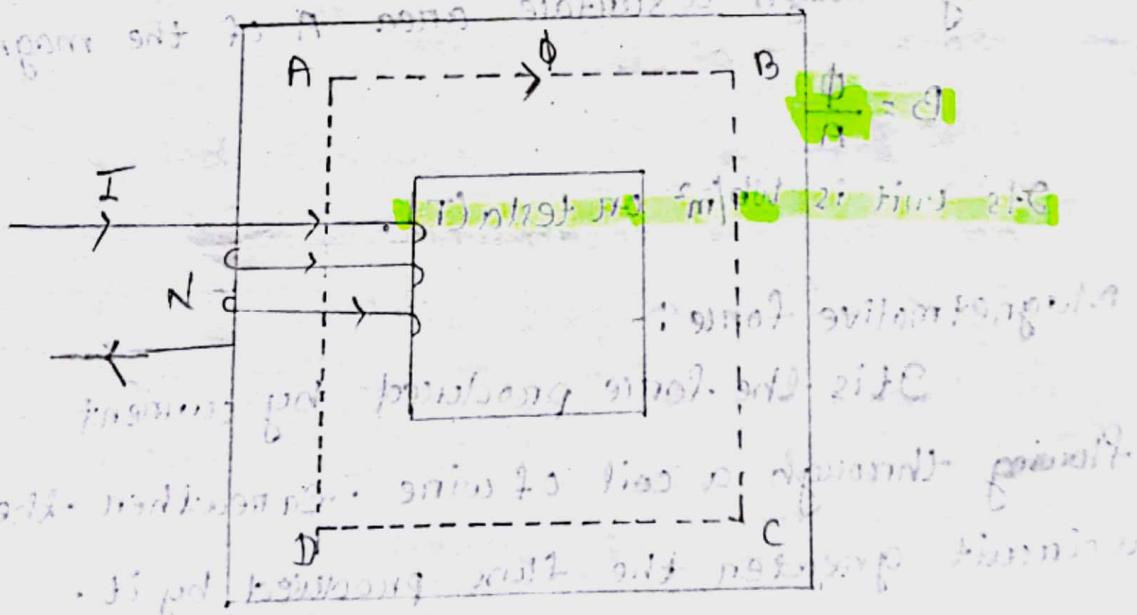


## Magnetic Circuit :-

The closed path followed by magnetic flux is called magnetic circuit. A magnetic circuit usually consists of materials having high performance e.g. iron, soft steel etc. It is because these material offer very small opposition to the 'flow' of magnetic flux. Consider a coil of  $N$  turns wound on the iron core. When current  $I$  is passed through a coil, magnetic flux  $\Phi$  is set up in the core. The flux follows the closed path ABCDA and ABCDA is the magnetic circuit. The amount of magnetic flux set up in the coil depends upon current ( $I$ ) and no. of turns ( $N$ ). If we increase the current or number of turns, the amount of magnetic flux also increases and vice-versa. The product  $NI$  is called magnetomotive force (m.m.f) and determines the amount of magnetic flux set up in the magnetic circuit.

$$\text{m.m.f} = NI \text{ ampere turns}$$



$\text{TA} = \text{BA} = \text{m.m.f}$

Magnetic field strength :-

Magnetic field strength :- (H) ~~is~~ <sup>is</sup> ~~hard's~~ soft

It is the magnetising force that produces flux in a given magnetic circuit. It is defined as magnetomotive force per unit length of the magnetic circuit. If in a given magnetic circuit, mmf = NI and l, the average length of magnetic path then.

$$H = \frac{mmf}{length} = \frac{NI}{l} \text{ ampere/metre}$$

Magnetic flux ( $\Phi$ ) :-

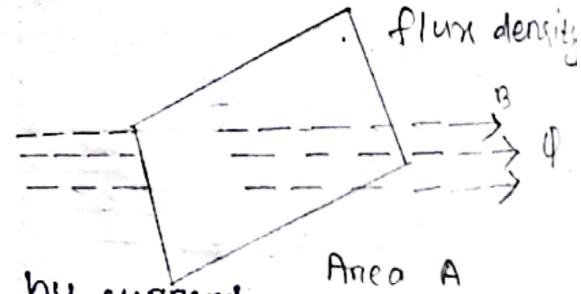
It equals the entire number of lines of force produced by a magnet. The more the lines of force, the greater the flux and stronger the magnetic field. The unit of magnetic flux is weber (Wb). One weber equals to  $10^8$  lines.

Flux density ( $B$ ) :-

The flux density  $B$  is given by the flux  $\Phi$  passing normally through a surface area  $A$  of the magnetic field.

$$B = \frac{\Phi}{A}$$

Its unit is  $\text{Wb}/\text{m}^2$  or tesla (T).



Magnetomotive force :-

It is the force produced by current flowing through a coil of wire underneath the mmf of a circuit greater than the flux produced by it.

$$mmf = NI AT$$

For example, if a coil has 500 turns and carries a current of 10 mA, then mmf produced

$$= 5000 \times 10 \times 10^{-3} = 50 \text{ AT}$$

## Reluctance ( $S$ ) :-

It is the opposition that a magnetic circuit offers to the passage of magnetic flux through it. It is given by the ratio of the mmf and the amount of flux produced.

$$S = \frac{\text{mmf}}{\phi} = \frac{NI}{\phi} \text{ AT/Wb},$$

In terms of the magnetic quantities, it is given by,

$$S = \frac{1}{\mu A} = \frac{1}{\mu_0 \mu_r A} \text{ AT/Wb}$$

Since  $1 \text{ AT/Wb} = 1/\text{Henry}$ , Hence unit of reluctance is also 'reciprocal' Henry.

## Permeance :-

It is reciprocal of reluctance and means the ease with which magnetic flux is produced. Its unit is Henry or  $\text{Wb/AT}$ .

## Permeability of a medium :-

The ability of a material to concentrate magnetic flux is called permeability. Any material that is easily magnetised tends to concentrate magnetic flux. Because of this soft iron is easily magnetised. We say that if it has a high permeability, since the permeability of every magnetic medium is compared with the permeability of the material, has two permeabilities :-

### (i) Relative Permeability ( $\mu_r$ ):

It is given by the ratio of the permeability of the material and permeability of air:

$$\mu_r = \frac{\mu}{\mu_0}$$

Where  $\mu_0$  is the permeability of air or vacuum. It may be that  $\mu_r$  has no units.

Materials like iron, steel, cobalt and nickel have values of  $\mu_r$  ranging from 100 to 1,00,000.

