 & 4 & 3 \\ 1 & 1 & 4 \end{bmatrix}$ *and* $A - 2B = \begin{bmatrix} 1 & 6 & 5 \\ 5 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix}$

Now $2(2A + B) + A - 2B = \begin{bmatrix} 4 & 4 & 10 \\ 10 & 8 & 6 \\ 2 & 2 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 5 \\ 5 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 15 \\ 15 & 10 & 5 \\ 0 & 0 & 10 \end{bmatrix}$

$\Rightarrow 5A = \begin{bmatrix} 5 & 10 & 15 \\ 15 & 10 & 5 \\ 0 & 0 & 10 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

Therefore

$2B = A - \begin{bmatrix} 1 & 6 & 5 \\ 5 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1-1 & 2-6 & 3-5 \\ 3-5 & 2-2 & 1+1 \\ 0+2 & 0+2 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & -4 & -2 \\ -2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} 0 & -2 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

8. Find the Adjoint of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 2 & -1 \end{bmatrix}$.

Level-3(Apply)

Answer :Now co-factors of the given matrix are

$C_{11}= M_{11}=$ $\begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix}$ = -1

C_{12}=- M_{12} = $-\begin{vmatrix} 3 & 0 \\ -2 & -1 \end{vmatrix}$ = 3

$C_{13}= M_{13}$ = $\begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}$ = 8

C_{21}= -M_{21} = $-\begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix}$ = 8

$C_{22}= M_{22}$ = $\begin{vmatrix} 1 & 3 \\ -2 & -1 \end{vmatrix}$ = 5

C_{23} = -M_{23}= $-\begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix}$ = -6

$C_{31}= M_{31}$ = $\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix}$ = -3

C_{32}= -M_{32} = $-\begin{vmatrix} 1 & 3 \\ 3 & 0 \end{vmatrix}$ = 9

$C_{33}= M_{33}$ = $\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$ = -5

Adjoint of A is the transpose of $[\, C_{ij}]_{3x3}$

Hence adj A= $\begin{bmatrix} -1 & 8 & -3 \\ 3 & 5 & 9 \\ 8 & -6 & -5 \end{bmatrix}$.

9. Using Cramer's rule, solve the system of equations

$5x - y + 4z = 5$

$2x + 3y + 5z$ =2

$5x - 2y + 6z = -1$

Level-3(Apply)

Answer : We have $\Delta = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$

$= 5\,(18 + 10) + 1(12 - 25) + 4(-4 - 15) = 51$

$\Delta_1 = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ -1 & -2 & 6 \end{vmatrix}$

$= 5(18+10)+1(12+5)+4(-4+3)=153$

$$\Delta_2 = \begin{vmatrix} 5 & 5 & 4 \\ 2 & 2 & 5 \\ 5 & -1 & 6 \end{vmatrix}$$

$= 5(12+5)-5(12-25)+4(-2-10)=102$

$$\Delta_3 = \begin{vmatrix} 5 & -1 & 5 \\ 2 & 3 & 2 \\ 5 & -2 & -1 \end{vmatrix}$$

$= 5(-3+4)+1(-2-10)+5(-4-15)=-102$

Since $\Delta \neq 0,$ we have by Cramer's rule,

$$\therefore x = \frac{\Delta_1}{\Delta} = \frac{153}{51} = 3$$

$$y = \frac{\Delta_2}{\Delta} = \frac{102}{51} = 2$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-102}{51} = -2$$

$\therefore x = 3, y = 2, z = -2$

10. Solve the system of linear equations by using matrix method. **Level-3(Apply)**

x+y+z=3

x+2y+3z=4

x+4y+9z=6

The given system of equations can be written as AX=B

Where $= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} - 1 \begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}$$

$$= 1(18-12) - 1(9-3) + 1(4-2)$$

=6-6+2

=2 $\neq$0

Since $|A| \neq 0$, A is not singular,

Hence, A^{-1} exists and the unique solution is $X = A^{-1}B$.

Cofactors of the elements of $|A|$ are given by

$A_{11} = \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 6,\quad A_{12} = -\begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -6,\quad A_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 2,$

$A_{21} = -\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -5,\quad A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 8,\quad A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -3,$

$A_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1,\quad A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -2,\quad A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1,$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\text{ adj A} = \frac{1}{2}\begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$\Rightarrow X = A^{-1}B$$

$$= \frac{1}{2}\begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 18 - 20 + 6 \\ -18 + 32 - 12 \\ 6 - 12 + 6 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \Rightarrow x = 2, y = 1, z = 0$$

11. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c})$ **Level-3(Apply)**

Solution: L.H.S = $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

= $\begin{vmatrix} 1+a-1 & 1-1 & 1 \\ 1-1-b & 1+b-1 & 1 \\ 1-1 & 1-1-c & 1+c \end{vmatrix}$ (replacing $C_1=C_1-C_2, C_2=C_2-C_3$)

$$= \begin{vmatrix} a & 0 & 1 \\ -b & b & 1 \\ 0 & -c & 1+c \end{vmatrix}$$

Expanding we get

$$= a\begin{vmatrix} b & 1 \\ -c & 1+c \end{vmatrix} + 1\begin{vmatrix} -b & b \\ 0 & -c \end{vmatrix}$$

$$= a(b+bc+c)+bc$$

$$= ab+abc+ac+bc$$

$$= abc(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}) = \text{R.H.S}$$

12. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix} = (b-c)(c-a)(a-b)$ **Level-3(Apply)**

Solution: L.H.S $= \begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1-1 & 1-1 \\ b+c & c+a-b-c & a+b-b-c \\ b^2+c^2 & c^2+a^2-b^2-c^2 & a^2+b^2-b^2-c^2 \end{vmatrix}$ (replacing C_2 by C_2-C_1 and C_3 by C_3-C_1)

$$= \begin{vmatrix} 1 & 0 & 0 \\ b+c & a-b & a-c \\ b^2+c^2 & a^2-b^2 & a^2-c^2 \end{vmatrix}$$

$= (a-b)(a-c)\begin{vmatrix} 1 & 0 & 0 \\ b+c & 1 & 1 \\ b^2+c^2 & a+b & a+c \end{vmatrix}$ (Taking common $(a-b)(a-c)$ from C_2&C_3 respectively)

$$= (a-b)(a-c)\begin{vmatrix} 1 & 1 \\ a+b & a+c \end{vmatrix}$$

$$= (a-b)(a-c)(a+c-a-b)$$

$$=(a-b)(a-c)(c-b)$$

$=(b-c)(c-a)(a-b) =$ R.H.S

QUESTION BANK WITH SOLUTIONS IN MATHEMATICS-II

FOR 2ND SEMESTER DIPLOMA COURSES FROM SUMMER 2025 ONWARDS

UNIT-II

A. 02 marks Questions & Solutions

1. Evaluate $\int a^{mx}\, dx$ ……………. **(L-1 - REMEMBER)**

Soln: $\int a^{mx}\, dx = \frac{a^{mx}}{\log a}\frac{1}{m} + c$

$= \frac{a^{mx}}{m \log a} + c$ (Ans)

2. **Evaluate** $\int \sqrt{1+\ cos2x}\, dx$ …………… **(L-2 - UNDERSTANDING)**

Soln: $\int \sqrt{1+\ cos2x}\, dx = \int \sqrt{2cos^2 x}\, dx = \int \sqrt{2}\, cosx\, dx = \sqrt{2}\int cosx\, dx = \sqrt{2}\ sinx + c$ (Ans)

3. **Evaluate** $\int e^{\sin^{-1}x+\cos^{-1}x}\, dx$………….. **(L-2 - UNDERSTANDING)**

Soln: $\int e^{\sin^{-1}x+\cos^{-1}x}\, dx = \int e^{\pi/2}\, dx = e^{\pi/2}\int dx = e^{\pi/2}x + c$ (Ans)

4. **Evaluate** $\int a^x e^x\, dx$ ……….. **(L-2 - UNDERSTANDING)**

Soln: $\int a^x e^x\, dx = \int (ae)^x\, dx = \frac{(ae)^x}{\log (ae)} + c = \frac{(ae)^x}{\log a+\log e} + c = \frac{(ae)^x}{\log a+1} + c$ (Ans)

5. **Evaluate** $\int \frac{dx}{x^2+4}$…………… **(L-1 - REMEMBER)**

Soln: $\int \frac{dx}{x^2+4} = \int \frac{dx}{x^2+2^2} = \frac{1}{2}\tan^{-1}\frac{x}{2} + c$ (Ans)

6. $Evaluate \int 2\, e^{\ln x} dx$ ((L-2 - UNDERSTANDING))

Solution:- $\int 2\, e^{\ln x} dx = \int 2\, x dx = 2 \int x\, dx = 2.\frac{x^2}{2} + c = x^2 + c$

7. Evaluate $\int 2\, x e^{x^2} dx$ ((L-2 - UNDERSTANDING))

Solution: - $\int 2\, x e^{x^2} dx$ [Put $x^2 = t => 2xdx = dt => dx = \frac{dt}{2x}$]

$= \int e^t dt = e^t + c = e^{x^2} + c$

8. Evaluate $\int e^x \cos e^x\, dx$ ((L-2 - UNDERSTANDING))

Solution: - $\int e^x \cos e^x\, dx$ [Put $e^x = t => e^x dx = dt$]

$= \int \cos t\, dt = sint + c = \sin e^x + c$

9. Evaluate $\int \tan^2 x\, dx$ ((L-2 - UNDERSTANDING))

Solution: - $\int \tan^2 x\, dx = \int (\sec^2 x - 1) dx = \int \sec^2 x\, dx - \int dx = \tan x - x + c$

10. Define integration ((L-1 -REMEMBER))

Solution: - The process of finding anti-derivative of a function is called as integration.