### Absolute Permeability ( $\mu$ ):

It is given by  $\mu = \mu_0 \mu_r$

It is also given by  $\mu = B/H$

Where 'H' is the magnetising force and 'B' is the flux density in the material.

It is called the absolute permeability of a material. Since relative permeability of air with respect to itself is unity, hence its absolute permeability is  $\mu_0$ . Its value is  $4\pi \times 10^{-7}$  henry/metre.

## Relation between the Magnetic Circuit:-

Consider the magnetic circuit shown in figure. Suppose the mean length of the magnetic circuit is  $l$  mtes, cross-section area of the core is  $a$   $m^2$  and the relative permeability of core material is  $\mu$ . When current  $I$  is passed through the coil, it will set up magnetic flux  $\Phi$  in the material.

$$\text{Flux density in the medium, } B = \frac{\Phi}{a} \text{ Wb/m}^2$$

$$\text{Magnetising force in the material, } H = \frac{B}{\mu_0 M_r} = \frac{\Phi}{a \mu_0 M_r} \text{ AT/m}$$

Now magnetising force  $H$  is equal to mmf per unit length of the magnetic circuit

$$H = \frac{NI}{l} \Rightarrow Hl = NI$$

$$\frac{\Phi}{a \mu_0 M_r} \times l = NI$$

$$\Rightarrow \Phi = \frac{NI}{(l/a \mu_0 M_r)}$$

$$\Rightarrow \Phi = \frac{\text{mmf}}{\text{reluctance}}$$

Analogy b/w Electric & Magnetic Circuit :-

Magnetic circuit :-

$$1 \rightarrow \text{flux} = \frac{\text{mmf}}{\text{reluctance}}$$

$$2 \rightarrow \text{mmf (ampere-turn)} = \text{current} \times \text{number of turns}$$

$$3 \rightarrow \text{flux } \phi \text{ (weber)}$$

$$4 \rightarrow \text{flux density } B \text{ (wb/m}^2\text{)}$$

$$5 \rightarrow \text{Reluctance, } S = \frac{l}{\mu A} = \frac{l}{\mu_0 A n A} \text{ (AT/wb)}$$

$$6 \rightarrow \text{Magnetic field intensity (AT/m)}$$

$$7 \rightarrow \text{Permeance} = \frac{1}{\text{reluctance}} \text{ (henry)}$$

$$8 \rightarrow \text{Reluctance (AT-m/wb)}$$

$$9 \rightarrow \text{Permeability} = \frac{1}{\text{reluctivity}} \text{ (wb/AT-m)}$$

Electric Circuit :-

$$1 \rightarrow \text{Current} = \frac{\text{emf}}{\text{resistance}}$$

$$2 \rightarrow \text{emf (volt)}$$

$$3 \rightarrow \text{Current (ampere)}$$

$$4 \rightarrow \text{Current density (A/m}^2\text{)}$$

$$5 \rightarrow \text{Resistance, } R = \rho \frac{l}{A} = \frac{l}{\sigma A} \text{ (Ohm)}$$

$$6 \rightarrow \text{electrical field intensity (V/m)}$$

$$7 \rightarrow \text{Conductance} = \frac{1}{\text{resistance}} \text{ (Siemens)}$$

$$8 \rightarrow \text{resistivity (ohm-meter)}$$

$$9 \rightarrow \text{Conductivity} = \frac{1}{\text{resistivity}} \text{ (Siemens/metre)}$$

## Calculation of Ampere turns :-

$$\text{Flux}(\Phi) = \frac{\text{mmf}}{\text{reluctance}}$$

$$= \frac{NI}{\frac{l}{a \mu_0 H_r}}$$

$$\text{Ampere turns required} = \frac{\Phi \times l}{a \mu_0 H_r} = \frac{\Phi}{a} \times \frac{l}{\mu_0 H_r}$$

$$= \frac{BL}{a \mu_0 H_r} = dH \quad \left( \frac{B}{\mu_0 H_r} = H \right)$$

~~Ampere turns required for any part~~

Ampere turn required is equals to magnetic strength in that part into length of that part.

Some magnetic Circuits:-

We shall discuss some magnetic circuits that the

reader may encounter in many practical situations.

(i) Simplest magnetic circuit.

The simplest magnetic circuit is that which is of the same cross-section and material throughout.

It is in the form of an iron ring.

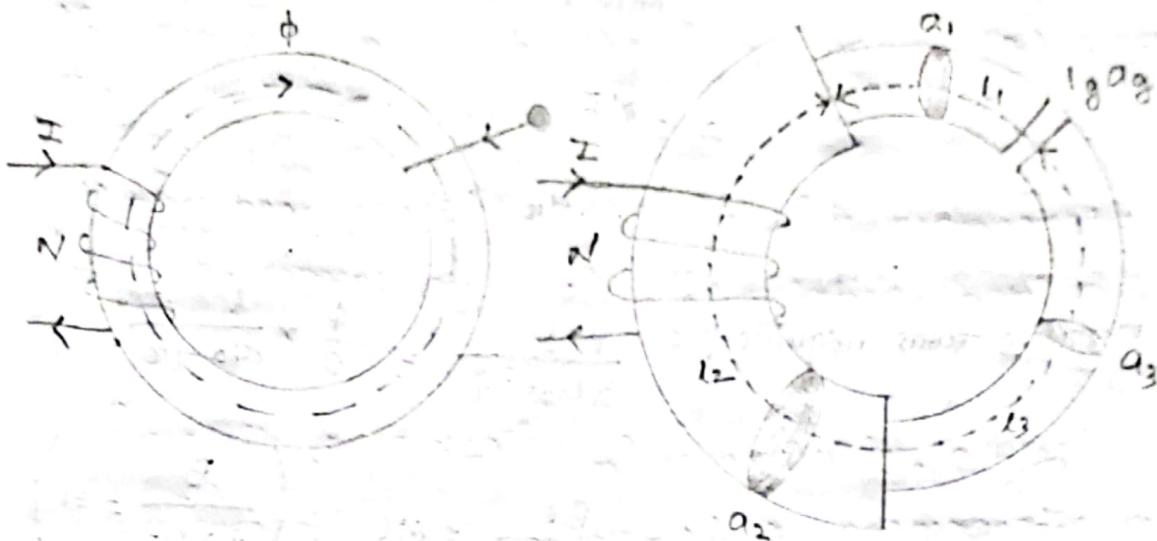
At the flux density in the ring,  $B = \frac{d}{a}$

$$dH = \frac{B}{\mu_0 H_r}$$

It is in the form of a ring

∴ AT required to produce flux  $\phi$  in the ring

$$= \delta I \times \text{mean length of flux path} = \delta I \times Z$$



(ii) Series magnetic circuit :-

In a series magnetic circuit, the same flux ' $\phi$ ' flows through each path of the circuit. It can just be compared to a series electric circuit which carries the same current throughout. Consider a composite magnetic circuit consisting of three different magnetic materials of different relative permeability along with an air gap. Since, the parts do not have the same cross-sectional area, the flux density will be different in different parts. If we are given the flux and it is required to find the m.m.f. we take one part at a time and find the flux density  $B$ , the magnetising force  $\delta H$  ( $\delta H = B/\mu_0 M_r$  or more accurately from  $B-H$  curve for magnetic materials; not-air-gap) and product  $\delta Hl$  for that part. The total m.m.f required will be the sum of these products.

$$\text{Required } \text{m.m.f.} = \delta H_1 l_1 + \delta H_2 l_2 + \delta H_3 l_3 + \delta H g l g$$

(iii) Magnetic core having parallel branches:-

A magnetic circuit which has more than one path for flux is called a parallel magnetic circuits. It can just be compared to a parallel electrical circuit which has more than one path for electric current. The concept of parallel magnetic circuit is illustrated.

Here a coil of  $N$  turns wound on limb AF carries a current of  $I$  amperes. The flux  $\Phi_1$ , set up by the coil

divides at B into two parts,

namely (i) flux  $\Phi_2$  passes along the path BE.

(ii) flux  $\Phi_3$  follows the path BCDE.

The magnetic paths BE and BCDE are in parallel and from a parallel magnetic circuit. The AT required for this parallel circuits is equal to AT required for any one of the paths.

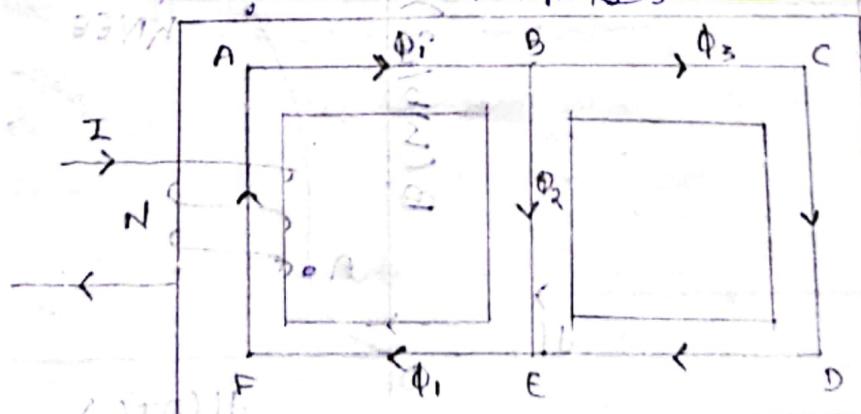
Let  $S_1$  = reluctance of path EFAB

$S_2$  = reluctance of path BE

$S_3$  = reluctance of path BCDE

Total mmf required = mmf for path EFAB + mmf for path BE on path BCDE.

$$NI = \Phi_1 S_1 + \Phi_2 S_2 + \Phi_3 S_3 = NI$$



Chaitanya

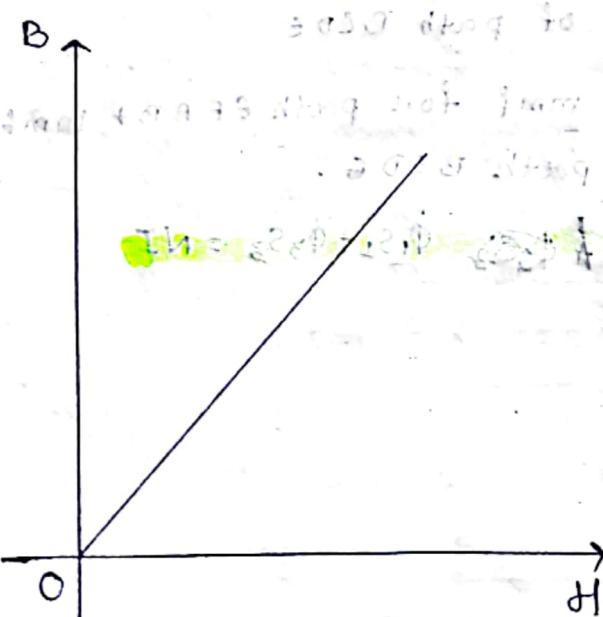
## BH CURVE :-

The B-H curve (or magnetisation curve) indicates the manner in which the flux density ( $B$ ) varies with the magnetising force ( $H$ ). For non-magnetic material (e.g. air, copper, wood etc.) the relation between  $B$  &  $H$  is

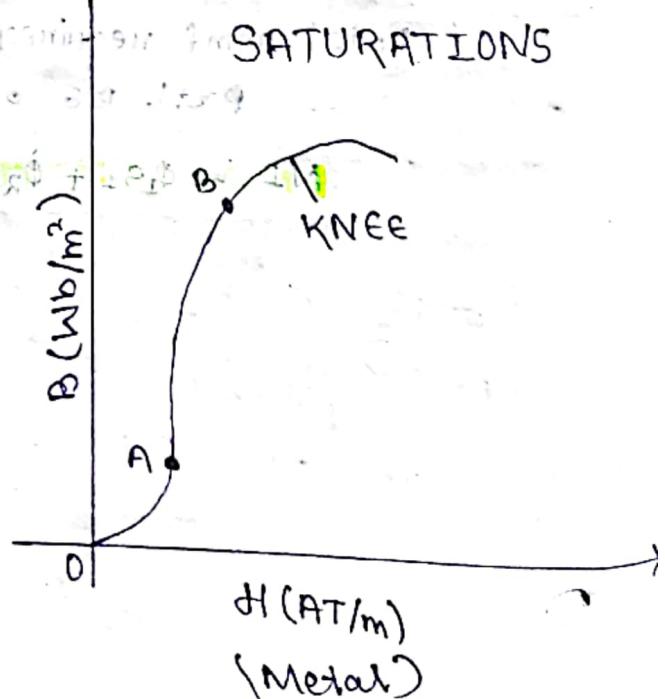
$$B = \mu_0 H$$

Since  $\mu_0 (= 4\pi \times 10^{-7} \text{ AT/m})$  is constant,  $B \propto H$

Hence the B-H curve of a non-magnetic material is a straight line passing through the origin as shown in fig. Two things are worth noting. First, the curve never saturates no matter how great the flux density may be. Secondly, a large mmf is required to produce a given flux in the non-magnetic material.