11. Evaluate $\int \frac{\cos x}{1+sin\, x} dx$ (L-2 - UNDERSTANDING)

Solution:- $\int \frac{cosx}{1+sinx} dx = \int \frac{dt}{t}$ [Put $1 + sinx = t \Rightarrow cosx\, dx = dt$]

$= \log t + c = \log(1 + sinx) + c$

12. Evaluate $\int \frac{e^{2x}+1}{e^x} dx$ (L-2 - UNDERSTANDING)

Solution:- $\int \frac{e^{2x}+1}{e^x} dx = \int (\frac{e^{2x}}{e^x} + \frac{1}{e^x})dx = \int (e^x + e^{-x})dx = \int e^x dx + \int e^{-x} dx$

$= e^x - e^{-x} + c$

13. Evaluate $\int xsinx\ dx$ (L-2 - UNDERSTANDING)

Solution:- $\int xsinx\ dx = x \int sinx\ dx - \int \left(\frac{d}{dx} x \int sinx\ dx\right) dx$

$= x(-cosx) - \int 1(-cosx)dx = -xcosx + \int cosx\ dx = -xcosx + sinx + c$

14. Evaluate $\int sinx\ cosx\ dx$ (L-2 - UNDERSTANDING)

Solution:- $\int sinx\ cosx\ dx = \int \frac{2sinx\ cosx}{2} dx = \int \frac{\sin 2x}{2} dx$

$= \frac{1}{2} \int \sin 2x dx = \frac{1}{2}\left(\frac{-cos 2x}{2}\right) + c = -\frac{cos 2x}{4} + c$

15. Evaluate $\int cos^2 x\ dx$ (L-2 - UNDERSTANDING)

Solution:- $\int cos^2 x\ dx = \int \left(\frac{1+cos2x}{2}\right) dx = \frac{1}{2} \int (1 + cos2x)dx$

$= \frac{1}{2}\left[\int dx + \int cos2x\ dx\right] = \frac{1}{2}\left(x + \frac{sin2x}{2}\right) + C$

16. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ (L-2 - UNDERSTANDING)

Solution:- $\int_0^1 \frac{dx}{1+x^2} = \left[tan^{-1}x\right]_0^1 = \tan^{-1} 1 - \tan^{-1} 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

17. Evaluate $\int_{-1}^{1} |x| dx$ (L-2 - UNDERSTANDING)

Solution:- $= \int_{-1}^{1} |x| dx = \int_{-1}^{0} |x|\, dx + \int_{0}^{1} |x| dx = \int_{-1}^{0} -x\, dx + \int_{0}^{1} x\, dx$

$= \left[\frac{-x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1} = \left(0 + \frac{1}{2}\right) + \left(\frac{1}{2} - 0\right) = \frac{1}{2} + \frac{1}{2} = 1$.

B .05 MARKS QUESTIONS & SOLUTIONS :

1. Evaluate $\int e^{cos^2 x} \sin 2x\, dx$ ………………… **(L-3 APPLY)**

Soln: $\int e^{cos^2 x} \sin 2x\, dx$ Substituting $cos^2 x = t$

$= \int e^t(-dt)$ => 2cosx (-sinx) dx=dt

$= -\int e^t dt$ => -sin2x dx = dt

= - $e^t + c$

= - $e^{cos^2 x} + c$ (Ans)

2. Evaluate $\int \frac{dx}{1+e^{-x}}$ ……………… **(L-3 APPLY)**

Soln: $\int \frac{dx}{1+e^{-x}} = \int \frac{dx}{1+\frac{1}{e^x}} = \int \frac{dx}{\frac{e^x+1}{e^x}} = \int \frac{e^x dx}{e^x+1}$

$= \int \frac{e^x dx}{1+e^x}$ Substituting $1 + e^x = t$

$= \int \frac{dt}{t}$ => $e^x dx = dt$

$= logt + c$

$= \log(1 + e^x) + c$

(Ans)

3. Evaluate $\int \tan^{-1} x \, dx$ ....(L-3 APPLY)

Soln: $\int \tan^{-1} x \, dx$

$= \tan^{-1} x \int dx - \int \left(\frac{d}{dx}(\tan^{-1} x) \int dx\right) dx$

$= \tan^{-1} x \, . \, x - \int \frac{1}{1+x^2} \, x \, dx$

Substituting $1 + x^2 = t$

$=> 2xdx = dt$

$= x \tan^{-1} x - \int \frac{x}{1 + x^2} \, dx$

$= x \tan^{-1} x - \int \frac{1}{t} \frac{dt}{2}$

$= x \tan^{-1} x - \frac{1}{2} \int \frac{1}{t} \, dt$

$= x \tan^{-1} x - \frac{1}{2} \log t + c$

$= x \tan^{-1} x - \frac{1}{2} \log(1 + x^2) + c$ (Ans)

4. Evaluate $\int e^x sinx \, dx$ (L-3 APPLY)

Solution: - $I = \int e^x sinx \, dx = sinx \int e^x \, dx - \int \left\{\frac{d}{dx}(sinx) \int e^x dx\right\} dx$

$= sinx \, e^x - \int e^x cosx \, dx$

$$= sinx\, e^x - cosx \int e^x\, dx - \int \left\{\frac{d}{dx}(cosx) \int e^x dx\right\} dx$$

$$= sinx\, e^x - cosx\, e^x - \int sinx\, e^x dx$$

$$= sinx\, e^x - cosx\, e^x - I$$

$$=> 2I = e^x(sinx - cosx) => I = \frac{e^x}{2}(sinx - cosx)$$

5. Evaluate $\int e^x(sinx + \cos x)\ dx$ (L-3 APPLY)

Solution: - $\int e^x(sinx + \cos x)\ dx$

$$Take\ f(x) = sinx\ then\ f^{'}(x) = cosx$$

$$[\ \int e^x(\{ f(x) + f^{'}(x)\}dx = e^x f(x) + c\,]$$

$$= e^x sinx\, dx + c$$

6. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x + \sin x}} dx$ (L-3 APPLY)

$$Solution: - let\ I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}} dx \qquad [\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = I$$

$$=> 2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$= \int_0^{\frac{\pi}{2}} 1\, dx = x]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

$$=> I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx = \frac{\pi}{4}$$

7. Evaluate $\int \frac{3x+1}{(x+1)(x-2)} dx$ (L-3 APPLY)

Solution:-Let $\frac{3x+1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$--------------------(i)

$$\Rightarrow \frac{3x+1}{(x+1)(x-2)} = \frac{A(x-2)+B(x+1)}{(x+1)(x-2)}$$

$$\Rightarrow 3x + 1 = A(x-2) + B(x+1) \text{----------}(ii)$$

Putting $x + 1 = 0 \; in \; (ii)$, we get,

$$3(-1) + 1 = A(-1-2) + B(-1+1)$$

$$\Rightarrow -3 + 1 = A(-3) + B(0)$$

$$\Rightarrow -2 = -3A$$

$$\Rightarrow A = \frac{2}{3}$$

Putting $x - 2 = 0 \; in \; (ii)$, we get,

$$3(2) + 1 = A(2-2) + B(2+1)$$

$$\Rightarrow 6 + 1 = A(0) + B(3)$$

$$\Rightarrow 7 = 0 + 3B$$

$$\Rightarrow 7 = 3B$$

$\Rightarrow B = \frac{7}{3}$

Putting the values of A and B $\overset{in}{}$ (i), we get,

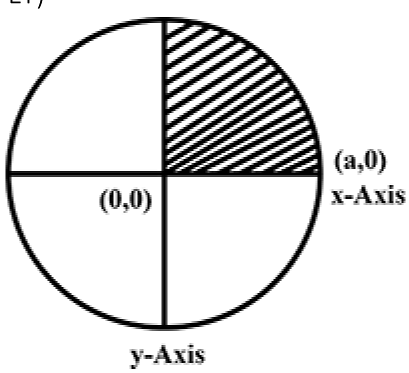
$$\frac{3x+1}{(x+1)(x-2)} = \frac{\frac{2}{3}}{(x+1)} + \frac{\frac{7}{3}}{(x-2)}$$

$$\therefore \int \frac{3x+1}{(x+1)(x-2)} dx = \int \left(\frac{\frac{2}{3}}{(x+1)} + \frac{\frac{7}{3}}{(x-2)} \right) dx$$

$$= \frac{2}{3} \int \frac{dx}{x+1} + \frac{7}{3} \int \frac{dx}{x-2}$$

$$= \frac{2}{3} \log(x+1) + \frac{7}{3} \log(x-3) + c$$

8. Find the area enclosed by the circle $x^2 + y^2 = a^2$. (L-3 APPLY)



Solution:- Given equation of the circle is $x^2 + y^2 = a^2$

$$\Rightarrow y = \pm \sqrt{a^2 - x^2}$$

$$\Rightarrow y = \sqrt{a^2 - x^2} \quad (\because y > 0 \text{ in first quadrant})$$