(Non-metal)

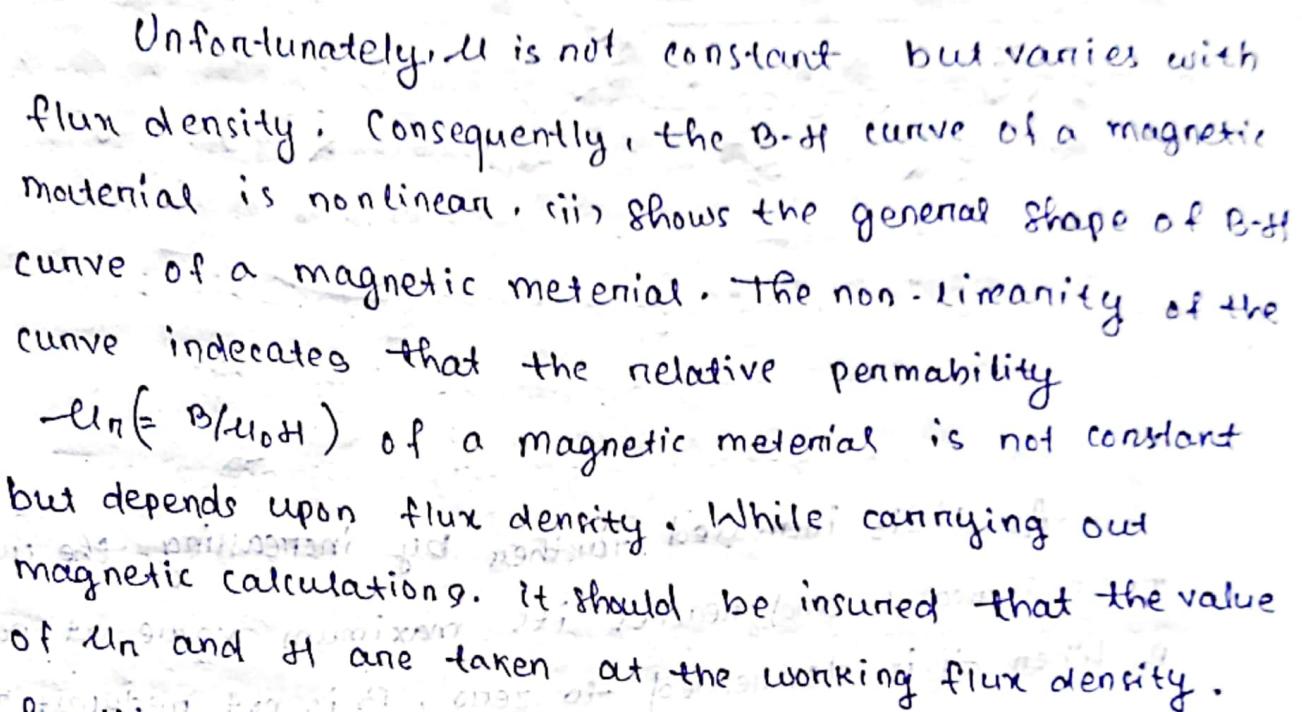


$B (\text{Wb}/\text{m}^2)$   
(Metal)

(iii) for magnetic material :-

for magnetic material the relation  
between  $B$  &  $H$  is given by;

$$B = \mu_0 H$$

Unfortunately,  $\mu$  is not constant but varies with flux density. Consequently, the  $B$ - $H$  curve of a magnetic material is nonlinear. 

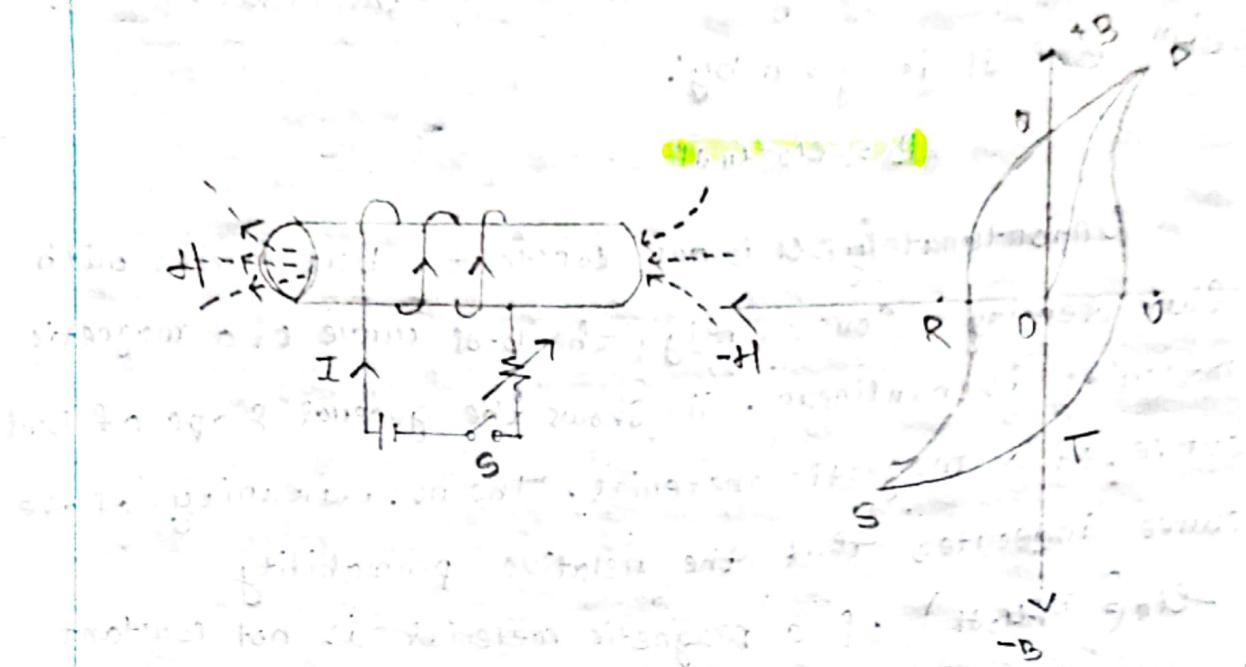
for this purpose, the  $B$ - $H$  curve of the material in question may be helpful.

(a) Explain the term magnetic hysteresis? What is it due to? What is meant by hysteresis loop and what is its significance?

Ans:- Let us take an arrangement bar of iron and magnetise it by placing it within the magnetic field of a solenoid.

We will magnetise the bar, first in one direction and then in the opposite direction by reversing the direction of current through the coil. The field produced by the solenoid is called the magnetising field or force  $H$ . If produced magnetic flux density  $B$  in the iron bar, the value of  $B$  can be increased or decreased by increasing or decreasing the current.

We will now plot values of  $B$  for different values of  $H$ . Let us start from point  $O$  where  $H=0$ .



As  $H$  is increased further by increasing the current source  $I$ ,  $B$  increases and reaches its maximum value at point  $P$ . When  $H$  is decreased to zero,  $B$  is not reduced to zero. It has a value of  $OQ$ . Let us now reverse the direction of  $H$  by reversing the direction of current  $I$ . When  $H = -OP$ ,  $B = 0$  and the iron rod is demagnetised. When  $H$  is decreased to zero,  $B$  is not reduced to zero and has a value equal to  $OT$ . From the above discussion, we find that  $B$  always lags behind  $H$ . The two never becomes zero or maximum at the same time.

The value of  $B$  becomes zero much after  $H$  has become zero. This lagging of flux density  $B$  behind the magnetising force  $H$  is called hysteresis.

The hysteresis (or lagging) is produced because the magnetic domains inside the iron bar take time to align with the external magnetising force  $H$ . Also the domains do not return exactly to their original positions when  $H$  is removed.

The  $B/H$  curve shown in fig. is called the hysteresis loop. The shape of the hysteresis loop shows the qualities of the magnetic material from which the bar is made. A material having a high retentivity produces a nearly square hysteresis loop. However, the area of the hysteresis loop represents the loss of energy when a unit volume of the magnetic material is carried through one reversal of magnetisations. Due to this loss of energy, the magnetic bar is heated up if carried through many cycles of reversals. This heat is produced due to friction between the elementary magnets in the bar as they turn from one direction to another.

(b) The hysteresis loop of a sample of sheet subjected to a maximum flux density of  $1.3 \text{ wb/m}^2$  has an area of  $93 \text{ cm}^2$  the scale being  $1 \text{ cm} = 0.1 \text{ wb/m}^2$  and  $1 \text{ cm} = 50 \text{ AT/m}$ . Alternating flux density of  $1.3 \text{ wb/m}^2$  peak value at a frequency of  $65 \text{ Hz}$ .

Ans:-

## Hysteresis loss :-

When a magnetic material is subjected to a cycle of magnetisation (i.e. it is magnetised first one direction and then in the other) an energy loss takes place due to the domain friction in the material. That is the domain of the material resist being turned first in one direction and then in the other. Energy is thus expended in the material in over coming this opposition. This loss is in the form of heat and is called hysteresis loss. It is so called because it results due to the hysteresis effect in a magnetic material.

Hysteresis loss is present in all those electrical machines whose iron parts are subjected to cycles of magnetisation.

The obvious effect of hysteresis loss is the rise of temperature of the machine. It can be shown that hysteresis energy loss per cycle is directly proportional to the area of the hysteresis loop.

## Importance of Hysteresis loop :-

The shape and size of the hysteresis loop largely depends upon the nature of the material. The choice of a magnetic material for a particular application often depends upon the shape and size of the hysteresis loop. A few uses are discussed below by way of illustration :-

- i) The smaller the hysteresis loop area of a magnetic material the less is the hysteresis loss. The hysteresis loop for silicon steel has a very small area. For this reason, silicon steels is widely used for making transformer cores and rotating machines which are subjected to rapid

## Magnetic Section.

- (i) The hysteresis loop for Hard steel indicates that this material has high retentivity and coercivity. Therefore hard steel is quite suitable for making permanent magnets. But due to the large area of the loop, there is greater hysteresis loss. For this reason, hard steel is not suitable for the construction of electrical machines.
- (ii) The hysteresis loop for Concreet iron shows that this material has fairly good residual magnetism and coercivity. Hence it is suitable for making cores of electro-magnets.

~~After writing the question, the student has written the following notes:~~

~~Explanation of the last two headings~~

Imp

(Comparison between Hysteresis Loop)

High Retentivity & High Coercivity

Low Remained Magnetic Flux

Large Area of the loop

High Remained Magnetic Flux

Small Area of the loop

It is (i) i.e., the first found

Retentivity & Coercivity

High & High

Low & Low

It is (ii) i.e., Hudd's Iron

As it is soft

## ANALYSIS OF COUPLE CIRCUIT.

Induction is the magnetic field which is proportional to the rate of change of the magnetic field which is called inductance ( $L$ ) and S.I unit of inductance is Henery.

According to Faraday's Law of electromagnetic induction, induction is of types :-

- (1) Self induction
- (2) Mutual induction.

Self inductance :- ( $L$ )

When there is a change in the current or in magnetic flux of the coil an oppose emf is produced which is called self inductance.

$$\Phi \propto I$$

$$\Rightarrow \Phi = LI \quad (\because L - \text{Self inductance coefficient})$$

Self inductance depends upon

- (i) Cross-Sectional Area.
- (ii) The permeability of material.
- (iii) Number of turn in the coil.

$$\text{Induced e.m.f} = \frac{d\Phi}{dt} = \frac{d(LI)}{dt} = L \frac{dI}{dt}$$

$$\therefore L = \frac{n\Phi}{I}$$

Where  $n$  = turns

$\Phi$  = flux

$I$  = Current . self

$L$  = co-efficient of inductance.

$$\text{Opposed e.m.f} = -\frac{d\Phi}{dt} = -\frac{d(LI)}{dt} = -L \frac{dI}{dt}$$

## Mutual Inductance :- (M)

When there is a change in current or magnetic flux linked with two coils and opposing emf is produced across each coil and this phenomenon is called mutual inductance.

The relation is given by

$$\Phi \propto I$$

$$\Phi = mI \quad (\text{where } m \text{ is termed as mutual inductance of two coil.})$$

The rate of change of magnetic flux in a coil is given as e (opposing emf) =  $-\frac{d\Phi}{dt}$

$$= -\frac{dMI}{dt} = -m \frac{dI}{dt}$$

$$\text{Mutual inductance } m = \frac{\mu_0 \mu_r \cdot N_1 \cdot N_2 \cdot a}{l}$$

$$\Rightarrow M = \frac{\mu_0 N_1 N_2 a}{l}$$

Where  $\mu_0$  = absolute permeability

$\mu_r$  = relative permeability

$N$  = Number of turns.