$\therefore$ *Area of circle* $= 4 \times$ *Area of first quadrant*

$$= 4\int_0^a y dx = 4\int_0^a \sqrt{a^2 - x^2}\, dx$$

$$= 4\int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 sin^2\theta}\; a\, cos\theta d\theta$$

(Put $x = sin\theta \Rightarrow dx = a\cos\theta d\theta$,

As $x = 0 \Rightarrow \theta = 0, x = a \Rightarrow \theta = \frac{\pi}{2}$)

$$= 4\int_0^{\frac{\pi}{2}} \sqrt{a^2(1 - sin^2\theta)}\; a\, cos\theta d\theta$$

$$= 4a\int_0^{\frac{\pi}{2}} \sqrt{a^2 cos^2\theta}\;\; cos\theta d\theta$$

$$= 4a^2\int_0^{\frac{\pi}{2}} cos^2\,\theta d\theta$$

$$= 4a^2\int_0^{\frac{\pi}{2}} \frac{(1+cos2\theta)}{2} d\theta$$

$$= 2a^2\int_0^{\frac{\pi}{2}} (1 + \; cos2\theta\,)\; d\theta$$

$$= 2a^2[\theta + \frac{sin2\theta}{2}]_0^{\frac{\pi}{2}}$$

$$= 2a^2(\frac{\pi}{2} + \frac{sin2\frac{\pi}{2}}{2} - 0 - \frac{sin0}{2})$$

$$= 2a^2\left(\frac{\pi}{2} + 0\; - 0\right)$$

$$= 2a^2\frac{\pi}{2} = \pi a^2$$

$\therefore$ *Area enclosed by the circle* $x^2 + y^2 = a^2$ *is* $\pi a^2 sq.\,unit.$

QUESTION BANK WITH SOLUTIONS IN MATHEMATICS-II

FOR 2ND SEMESTER DIPLOMA COURSES FROM SUMMER 2025 ONWARDS

UNIT-III

A. 02 marks Questions & Solutions

Q1. Prove that the lines 3x+4y+7=0 and 4x-3y+1=0 are perpendicular to each other. (L-2 Understanding)

Ans: Let L_1 : 3x+4y+7=0 and L_2 : 4x-3y+1=0

Slope $m_1 = -\frac{a}{b} = -\frac{3}{4}$ and $m_2 = -\frac{a}{b} = -\frac{4}{-3} = \frac{4}{3}$

$$m_1 \times m_2 = -\frac{3}{4} \times \frac{4}{3} = -1$$

$\Rightarrow L_1 \perp L_2$.

Q2. What is the slope of a line making an inclination of 45^0 with positive direction of X-axis ?(L-1 Remembering)

Ans: Given $\theta = 45°$

Slope = $\tan\theta = \tan 45° = 1$

Q3. Find the equation of circle with centre at (0,0) and radius 4.

(L-1 Remembering)

Ans: By Centre and radius formula, $(x-0)^2 + (y-0)^2 = 4^2$

$$\Rightarrow x^2 + y^2 = 16$$

Q4. Find the centre and radius of the circle, $\boldsymbol{2x^2 + 2y^2 + 4x + 8y + 2 = 0}$.

(L-2 Understanding)

Ans: The equation of the circle can be written as, $x^2 + y^2 + 2x + 4y + 1 = 0$

Equating with general formula of circle $x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 2 \Rightarrow g = 1$

$2f = 4y \Rightarrow f = 2$

c= 1

Centre= (-g, -f) = (-1, -2)

Radius= $\sqrt{g^2 + f^2 - c} = \sqrt{1^2 + 2^2 - 1}$ = 2.

Q5. Find the area of triangle whose vertices are given by (1, 1), (3, 2), (3, 3).

(L-2 Understanding)

Ans: Area= $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[1(2-3) + 3(3-1) + 3(1-2)]$

$= \frac{1}{2}[1(-1) + 3(2) + 3(-1)] = \frac{2}{2} = 1$ sq unit

Q6.Find the perpendicular distance from the point $(2, 1)$ to the straight line $12x - 5y + 9 = 0$.(L-1 Remembering)

Solution: We know that the length of the perpendicular from the point (x_1, y_1) on $ax + by + c = 0$ is $\left|\frac{ax_1+by_1=c}{\sqrt{a^2+b^2}}\right|$

So, the length of the perpendicular from the point $(2,1)$ on $12x - 5y + 9 = 0$ is

$$= \left|\frac{12 \times 2 - 5 \times 1 + 9}{\sqrt{12^2 + (-5)^2}}\right| = \left|\frac{28}{13}\right| = \frac{28}{13}.$$

Q7. Find the value of k if the lines $2x - 3y + 7 = 0$ and $3x + ky + 2 = 0$ are perpendicular each other.**(L-2 Understanding)**

Solution:The lines $2x - 3y + 7 = 0$ and $3x + ky + 2 = 0$

$$m_1 = \frac{-A_1}{B_1} = \frac{2}{3}$$

and $m_2 = \frac{-A_2}{B_2} = \frac{-3}{k}$.

Since the lines are perpendicular to each other

$\Rightarrow m_1.m_2 = -1$

$\Rightarrow \frac{2}{3}.\frac{-3}{k} = -1$

$\Rightarrow k = 2$. (Ans.)

Q8. Find the length of latus rectum of the parabola $y^2 = 4ax$ if it passes through the point $(3,2)$.(L-2 Understanding)

Solution: Given that the equation of parabola is $y^2 = 4ax$.

It passes through the point $(3,2)$.

So, $4 = 12a \Rightarrow a = \frac{1}{3}$

Hence, length of the latus rectum $= 4a = 4 \times \frac{1}{3} = \frac{4}{3}$**(Ans.)**

Q9.Find the equation of the parabola whose vertex is at $(0,0)$ and which passes through the point $(2,3)$ with the axis $y = 0$.(L-2 Understanding)

Solution: Here the equation of parabola be $y^2 = 4ax$

Since $(2,3)$ lies on it, $\Rightarrow 9 = 4a.2$

$\Rightarrow a = \frac{9}{8}$

So, the required equation is $y^2 = 4.\frac{9}{8}x$

$\Rightarrow y^2 = \frac{9}{2}x$

$\Rightarrow 2y^2 = 9x$

$\Rightarrow 2y^2 - 9x = 0$ **(Ans.)**

Q10. Find the equation of the circle whose end points of the diameter are $(1,0)$ and $(0,1)$.(L-1 Remembering)

Solution: Equation of the circle whose end points of the diameter are $(1,0)$ and $(0,1)$ is given by

$(x-1)(x-0)+(y-0)(y-1)=0$

$\Rightarrow x^2-x+y^2-y=0$

$\Rightarrow x^2+y^2-x-y=0$ (Ans.)

Q11.Find the length of the major axis and coordinates of the vertices of the ellipse $16x^2+25y^2=400$.(L-2 Understanding)

Solution: We have $16x^2+25y^2=400$

$\Rightarrow \frac{x^2}{25}+\frac{y^2}{16}=1$

Where $a^2=25$ and $b^2=16$ *i.e.* $a=5, b=4$

Clearly, $a>b$, so X-axis is the major axis.

Length of the major axis $=2a=2\times 5=10$

The coordinates of vertices are $(5,0)$ and $(-5,0)$. (Ans.)

Q12.Find the equation of ellipse whose vertices are $(\pm 10,0)$ and foci are $(\pm 4,0)$. (L-2 Understanding)

Solution: Since the vertices are on the x-axis, the equation of ellipse will be

of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is semi major axis.

It is given $a=10, c=4$

Using $c^2=a^2-b^2 \Rightarrow b=\sqrt{100-16}=\sqrt{84}$

Equation of ellipse is $\frac{x^2}{100}+\frac{y^2}{84}=1$ (Ans.)

Q13. Find the equation of the line passing through $(-1, 2)$ and making intercept 2 on the Y-axis.(L-2 Understanding)

Solution: Let the equation of line be $y = mx + c$

Here, y-intercept $c = 2$.

So, $y = mx + 2$

Again, it passes through $(-1,2)$, its equation is given by,

$2 = -m + 2 \qquad \Rightarrow m = 0$

So, equation of straight line is $y = 0 + 2$

$\Rightarrow y = 2$. (Ans.)

Q14. Determine the equation of straight line parallel to x-axis and pasing through $(3, 4)$.(L-2 Understanding)

Solution: The equation of straight line parallel to x-axis is given by $y = k$ ------(1)

Since the line passes through the point $(3,4)$

∴The required equation is $y = 4$. (Ans.)

Q15. Reduce the equation $4x + 7y = 12$ to the intercept form.

(L-2 Understanding)

Solution: Given equation is $4x + 7y = 12$

$\Rightarrow \frac{4x}{12} + \frac{7y}{12} = 1$

$\Rightarrow \frac{x}{3} + \frac{y}{\frac{12}{7}} = 1$ (Ans.)

Q16. Find the angle between the lines $x + y + 7 = 0$ and $x - y + 1 = 0$.