$l$  = length

$a$  = cross sectional area.

## Difference b/w Self induction & Mutual inductance

### Self Inductance

- (i) Self inductance is the characteristic of a single coil.
- (ii) The induced emf opposes the decrease of current in the coil when the main current in the coil decreases.
- (iii) The induced emf opposes the growth of current in the coil when the main current in the coil when the main current of the coil increases.

### Mutual Inductance

- (i) Mutual induction is the characteristic of a pair of coils.
- (ii) The induced emf developed in the neighbouring coil opposes the decrease of current in the coil when the main current in the coil decreases.
- (iii) The induced emf developed in the neighbour coil opposes the growth of current in the coil when the main current of the coil increases.

### Derivations

$$L = \frac{N\phi}{I}$$

$$LI = N\phi = N\Phi_{BA}$$

$$\Phi = NI$$

$$B = \mu_0 H$$

$$L = \frac{N^2 \mu_0 A}{l}$$

Self inductance formula:-

$$L = \frac{N\phi}{I}$$

$$LI = N\phi = NBA \quad \boxed{\therefore \phi = BA}$$

$$\boxed{Hl = NI}$$

$$\boxed{B = \mu H}$$

$$\Rightarrow LI = NBA$$

$$\Rightarrow L = \frac{NBA}{I}$$

$$\Rightarrow L = \frac{N \times NBA}{NI} \quad (\because \text{cancel } N \text{ in both D & N})$$

$$\Rightarrow L = \frac{N^2 BA}{NI}$$

$$\Rightarrow L = \frac{N^2 BA}{Hl} \quad (\because NI = Hl)$$

$$\Rightarrow L = \frac{N^2 \mu H A}{Hl} \quad (\because B = \mu H)$$

$$\Rightarrow L = \frac{N^2 \mu A}{l}$$

$$\Rightarrow L = \frac{N^2 \mu_0 l n A}{l}$$

$$\Rightarrow \boxed{L = \frac{N^2 \mu A}{l}}$$

## Mutual Inductance :-

The magnetic flux emanating from coil 1 has two components: One component links only coil 1, and another component links both coils.

$$\Phi_1 = \Phi_{l1} + \Phi_{l2}$$

Leakage flux + Linkage flux

(1) The induced voltage in the first coil

$$V_1 = N_1 \frac{d\Phi_1}{di_1} \times \frac{di_1}{dt} = L_1 \frac{di_1}{dt} \quad \left[ V_1 = N_1 \frac{d\Phi_1}{dt} \right]$$

(2) The induced voltage in the 2<sup>nd</sup> coil

$$V_2 = N_2 \frac{d\Phi_{l2}}{dt}$$

$$V_2 = N_2 \frac{d\Phi_{l2}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt} \quad \left[ M_{21} = N_2 \frac{d\Phi_{l2}}{di_1} \right]$$

Where  $M_{21}$  is known as the mutual inductance of coil 2 with respect to coil 1.

- $M_{21}$  relates the induced voltage in coil 2 to the current in coil 1.
- Thus, the open-circuit mutual voltage (or induced voltage) across coil 2 is  $V_2$ .

Similarly :-

$$\Phi_2 = \Phi_{21} + \Phi_{22}$$

$$M_{12} = N_1 \frac{d\Phi_1}{di_2}$$

$$V_2 = M_{12} \frac{di_2}{dt}$$

Mutual inductance is bilateral :-

$$M_{12} = M_{21} = M$$

Coupling Coefficient :-

Coupling coefficient is the fraction of the total flux that links to both coils.

$$K = \frac{\Phi_{12}}{\Phi_1} = \frac{\Phi_2}{\Phi_1}$$

$$M^2 = \left( N_2 \frac{d\Phi_{12}}{di_1} \right) \left( N_1 \frac{d\Phi_{21}}{di_2} \right) = \left\{ N_2 \left( \frac{d(K\Phi_1)}{di_1} \right) \right\} \left\{ N_1 \frac{d(K\Phi_2)}{di_2} \right\}$$

$$= K^2 \left( N_1 \frac{d\Phi_1}{di_1} \right) \left( N_2 \frac{d\Phi_2}{di_2} \right) = K^2 L_1 L_2$$

$$\therefore M = K \sqrt{L_1 L_2} \quad \text{or} \quad \Phi_m = K \sqrt{X_1 X_2}$$

If all of the flux links the coil without any leakage flux then  $K=1$ .

\* The term close coupling is used when most of the flux links the coil, either by way of a magnetic core to contain the flux or by interleaving the turns of the coils directly over one another.

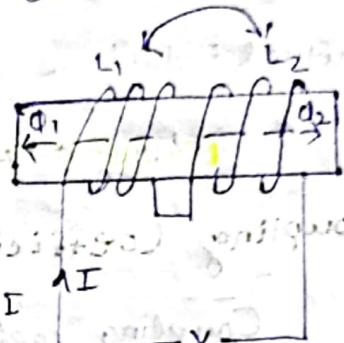
\* The term loose coupling is used when coils placed side-by-side, without a core and have correspondingly low values of  $K$ .

## Series-Aiding and Series opposing coils :-

### (1) Series Aiding Coils :-

$$V = j\omega L_1 I + j\omega M I + j\omega L_2 I + j\omega M I \quad M.$$

$$= j\omega L_{eq} I$$



### (2) Series opposing coils :-

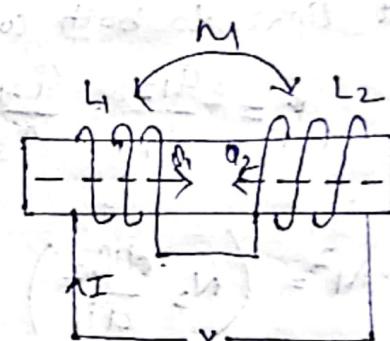
$$V = j\omega L_1 I - j\omega M I + j\omega L_2 I - j\omega M I$$

$$= j\omega L_{eq} I$$

Where  $L_{eq} = L_1 + L_2 - 2M$

Subtract both eq<sup>n</sup> :-

$$M = \frac{1}{4} (L_A - L_B)$$



## Parallel aiding and parallel opposing coils :-

### (1) Parallel aiding coils :-

$$V = j\omega L_1 I_1 + j\omega M I_2$$

$$V = j\omega M I_1 + j\omega L_2 I_2$$

Solving these equations for  $I_1$  &  $I_2$  yields

$$I_1 = \frac{V(L_2 - M)}{j\omega(L_1 L_2 - M^2)}$$

$$I_2 = \frac{V(L_1 - M)}{j\omega(L_1 L_2 - M^2)}$$

Using KCL gives us

$$I = I_1 + I_2 = \frac{V(L_1 + L_2 - 2M)}{j\omega(L_1 L_2 - M^2)} = \frac{V}{j\omega L_{eq}} \quad \boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$

(2) Parallel opposing coils :-

$$L_{eq} = \frac{L_1 L_2 - m^2}{L_1 + L_2 + 2m}$$

① Dot conventions :-

→ Since it is inconvenient to show the construction details of coils on a circuit schematic, we apply the dot convention in circuit analysis.