(L-2 Understanding)

Solution: Angle between two intersecting lines is given by $\theta = tan^{-1}\left|\frac{a_2b_1-a_1b_2}{a_1a_2+b_1b_2}\right|$

Here $a_1 = 1, a_2 = 1, b_1 = 1 \ and \ b_2 = -1 \ \ \Rightarrow \ a_1a_2 + b_1b_2 = 1 - 1 = 0$

So, the angle between the two lines is 90°. (Ans.)

Q17.Find the equation of straight through the points $(-1, 1)$ and $(2, 1)$.

(L-1 Remembering)

Solution: The equation of straight line through the points $(-1,1)$ and $(2,1)$ is

$y - 1 = \frac{1-1}{2+1}(x+1)$ {Use Two point formula y-$y - y_1 = (\frac{y_2-y_1}{x_2-x_1})(x - x_1)$}

$\Rightarrow y - 1 = 0 \quad \Rightarrow y = 1$ (Ans.)

Q18. Find the point of intersection of the lines $2x - 3y - 7 = 0$ and $3x - 4y - 13 = 0$. (L-1 Remembering)

Solution: Let $P(x, y)$ be the point of intersection of $2x - 3y - 7 = 0$

and $3x - 4y - 13 = 0$.

Then , $P(x,y) = \left(\frac{-3\times(-13)-(-4)\times(-7)}{2\times(-4)-3\times(-3)}, \frac{-7\times3-(-13)\times2}{2\times(-4)-3\times(-3)}\right)$

$= \left(\frac{39-28}{-8+9}, \frac{-21+26}{-8+9}\right) = (11,5)$(Ans.)

Q19. Find the centre and radius of the circle $x^2 + y^2 + 6x - 4y = 12$.

(L-2 Understanding)

Solution: Given equation of circle is $x^2 + y^2 + 6x - 4y - 12 = 0$.

Here, $2g = 6 \quad \Rightarrow g = 3$

$2f = -4$ .So $f = -2$

And $c = -12$

Hence, center$= (-g, -f) = (-3, 2)$.

Radius$\sqrt{g^2 + f^2 - c} = \sqrt{9 + 4 + 12} = \sqrt{25} = 5$ unit. (Ans.)

Q20. Find the equation of the circle which touches the X-axis and whose centre is at $(3, 4)$.(L-3 Applying)

Solution: Since the centre of the circleis $(3,4)$ and it touches the X-axis

So, radius of the circle is r$= 4$.

∴Equation of circle is $(x-3)^2 + (y-4)^2 = 16$

$\Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y - 16 = 0$

$\Rightarrow x^2 + y^2 - 6x - 8y + 9 = 0$.(Ans.)

B. 5 Marks Questions with Solutions

Q1. Find the equation of straight line passing through point (-4, 2) and parallel to the line 4x-3y-10=0.(L-3 Applying)

Ans: Let L_1 : 4x-3y -10=0

slope $m_1 = -\frac{a}{b} = -\frac{4}{-3} = \frac{4}{3}$

Since two lines are parallel to each other, $m_1 = m_2 = \frac{4}{3}$

Line passes through (-4, 2)

So equation of required line is $y - y_1 = m_2(x - x_1)$

$$\Rightarrow y - 2 = \frac{4}{3}(x + 4)$$

$$\Rightarrow 4x - 3y + 22 = 0$$

Q2. If latus rectum of an ellipse is half of its minor axis, calculate its eccentricity 'e' ? (L-3 Applying)

Ans: Let the equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Minor axis= 2b and latus rectum = $\frac{2b^2}{a}$

Given latus rectum = half of Minor axis

$$\Rightarrow \frac{2b^2}{a} = \frac{2b}{2} \quad \Rightarrow \frac{b}{a} = \frac{1}{2} \quad \Rightarrow \frac{b^2}{a^2} = \frac{1}{4}$$

Eccentricity, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$.

Q3. Find the equation of line passing through the intersection of lines $2x - y - 1 = 0$ and $3x - 4y + 6 = 0$ and parallel to the line $x + y - 2 = 0$.

(L-3 Applying)

Solution: Point of intersection of $2x - y - 1 = 0$ and $3x - 4y + 6 = 0$ is

$$\left(\frac{-1 \times 6 - (-4)(-1)}{2(-4) - 3(-1)}, \frac{(-1) \times 3 - 6(2)}{2(-4) - 3(-1)}\right) = \left(\frac{-6-4}{-8+3}, \frac{-3-12}{-8+3}\right)$$

$$= \left(\frac{-10}{-5}, \frac{-15}{-5}\right) = (2,3)$$

Any line parallel to the line $x + y - 2 = 0$ is given by $x + y + k = 0$------------(1)

Since the line passes through $(2,3)$ hence it satisfies the equation (1)

So, $2 + 3 + k = 0$

$\Rightarrow k = -5$

Now, putting the value of k in equation (1) , we get $x + y - 5 = 0$.

∴ Required equation of line is $x + y - 5 = 0$. (Ans.)

Q4. Find the equation of the circle which passes through the points $(0, 0)$, $(3, 0)$ and $(0, 4)$.(L-3 Applying)

Solution: Let the required equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$------------------(1)

Since the circle passes through $(0,0)$

$\Rightarrow 0 + 0 + 0 + 0 + c = 0$

$\Rightarrow c = 0$-----------------(2)

Also, the circle passes through $(3,0)$

$\Rightarrow 9 + 0 + 6g + c = 0$ (put $c = 0$)

$\Rightarrow 6g = -9$

$\Rightarrow g = \frac{-9}{6} = \frac{-3}{2}$--------------(3)

Again the circle passes through $(0,4)$

$\Rightarrow 0+16+0+8f+0=0$

$\Rightarrow 8f=-16$

$\Rightarrow f=-2$----------(4)

Substituting the values of c, g, f in (1) we get

$x^2+y^2+2\left(\frac{-3}{2}\right)x+2(-2)y+0=0$

$x^2+y^2-3x-4y=0$ (Ans.)

Q5.Find the distance of the point $(3,2)$ from the line $x+3y-1=0$, measured parallel to the line $3x-4y+1=0$.

Solution: Any line passing through the point $(3,2)$ is given by

$y-2=m(x-3)$------------(1)

Slope of the line $3x-4y+1=0$--------(2) is $\frac{3}{4}$

Since equation (1) is parallel to the line (2)

$\therefore m=\frac{3}{4}$

Now putting the value of m in equation (1) we get,

$y-2=\frac{3}{4}(x-3)$

$\Rightarrow 4y-8=3x-9$

$\Rightarrow 3x-4y-1=0$ ---------(3)

Let Q be the point of intersection of the line (3) and $x+3y-1=4$

So, coordinates of Q is $\left(\frac{4+3}{9+4}, \frac{-1+3}{9+4}\right) = \left(\frac{7}{13}, \frac{2}{13}\right)$

$$\therefore PQ = \sqrt{\left(3-\frac{7}{13}\right)^2 + \left(2-\frac{2}{13}\right)^2}$$

$$= \sqrt{\left(\frac{39-7}{13}\right)^2 + \left(\frac{26-2}{13}\right)^2}$$

$$= \sqrt{\left(\frac{32}{13}\right)^2 + \left(\frac{24}{13}\right)^2}$$

$= \sqrt{\frac{1024}{169} + \frac{576}{169}} = \sqrt{\frac{1600}{169}} = \frac{40}{13}$. (Ans.)

Q6.Find the equation of straight line passing through (-2,3) and sum of whose intercept is 2. (L-3 Applying)

Solution: Let the equation of the line in intercept form be ,

$\frac{x}{a} + \frac{y}{b} = 1$ where a and b are x-intercept and y intercept respectively.

Given that a+b =2 $\Rightarrow$ b = 2-a.

Putting this in above equation we have,

$$\frac{x}{a} + \frac{y}{2-a} = 1$$

Line passes through (-2,3) ,

$$\Rightarrow \frac{-2}{a} + \frac{3}{2-a} = 1$$

$$\Rightarrow \frac{-2(2-a)+3a}{a(2-a)} = 1$$

$\Rightarrow$ -4+5a=2a-a^2 $\Rightarrow$ a^2+3a-4=0

$\Rightarrow$ (a+4)(a-1)=0

$\Rightarrow$a=-4 or a=1

If a=-4 then b= 6, hence equation of the line is,

$\frac{x}{-4}+\frac{y}{6}=1 \Rightarrow \frac{3x-2y}{-12}=1$

$\Rightarrow$ 3x-2y+12=0

If a=1 then b= 1 and hence equation of the line is,

x+y -1=0. (Ans)

Q7.Find the equation of the line which passes through the point $(1,2)$ and perpendicular to the line $4x+3y+5=0$.(L-2 Understanding)

Solution: Equation of the line perpendicular to the line $4x+3y+5=0$ is given by $-3x+4y+d=0$-----------(1)

Since equation (1) passes through $(1,2)$,

$\Rightarrow -3\times 1+4\times 2+d=0$

$\Rightarrow d=3-8=-5$

So, the required equation of line is $-3x+4y-5=0$

$\Rightarrow 3x-4y+5=0$. (Ans.)

Q8.Find the coordinates of foci and vertices of the hyperbola $16x^2-9y^2=144$.(L-2 Understanding)

Solution: We have $16x^2-9y^2=144$.

$\Rightarrow \frac{x^2}{9}-\frac{y^2}{16}=1$

Where $a^2 = 9, b^2 = 16$

i.e. $a = 3, \quad b = 4$

The eccentricity $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

The coordinates of foci are $(ae.0) and (-ae,0) i.e. (5,0)$ and $(-5,0)$.

The coordinates of vertices are $(3,0)$ and $(-3,0)$. (Ans.)

Q9.Find the equation of line passing through the intersection of lines x+3y+2=0 and $x - 2y - 4 = 0$ and perpendicular to the line $2y + 5x - 9 = 0$.

(L-3 Applying)

Solution: Point of intersection of $2x - y - 1 = 0$ and $3x - 4y + 6 = 0$ is

$$\left(\frac{3 \times (-4) - (-2)2}{1(-2) - 1 \times 3}, \frac{2 \times 1 - (-4)1)}{1(-2) - 1 \times 3)}\right) = \left(\frac{-12+4}{-2-3}, \frac{2+4}{-2-3}\right)$$

$$= \left(\frac{-8}{-5}, \frac{6}{-5}\right) = \left(\frac{8}{5}, -\frac{6}{5}\right)$$

Any line perpendicular to the line $2y + 5x - 9 = 0$ is given by $2x - 5y + k = 0$-------(1)

Since the line passes through $\left(\frac{8}{5}, -\frac{6}{5}\right)$, hence it satisfies the equation (1)

So, $2 \times \frac{8}{5} - 5 \times \frac{-6}{5} + k = 0$

$\Rightarrow 16 + 30 + 5k = 0 \;\Rightarrow k = \frac{-46}{5}$

Now, putting the value of k in equation (1) , we get $2x - 5y - \frac{46}{5} = 0$.

$\therefore$ Required equation of line is $10x - 25y - 46 = 0$. (Ans.)

Q10.Find the equation of the ellipse with its Centre at origin , axes along the coordinate axes and which passes through the points $(2, 2)$ and $(3, 1)$.

(L-3 Applying)

Solution: Let the equation of ellipse be $Ax^2 + By^2 = 1$

This passes through the points $(2,2)$ and $(3,1)$.

Hence, $4A + 4B = 1$

And $9A + B = 1$

Solving we get $A = \frac{3}{32}, B = \frac{5}{32}$

Hence the equation of ellipse is $\frac{3}{32}x^2 + \frac{5}{32}y^2 = 1$

$\Rightarrow 3x^2 + 5y^2 = 32$. (Ans.)

QUESTION BANK WITH SOLUTIONS IN MATHEMATICS-II

FOR 2ND SEMESTER DIPLOMA COURSES FROM SUMMER 2025 ONWARDS

UNIT-IV

Sl No	A. 02 marks Questions & Solutions	BT Level
1	**Define Unit Vector and Null Vector**	L-1, Remembering
Ans.	Unit Vector: A vector is said to be unit vector if its magnitude is 1 or unit. It is denoted by $\hat{a}$ Null Vector: A vector is said to be null vector if its magnitude is zero or null and has infinitely many directions. It is denoted as $\vec{0}$	
2	**Find the unit vector of the vector $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$.**	L-2, Understanding
Ans.	**W.k.t.** If $\vec{a}$ is any vector, then its unit vector is given by $\hat{a} = \pm\frac{\vec{a}}{\lvert\vec{a}\rvert}$. $\therefore \lvert\vec{a}\rvert = \sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{49} = 7$ So $\hat{a} = \pm\left(\frac{2}{7}\hat{\imath} - \frac{3}{7}\hat{\jmath} + \frac{6}{7}\hat{k}\right)$	
3	**For what value of 'α' the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = \alpha\hat{\imath} + \hat{\jmath} + 5\hat{k}$ are perpendicular to each other.**	L-2, Understanding
Ans.	**ATQ** $\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \bullet \vec{b} = 0$ $\Rightarrow 1.\alpha + 2.1 + (-1).5 = 0 \Rightarrow \alpha = 3$	
4	**For what values of '$m\&n$' the vectors $\vec{a} = m\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 4\hat{\imath} + 2\hat{\jmath} + n\hat{k}$ are parallel to each other.**	L-2, Understanding
Ans.	**W.K.T.** $\vec{a} \parallel \vec{b} \Leftrightarrow \frac{a_x}{b_x} = \frac{a_y}{b_y} = \frac{a_z}{b_z}$ $\Rightarrow \frac{m}{4} = \frac{4}{2} = \frac{-3}{n} \Rightarrow m = 8\&n = -\frac{3}{2}$	
5	**If $\vec{p} = \hat{\imath} + 2\hat{\jmath}, \vec{q} = \hat{\jmath} - 3\hat{k}$ and $\vec{r} = \hat{k} - 4\hat{\imath}$ then find $2(\vec{p} + 3\vec{q}) - 5\vec{r}$**	L-2, Understanding
Ans.	$2(\vec{p} + 3\vec{q}) - 5\vec{r}$ $= 2\vec{p} + 6\vec{q} - 5\vec{r}$ $= 2\hat{\imath} + 4\hat{\jmath} + 6\hat{\jmath} - 18\hat{k} - 5\hat{k} + 20\hat{\imath}$ $= 22\hat{\imath} + 10\hat{\jmath} - 23\hat{k}$	
6	**If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, then show that $(\vec{a} + \vec{b})^2 = (\vec{a} - \vec{b})^2$**	L-3, Applying
Ans.	**W.K.T.** $\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \bullet \vec{b} = 0 \Rightarrow \pm 2\vec{a} \bullet \vec{b} = 0$ LHS: $(\vec{a} + \vec{b})^2 = a^2 + 2\vec{a} \bullet \vec{b} + b^2 = a^2 - 2\vec{a} \bullet \vec{b} + b^2 = (\vec{a} - \vec{b})^2$ =RHS. Proved	

7	**Find the work done by the force $\hat{\imath} - 3\hat{k}$ on a particle to displace it from (1,2,0) to (0,2,3).**	**L-2, Understanding**
Ans.	Given force $\vec{F} = \hat{\imath} - 3\hat{k}$ Displacement $\vec{S} = (0,2,3) - (1,2,0) = (-1,0,3) = -\hat{\imath} + 3\hat{k}$ ∴Work done=$W = \vec{F} \bullet \vec{S} = 1(-1) + (-3).3 = -10$unit	
8	**The position vectors of A,B and C are $2\hat{\imath} + \hat{\jmath} - \hat{k}$, $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$and $\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ respectively . Show that A, B and C are collinear.**	**L-3, Applying**
Ans.	$\overrightarrow{AB} = PV of B - PV of A = (3-2)\hat{\imath} + (-2-1)\hat{\jmath} + (1-(-1))\hat{k} = \hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ Similarly $\overrightarrow{BC} = -2\hat{\imath} + 6\hat{\jmath} - 4\hat{k} = -2\left(\overrightarrow{AB}\right)$ As $\overrightarrow{AB}$ II $\overrightarrow{BC}$and B is common, therefore A,B, C are collinear	
9	**Show that the points (3,4) ,(1,7) and (-5,16) are collinear.**	**L-3, Applying**
Ans.	$\overrightarrow{AB}$=(1,7)-(3,4)=(-2,3) $\overrightarrow{BC}$=(-5,16)-(1,7)=(-6,9)=3 $\overrightarrow{AB}$ As $\overrightarrow{AB}$ II $\overrightarrow{BC}$and B is common, therefore A,B, C are collinear	
10	**If $\vec{F} = 2\hat{\imath}, \vec{r} = 3\hat{\jmath}$ then find momentum of force.**	**L-2, Understanding**
Ans.	Momentum of force= $\vec{M} = \vec{r} \times \vec{F} = 3\hat{\jmath} \times 2\hat{\imath} = 6(-\hat{k}) = -6\hat{k}$	
11	**If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ then calculate $\vec{a} \times \vec{b}$**	**L-2, Understanding**
Ans.	$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & -1 \\ 2 & -2 & 4 \end{vmatrix} = (12-2)\hat{\imath} - (8+2)\hat{\jmath} + (-4-6)\hat{k} = 10\hat{\imath} - 10\hat{\jmath} - 10\hat{k}$	
12	**What inference can you draw when $\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a} \bullet \vec{b} = 0$?**	**L-2, Understanding**
Ans.	Given $\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a} \bullet \vec{b} = 0$ ⇒ ($\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ or $\vec{a}$II$\vec{b}$) & ($\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ or $\vec{a} \perp \vec{b}$) $\Rightarrow \vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$	
13	**Determine the area of a parallelogram whose adjacent sides are the vectors $\vec{a} = 2\hat{\imath}$ and $\vec{b} = 3\hat{\jmath}$.**	**L-2, Understanding**
Ans.	Area of the parallelogram with adjacent sides given by $\vec{a}$ and $\vec{b}$ is given by area = $\lvert\vec{a} \times \vec{b}\rvert$=$\lvert 2\hat{\imath} \times 3\hat{\jmath}\rvert$= $\lvert 6\hat{k}\rvert$= 6 sq unit.	
14	**Find a vector which is perpendicular to both the vectors $\vec{a} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$and $\vec{b} = -\hat{\imath} + 3\hat{k}$**	**L-2, Understanding**
Ans.	A vector which is perpendicular to both $\vec{a}$and $\vec{b}$is given by $\vec{a} \times \vec{b}$ $\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix} = (9-0)\hat{\imath} - (3-2)\hat{\jmath} + (0+3)\hat{k} = 9\hat{\imath} - \hat{\jmath} + 3\hat{k}$	