→ A dot is placed in the circuit at one end of each of the two magnetically coupled.

Steps to assign the dots :-

→ Select a current direction in one coil and place a dot at the terminal where this current enters the winding.

→ Determine the corresponding flux by application of the right hand rule.

→ The flux of the other winding, according to Lenz's law, opposes the first flux.

→ Use the right hand rule to find the natural current direction corresponding to this second flux.

→ Now place a dot at the terminal of the second winding where the natural current leaves the winding.

The dot Rule :-

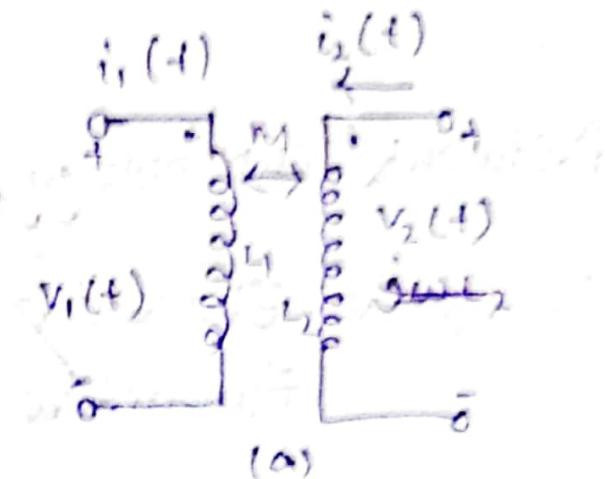
-: this principle follows 2 cases

- (1) When the assumed currents both enter or both leave a pair of coupled coil by the dotted terminals, the signs on the M-terms will be the same as the signs on the L-terms.
- (2) If one current enters by a dotted terminal while the other leaves by a dotted terminal, the signs on the M-terms will be opposite to the signs on the L-terms.

## The Dot Rule :-

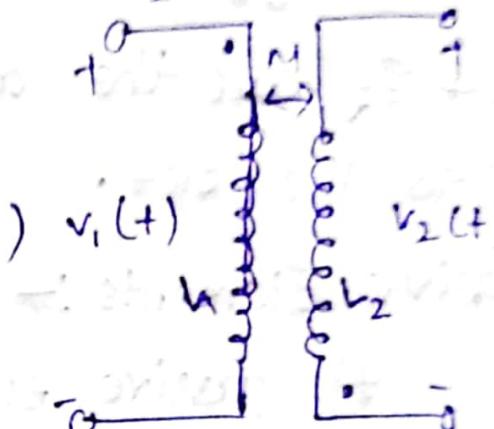
$$V_1(t) = L_1 \frac{d}{dt} i_1(t) + M \frac{d}{dt} i_2(t)$$

$$V_2(t) = L_2 \frac{d}{dt} i_2(t) + M \frac{d}{dt} i_1(t)$$



$$V_1(t) = L_1 \frac{d}{dt} i_1(t) - M \frac{d}{dt} i_2(t)$$

$$V_2(t) = L_2 \frac{d}{dt} i_2(t) - M \frac{d}{dt} i_1(t)$$



# Circuit Elements And Analysis:-

## Network Terminology:-

### (1) Active Element:-

An active element is one which supplies electrical energy to the circuit.

$E_1$  &  $E_2$  are the active elements because they supply energy to the circuit.

### (2) Passive Elements:-

A passive element is one which receives electrical energy and then either converts it into heat or stores in an electric field or magnetic field.  
There are three passive elements, namely  $R_1$ ,  $R_2$  &  $R_3$ . These passive elements convert it into heat.

### (3) Node:-

A node of a network is an equipotential surface at which two or more than two circuit elements are joined. Circuit elements  $R_1$  &  $E_1$  are joined at A and hence A is a node. Similarly B, C & D are nodes.

### (4) Junctions:-

A junction is that point in a network where three or more circuit elements are joined. There are only two junction points B & D. That B is a junction is clear from the fact that three circuit elements  $R_1$ ,  $R_2$  &  $R_3$  are jointed at it.

### (5) Branch:-

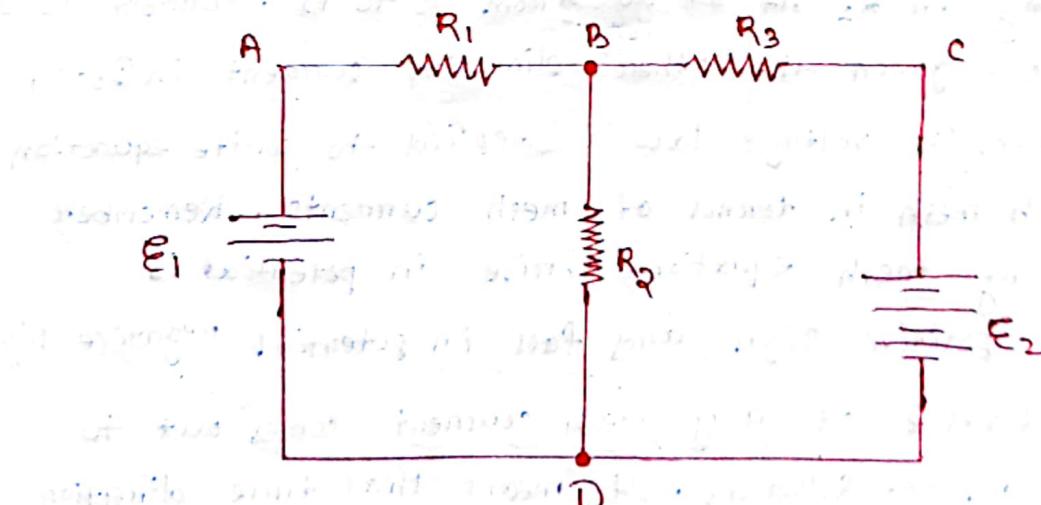
A branch is that part of a network which lies between two junction points. Thus referring to fig. there are a total of three branches i.e. AD, BC & BD. The branch BAD consists of  $R_1$  &  $E_1$ ; the branch BCD consists of  $R_2$  &  $E_2$  and branch BD merely consists of  $R_2$ .