15	**Prove that the sum of the vectors represented by the sides of a closed pentagon taken in order is a zero vector.**	**L-3, Applying**
Ans.	Let ABCDE is a pentagon. **A.T.Q** we have to show $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EA} = \vec{0}$ LHS: $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EA} = \vec{AC} + \vec{CD} + \vec{DE} + \vec{EA} = \vec{AD} + \vec{DE} + \vec{EA} = \vec{AE} + \vec{EA} = \vec{0}$	
16	**Find $\lvert\vec{a} - \vec{b}\rvert$, if two vectors $\vec{a}$& $\vec{b}$are such that $\lvert\vec{a}\rvert = 2, \lvert\vec{b}\rvert = 3 \& \vec{a} \bullet \vec{b} = 4$**	**L-2, Understanding**
Ans.	$\therefore \lvert\vec{a} - \vec{b}\rvert^2 = \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 - 2\vec{a} \bullet \vec{b} = 2^2 + 3^2 - 2 \cdot 4 = 5 \Rightarrow \lvert\vec{a} - \vec{b}\rvert = \sqrt{5}$	
17	**Define Collinear Vector?**	**L-1, Remembering**
Ans.	vectors which lie along the same line or same parallel support are known as collinear vectors.	
18	**find the magnitude of $\vec{u} = 2\hat{i} + 3\hat{j}$**	**L-2, Understanding**
Ans.	**Magnitude=** $\sqrt{x^2 + y^2} => \lvert\vec{u}\rvert = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$	
19	**Find the angle between the vectors $5\hat{i} + 3\hat{j} + 4\hat{k}$ and $6\hat{i}-8\hat{j}-\hat{k}$**	**L-2, Understanding**
	Let $\vec{a}= 5\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b}= 6\hat{i}-8\hat{j}-\hat{k}$ Let θ be the angle between $\vec{a}$ and $\vec{b}$. Then $\theta = cos^{-1}(\frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1{}^2+a_2{}^2+a_3{}^2}\sqrt{b_1{}^2+b_2{}^2+b_3{}^2}})$ $= cos^{-1}(\frac{5.6+3.(-8)+4.(-1)}{\sqrt{5^2+3^2+4^2}\sqrt{6^2+(-8)^2+(-1)^2}}) = cos^{-1}(\frac{30-24-4}{\sqrt{50}\sqrt{101}}) = cos^{-1}(\frac{2}{\sqrt{50}\sqrt{101}})$	

B. 5 MARKS QUESTIONS WITH SOLUTIONS

Sl No	Questions	BT Level
1	**Find the scalar & vector projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.**	L-2, Understanding
Ans.	Given $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ Scalar Projection of $\vec{a}$ on $\vec{b}$=$\frac{\vec{a}\bullet\vec{b}}{\lvert\vec{b}\rvert} = \frac{2.1+3.2+2.1}{\sqrt{1^2+2^2+1^2}} = \frac{10}{\sqrt{6}} = \frac{5\sqrt{6}}{3}$ Vector Projection of $\vec{a}$ on $\vec{b}$ =$\frac{\vec{a}\bullet\vec{b}}{\lvert\vec{b}\rvert}(\hat{b}) = \frac{\vec{a}\bullet\vec{b}}{\lvert\vec{b}\rvert}\left(\frac{\vec{b}}{\lvert\vec{b}\rvert}\right) = \frac{10}{\sqrt{6}}\left(\frac{\hat{\imath}+2\hat{\jmath}+\hat{k}}{\sqrt{6}}\right) = \frac{5}{3}\left(\hat{\imath} + 2\hat{\jmath} + \hat{k}\right) = \frac{5}{3}\hat{\imath} + \frac{10}{3}\hat{\jmath} + \frac{5}{3}\hat{k}$	
2	**Show that the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ & $3\hat{i} - 4\hat{j} - 4\hat{k}$ form a right angle triangle.**	L-3, Applying
Ans.	Let position vectors of the vertices are given by A=$2\hat{\imath} - \hat{\jmath} + \hat{k}$ B=$\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and C=$3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ $\therefore \overrightarrow{AB}$ =PV of B- PV of A=$-\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$ Similarly $\overrightarrow{BC} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\overrightarrow{AC} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ To prove ABC form a triangle it must obey triangle rule. So it is clear that $\overrightarrow{BC} + \overrightarrow{AB} = \overrightarrow{AC}$ Let $\overrightarrow{BC} \bullet \overrightarrow{AC} = 2.1 + (-1)(-3) + 1.(-5) = 2 + 3 - 5 = 0$ $\Rightarrow \overrightarrow{BC} \perp \overrightarrow{AC}$ Hence $ABC\Delta$ is right angle & C is right angle	
3	**Prove that** **i. $\vec{a} \bullet (\vec{b} - \vec{c}) + \vec{b} \bullet (\vec{c} - \vec{a}) + \vec{c} \bullet (\vec{a} - \vec{b}) = 0$** **ii. $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$**	L-3, Applying
Ans.	**i.** LHS $\vec{a} \bullet (\vec{b} - \vec{c}) + \vec{b} \bullet (\vec{c} - \vec{a}) + \vec{c} \bullet (\vec{a} - \vec{b})$ $= \vec{a} \bullet \vec{b} - \vec{a} \bullet \vec{c} + \vec{b} \bullet \vec{c} - \vec{b} \bullet \vec{a} + \vec{c} \bullet \vec{a} - \vec{c} \bullet \vec{b}$ $= 0(\because \vec{x} \bullet \vec{y} = \vec{y} \bullet \vec{x})$ =RHS (Proved) **ii.** RHS $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$ $= \vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times c + \vec{b} \times \vec{a} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$ $= \vec{a} \times \vec{b} - \vec{c} \times \vec{a} + \vec{b} \times c - \vec{a} \times \vec{b} + \vec{c} \times \vec{a} - \vec{b} \times \vec{c}(\because \vec{x} \times \vec{y} = -\vec{y} \times \vec{x})$ $= \vec{0}$	

4	**Show that** $\left(\vec{a} \times \vec{b}\right)^2 = a^2b^2 - \left(\vec{a} \bullet \vec{b}\right)^2$	**L-3, Applying**
Ans.	LHS $\left(\vec{a} \times \vec{b}\right)^2$ $= \left(\vec{a} \times \vec{b}\right) \bullet \left(\vec{a} \times \vec{b}\right)$ $= (ab\, sin\, \theta\, \hat{n}) \bullet (ab\, sin\, \theta\, \hat{n})$ (let θ be the angle between $\vec{a}$&$\vec{b}$) $= a^2b^2\, sin^2\, \theta\, (\hat{n} \bullet \hat{n})$ $= a^2b^2(1 - cos^2\, \theta)(\because \hat{n} \bullet \hat{n} = 1)$ $= a^2b^2 - (ab\, cos\, \theta)^2$ $= a^2b^2 - \left(\vec{a} \bullet \vec{b}\right)^2$ =RHS (Proved)	
5	**Obtain the area of the parallelogram whose diagonals are given by the vectors** $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ & $\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$	**L-2, Understanding**
Ans.	Let $\vec{d}_1 = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ $\vec{d}_2 = \hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ are two diagonals of a parallelogram Area of the parallelogram whose diagonals are $\vec{d}_1$ & $\vec{d}_2 = \frac{1}{2}\left\lvert \vec{d}_1 \times \vec{d}_2 \right\rvert$ Now, $\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = (4+6)\hat{\imath} - (12+2)\hat{\jmath} + (9-1)\hat{k} = 10\hat{\imath} - 14\hat{\jmath} + 8\hat{k}$ $\Rightarrow \frac{1}{2}\left\lvert \vec{d}_1 \times \vec{d}_2 \right\rvert = \frac{1}{2}\sqrt{100 + 196 + 64} = \frac{\sqrt{360}}{2} = \frac{6\sqrt{10}}{2} = 3\sqrt{10}$ sq. unit	
6	**Find the momentum about (1,0,1) of the force** $2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ **acting at (2,1,-1)**	**L-2, Understanding**
Ans.	Let $O = (1,0,1)$, $A = (2,1,-1)$ and $\vec{F} = 2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ We know moment of force about O is given by $\overrightarrow{OA} \times \vec{F}$ $\therefore \overrightarrow{OA} = (2,1,-1) - (1,0,1) = (1,1,-2) = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ $\overrightarrow{OA} \times \vec{F} = \begin{vmatrix} i & j & k \\ 1 & 1 & -2 \\ 2 & 3 & 5 \end{vmatrix} = (5+6)\hat{\imath} - (5+4)\hat{\jmath} + (3-2)\hat{k} = 11\hat{\imath} - 9\hat{\jmath} + \hat{k}$	
7	**Let** $\vec{a}, \vec{b}$&$\vec{c}$ **be three vectors such that** $\lvert\vec{a}\rvert = 3, \lvert\vec{b}\rvert = 4, \lvert\vec{c}\rvert = 5$ **and each one of them being perpendicular to the sum of two other vector, then find** $\left\lvert \vec{a} + \vec{b} + \vec{c} \right\rvert$	**L-2, Understanding**
Ans.	ATQ $\vec{a} \bullet \left(\vec{b} + \vec{c}\right) = 0, \vec{b} \bullet (\vec{c} + \vec{a}) = 0, \vec{c} \bullet \left(\vec{a} + \vec{b}\right) = 0$ Now $\left\lvert \vec{a} + \vec{b} + \vec{c} \right\rvert^2 = \left(\vec{a} + \vec{b} + \vec{c}\right)^2 = \left(\vec{a} + \vec{b} + \vec{c}\right) \bullet \left(\vec{a} + \vec{b} + \vec{c}\right)$ $= \vec{a} \bullet \vec{a} + \vec{a} \bullet \left(\vec{b} + \vec{c}\right) + \vec{b} \bullet \vec{b} + \vec{b} \bullet (\vec{c} + \vec{a}) + \vec{c} \bullet \vec{c} + \vec{c} \bullet \left(\vec{a} + \vec{b}\right)$ $= \lvert\vec{a}\rvert^2 + \left\lvert\vec{b}\right\rvert^2 + \lvert\vec{c}\rvert^2 = 9 + 16 + 25 = 50$ $\Rightarrow \left\lvert \vec{a} + \vec{b} + \vec{c} \right\rvert = \sqrt{50} = 5\sqrt{2}$	

8	**Find area of a triangle whose two sides are represented by the vectors $\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} + \hat{\jmath} + 2\hat{k}$**	**L-2, Understanding**
Ans.	We know that the area of a triangle is the magnitude of half of cross product of two vectors represented the sides of triangle. let $\vec{u} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ $\vec{v} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ are the two sides of a triangle So, vector area of the triangle is $\vec{u} X \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -3 & 5 \\ 1 & 1 & 2 \end{vmatrix}$ $= (\hat{\imath}(-3*2 - 5*1) - \hat{\jmath}(1*2 - 1*5) + \hat{k}(1*1 - 1*-3))$ $= (-11\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ Now , required area is $\lvert\vec{u} X \vec{v}\rvert = \sqrt{(-11)^2 + 3^2 + 4^2} = \sqrt{121 + 9 + 16} = \sqrt{146}$ sq unit.	
9	1. **Calculate the area of a triangle ABC by vector method, where A(1,1,2), B(2,2,3) and C(3,-1,-1).**	**L-2, Understanding**
Ans.	Let the position vector of the vertices A,B and C is given by $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ $\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ $\vec{c} = 3\hat{\imath} - \hat{\jmath} - \hat{k}$ Now $\overrightarrow{AB} = Position\ vector\ of\ B - Position\ vector\ of\ A$ $= 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$- $(\hat{\imath} + \hat{\jmath} + 2\hat{k})$ $= (2-1)\hat{\imath} + (2-1)\hat{\jmath} + (3-2)\hat{k}$ $= \hat{\imath} + \hat{\jmath} + \hat{k}$ Similarly, $\overrightarrow{AC} = Position\ vector\ of\ C - Position\ vector\ of\ A$ $= 3\hat{\imath} - \hat{\jmath} - \hat{k}$- $(\hat{\imath} + \hat{\jmath} + 2\hat{k})$ $= (3-1)\hat{\imath} + (-1-1)\hat{\jmath} + (-1-2)\hat{k}$ $=2\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$	

	Now $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -2 & -3 \end{vmatrix}$ $= (-3 + 2)\,\hat{i} - (-3 - 2)\,\hat{j} + (-2 - 2)\,\hat{k} = -\,\hat{i} + 5\,\hat{j} - 4\,\hat{k}$ Hence area of the triangle is given by $\Delta = \mathrm{I}\,\overrightarrow{AB} \times \overrightarrow{AC}\,\mathrm{I} = \sqrt{(-1)^2 + 5^2 + (-4)^2}$ $= \sqrt{1 + 25 + 16} = \sqrt{42}$ sq units. (Ans)	

QUESTION BANK WITH SOLUTIONS IN MATHEMATICS-II

FOR 2[ND] SEMESTER DIPLOMA COURSES FROM SUMMER 2025 ONWARDS

UNIT-V

A. 02 Mark Questions & Solutions

1. **Find the order and degree of the equation** $3\frac{d^2y}{dx^2}=\left\{2+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{5}{3}}$ **.Level-2(Understanding)**

Solution:

Given $3\frac{d^2y}{dx^2}=\left\{2+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{5}{3}}$

$$=>\left\{3\frac{d^2y}{dx^2}\right\}^3=\left\{\left\{2+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{5}{3}}\right\}^3$$

$$=>\left\{3\frac{d^2y}{dx^2}\right\}^3=\left\{2+\left(\frac{dy}{dx}\right)^2\right\}^5$$

∴ Order=2 and degree=3.

2. **Solve** $\mathbf{xdx-ydy=0}$. **Level-3(Applying)**

Solution:

$$xdx-ydy=0$$
$$=>xdx=ydy$$

Integrating both sides, we have

$$\int xdx=\int ydy$$
$$=>\frac{x^2}{2}=\frac{y^2}{2}+C$$
$$=>\frac{x^2}{2}-\frac{y^2}{2}=C.$$

3. **Solve** $\mathbf{\frac{dy}{dx}=xcosx}$. **Level-3(Applying)**

Solution:

$$\frac{dy}{dx}=xcosx.$$
$$=>dy=xcosxdx.$$

Integrating both sides and by applying by parts rule, we have

$$=>\int dy=\int xcosxdx.$$

$$=> y = x\int \cos x dx - \int \frac{d}{dx}(x)\left(\int \cos x dx\right)dx + c$$
$$= x\sin x - \int 1.\sin x\, dx + c$$
$$= x\sin x - \int \sin x\, dx + c$$
$$y = x\sin x - (-\cos x) + c$$
$$y = x\sin x + \cos x + c$$

4. Find the integrating factor of $\frac{dy}{dx} + y = e^{-x}$. **Level-3(Applying)**

Solution:

Given $\frac{dy}{dx} + y = e^{-x}$

$P(x) = 1,\ Q(x) = e^{-x}$

Integrating factor= $e^{\int P(x)dx} = e^{\int 1dx} = e^{x}$.

5. Show that $y = A\cos x + B\sin x$ **is a solution of the differential equation** $\frac{d^2y}{dx^2} + y = 0$.
Level-3(Applying)

Solution:

Given $y = A\cos x + B\sin x$

$$\frac{dy}{dx} = \frac{d}{dx}(A\cos x + B\sin x) = A\frac{d}{dx}(\cos x) + B\frac{d}{dx}(\sin x) = -A\sin x + B\cos x.$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-A\sin x + B\cos x) = -A\frac{d}{dx}(\sin x) + B\frac{d}{dx}(\cos x) = -A\cos x - B\sin x$$
$$= -(A\cos x + B\sin x) = -y.$$

$$=> \frac{d^2y}{dx^2} + y = 0.$$

6. Solve $\frac{dy}{dx} = y + 2$. **Level-3(Applying)**

Solution:

Given $\frac{dy}{dx} = y + 2$

$$=> dy = (y + 2)dx$$
$$=> \frac{dy}{y + 2} = dx.$$

Integrating both sides, we have

$$=> \int \frac{dy}{y + 2} = \int dx$$
$$=> \log(y + 2) = x + c$$
$$=> y + 2 = e^{x+c}$$
$$=> y = e^{x+c} - 2.$$

7. Solve $\frac{dx}{dy} + \sqrt{\frac{1-x^2}{1-y^2}} = 0.$ **Level-3(Applying)**

Solution:

Given $\frac{dx}{dy} + \sqrt{\frac{1-x^2}{1-y^2}} = 0$

$$=> \frac{dx}{dy} = -\sqrt{\frac{1-x^2}{1-y^2}} = -\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}.$$

$$=> \frac{dx}{\sqrt{1-x^2}} = -\frac{dy}{\sqrt{1-y^2}}$$

$$=> \frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} = 0$$

Integrating both sides, we have

$$=> \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{dy}{\sqrt{1-y^2}} = c$$

$$=> sin^{-1} x + \sin^{-1} y = c.$$

8. Solve $\frac{dy}{dx} = (x^2 + \sin 3x).$ **Level-3(Applying)**

Solution:

Given $\frac{dy}{dx} = (x^2 + \sin 3x).$

$$=> dy = (x^2 + \sin 3x)dx$$

Integrating both sides, we have

$$=> \int dy = \int (x^2 + \sin 3x) dx$$

$$=> \int dy = \int x^2 dx + \int \sin 3x dx$$

$$=> y = \frac{x^3}{3} - \frac{\cos 3x}{3} + c.$$

9. Solve $\frac{dy}{dx} = (e^x + 1)y.$ **Level-3(Applying)**

Solution:

Given $\frac{dy}{dx} = (e^x + 1)y$

$$=> dy = (e^x + 1)y dx$$

$$=> \frac{dy}{y} = (e^x + 1)dx$$

Integrating both sides, we have

$$=> \int \frac{dy}{y} = \int (e^x + 1)dx = \int e^x dx + \int 1.dx$$

$$=> \log y = e^x + x + c.$$

10. Find the order and degree of the equation $2\frac{d^2y}{dx^2} = \sqrt{\left(\frac{dy}{dx}\right)^3 + 5}$. **Level-2(Understanding)**

Solution:

Given $$2\frac{d^2y}{dx^2} = \sqrt{\left(\frac{dy}{dx}\right)^3 + 5}$$

$$\Rightarrow \left(2\frac{d^2y}{dx^2}\right)^2 = \left(\sqrt{\left(\frac{dy}{dx}\right)^3 + 5}\right)^2$$

$$\Rightarrow 4\left(\frac{d^2y}{dx^2}\right)^2 = \left(\left(\frac{dy}{dx}\right)^3 + 5\right)$$

∴ Order=2 and degree=2.

11. Solve $\frac{dy}{dx} = \frac{e^{2x}+1}{e^x}$. **Level-3(Applying)**

Solution:

Given $$\frac{dy}{dx} = \frac{e^{2x}+1}{e^x}.$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{2x}+1}{e^x} = \frac{e^{2x}}{e^x} + \frac{1}{e^x}.$$

$$\Rightarrow \frac{dy}{dx} = e^x + e^{-x}.$$

$$\Rightarrow dy = (e^x + e^{-x})dx.$$

$$\Rightarrow \int dy = \int (e^x + e^{-x})dx = \int e^x dx + \int e^{-x} dx.$$

$$\Rightarrow y = e^x - e^{-x} + c.$$

12. What is MATLAB? **Level-1(Remember)**

Solution:

MATLAB is an abbreviation for MATrixLABoratory. It is a high level programming language initially developed for mathematical calculation including matrix operation. It was created by Cleve Moler in 1970.

13. Write five basic functions used in MATLAB. **Level-1(Remember)**

Solution:

(a) disp()
(b) clear
(c) close all
(d) clc
(e) exp(x)

14. How to find the square root of 25 by using MATLAB. **Level-2(Understanding)**

Solution:

$\gg x = 25$
$\gg y = \text{sqrt}(25)$
$\gg y = 5.$

B. 5 mark Questions & Solutions

1. Solve $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x}$. Level-3(Applying)

Solution:

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x}$$

$$P = \frac{1}{x}, Q = \frac{1}{x}$$

Integrating factor $= e^{\int P dx} = e^{\int \frac{dx}{x}} = e^{\log x} = x.$

General solution: $y \times I.F = \int Q \times I.F\, dx + c$

$$=> y \times x = \int \frac{1}{x}.x dx + c$$

$$=> y \times x = \int 1.dx + c$$

$$=> xy = x + c$$

$$=> y = \frac{x}{x} + \frac{c}{x}$$

$$=> y = 1 + \frac{c}{x}$$

2. solve $(1+y^2)dx+(1+x^2)dy=0$. Level-3(Applying)

Solution:

$(1+y^2)dx+(1+x^2)dy=0$

$=> (1+y^2)dx=-(1+x^2)dy$

$=> \frac{dx}{1+x^2}=-\frac{dy}{1+y^2}$

$=> \frac{dx}{1+x^2}+\frac{dy}{1+y^2}=0$

Integrating both sides, we have

$=> \int\frac{dx}{1+x^2}+\int\frac{dy}{1+y^2}=c$

$=> \tan^{-1}x+\tan^{-1}y=\tan^{-1}c$

$=> \tan^{-1}\frac{x+y}{1-xy}=\tan^{-1}c$

$=> \frac{x+y}{1-xy}=c$

3. Solve $x\log x\frac{dy}{dx}+y=2\log x$. Level-3(Applying)

Solution:

$x\log x\frac{dy}{dx}+y=2\log x$

divide $x\log x$ on both sides

$=> \frac{dy}{dx}+\frac{y}{x\log x}=\frac{2\log x}{x\log x}$

$=> \frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}$

$P=\frac{1}{x\log x}, Q=\frac{2}{x}$

Integrating factor $= e^{\int Pdx}=e^{\int\frac{dx}{x\log x}}$

put $\log x=t => \frac{1}{x}=\frac{dt}{dx}$

$=> Integrating\ factor = e^{\int\frac{dt}{t}}=e^{\log t}=t=\log x$

General Solution: $y\times I.F=\int Q\times I.F\,dx+c$

$=> y\times\log x=\int\frac{2}{x}.\log x\,dx+c$

put $\log x = t \Rightarrow \frac{1}{x} = \frac{dt}{dx} \Rightarrow \frac{1}{x}dx = dt$

$\Rightarrow y \times \log x = \int 2t dt + c = 2\frac{t^2}{2} + c = (\log x)^2 + c$

$\Rightarrow y = \log x + \frac{c}{\log x}$.

4.Solve $\frac{dy}{dt} = \sin t \cos t\, e^{\sin t}$. **Level-3(Applying)**

<u>Solution:</u>

Given $\frac{dy}{dt}$ $\sin t \cos t\, e^{\sin t}$

$dy = \sin t \cos t e^{\sin t} dt$

Integrating both sides, we have

$\int dy = \int \sin t \cos t e^{\sin t} dt$

Let $p = \sin t$

$\frac{dp}{dt} = \cos t$

$dp = \cos t\, dt$

Substituting the value and integrate by using by parts method, we have

$\int dy = \int p\, e^p dp$

$\Rightarrow y = p \int e^p\, dp - \int \left(\frac{dp}{dp}\right)\left(\int e^p dp\right) dp$

$\Rightarrow y = pe^p - \int 1.e^p dp$

$\Rightarrow y = pe^p - \int e^p dp$

$\Rightarrow y = pe^p - e^p + c$

$\Rightarrow y = \sin t\, e^{\sin t} - e^{\sin t} + c$

$\Rightarrow y = e^{\sin t}(\sin t - 1) + c$

5. Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$. **Level-3(Applying)**

<u>Solution:</u>

Given $(1 + y^2)dx = (\tan^{-1} y - x)dy$.

$$=> \frac{dx}{dy} = \frac{(\tan^{-1}y - x)}{(1+y^2)} = \frac{\tan^{-1}y}{(1+y^2)} - \frac{x}{(1+y^2)}.$$

$$=> \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{\tan^{-1}y}{(1+y^2)}$$

Here $P = \frac{1}{(1+y^2)}$ $Q = \frac{\tan^{-1}y}{(1+y^2)}$

Integrating factor= $e^{\int P dy} = e^{\int \frac{1}{(1+y^2)}dy} = e^{\tan^{-1}y}$.

General Solution: $x \times I.F = \int Q \times I.F\, dy + c$

$$=> xe^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{(1+y^2)} e^{\tan^{-1}y} dy + c$$

Let $z = \tan^{-1}y$

$$\frac{dz}{dy} = \frac{1}{(1+y^2)}$$

$$dz = \frac{1}{(1+y^2)} dy$$

$$=> xe^{\tan^{-1}y} = \int ze^{z}dz + c$$

$$= z\int e^{z}dz - \int (\frac{dz}{dz})\left(\int e^{z}dz)\, dz\right) + c$$

$$= ze^{z} - \int 1.e^{z}dz + c$$

$$= ze^{z} - e^{z} + c$$

$$=> xe^{\tan^{-1}y} = \tan^{-1}y\, e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$$

$$=> xe^{\tan^{-1}y} = (\tan^{-1}y - 1)e^{\tan^{-1}y} + c$$

$$=> x = (\tan^{-1}y - 1) + \frac{c}{e^{\tan^{-1}y}}.$$

6. Solve $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$. Level-3(Applying)

Solution:

$$\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$$

$$=> \frac{dy}{dx} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} = \tan^2\frac{x}{2} = \sec^2\frac{x}{2} - 1$$

$$=> dy = \left(\sec^2\frac{x}{2} - 1\right) dx$$

Integrating both sides, we have

$$=> \int dy = \int \left(\sec^2\frac{x}{2} - 1\right) dx$$

$$=> y = \int \left(\sec^2\frac{x}{2}\right) dx - \int 1dx$$

$$=> y = 2\tan\frac{x}{2} - x + c.$$

7. Solve $e^x tanydx + (1 + e^x)\sec^2 ydy = 0$. **Level-3(Applying)**

Solution:

$$e^x tanydx + (1 + e^x)\sec^2 ydy = 0.$$

$$=> e^x tanydx = -(1 + e^x)\sec^2 ydy.$$

$$=> \frac{e^x dx}{(1 + e^x)} = -\frac{\sec^2 ydy}{tany}$$

$$\frac{e^x dx}{(1 + e^x)} + \frac{\sec^2 y\,dy}{tany} = 0$$

Integrating both sides, we have

$$\int \frac{e^x dx}{(1 + e^x)} + \int \frac{\sec^2 y\,dy}{tany} = c$$

Let $p = 1 + e^x$ and $q = tany$

$$\frac{dp}{dx} = e^x \quad \text{and} \quad \frac{dq}{dy} = \sec^2 y$$

$$dp = e^x dx \quad \text{and} \quad dq = \sec^2\ ydy$$

Substituting the value and integrate, we have

$$\int \frac{dp}{p} + \int \frac{dq}{q} = c$$

$$=> log(p) + log(q) = c$$

$$=> \log(pq) = c$$

$$=> \log\left((1 + e^x)tany\right) = c$$

$$=> (1 + e^x)tany = e^c.$$