### (6) Loop:-

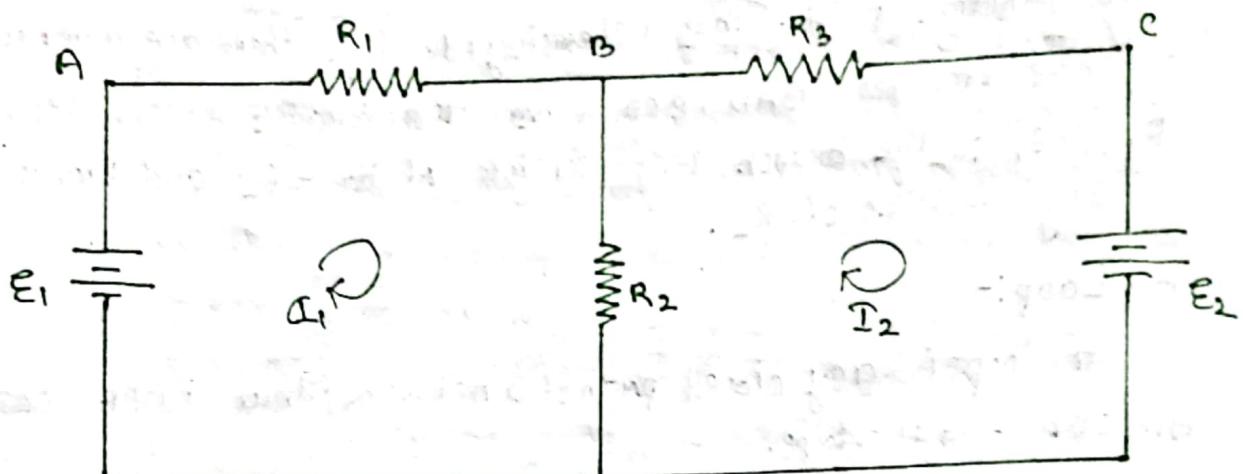
A loop is any closed path of a network. Thus ABDAA, BCDB & ABCDA are the loops.

### (7) Mesh:-

A mesh is the most elementary form of a loop and cannot be further divided into other loop. In the figure both loops ABDAA & BCDB qualify as meshes because they can't be further divided into other loops.



## Maxwell's Mesh Analysis:-



- (i) All mesh currents are assumed to flow in clockwise direction. For example meshes ABDA & BCDB have been assigned mesh current  $I_1$  &  $I_2$  respectively.
- (ii) If two mesh currents are flowing through a circuit elements. The actual current in the circuit elements is the algebraic sum of the two, there are two mesh current  $I_1$  &  $I_2$  flowing in  $R_3$ , if we go from B to D, current is  $I_1 + I_2$  and if we go in the other direction, current is  $I_2 - I_1$ .
- (iii) Kirchhoff's voltage law is applied to write equation for each mesh in terms of mesh currents. Remember while writing mesh equations, rise in potential is assigned positive sign and fall in potential negative sign.
- (iv) If the value of any mesh current comes out to negative in the solutions, it means that true direction of that mesh current is anticlockwise, opposite to the assumed clockwise direction.

Applying KVL we have

Mesh ADDA :-

$$I_1 R_1 + (I_1 - I_2) R_2 + E_1 = 0 \quad \text{Eqn 1}$$

$$\Rightarrow -I_1 R_1 + I_2 R_2 + I_3 R_2 = -E_1$$

$$\Rightarrow I_2 (R_2 + R_3) - I_3 R_2 = E_1 \quad \text{Eqn 1)$$

Mesh BCDB:-

$$-I_2 R_3 - E_2 - (I_2 - I_1) R_2 = 0$$

$$\Rightarrow -I_2 R_3 - E_2 - I_2 R_2 + I_1 R_2 = 0$$

$$\Rightarrow -I_2 R_2 + I_2 (R_1 + R_3) = -E_2$$

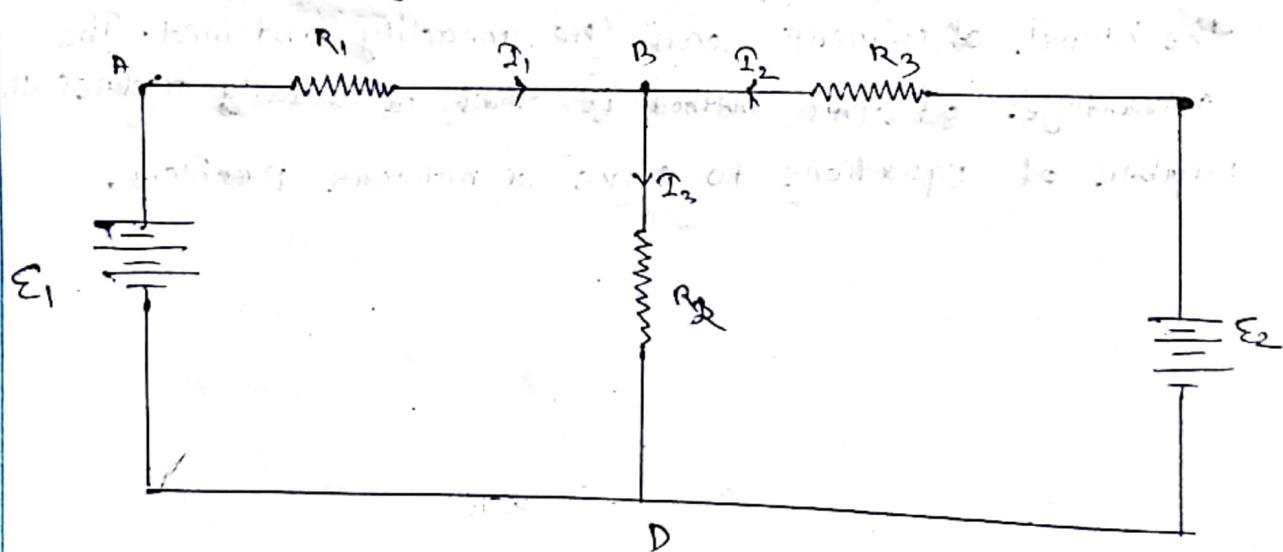
Solving eqn 1 & 2 simultaneously, mesh current  $I_1$  &  $I_2$

can be found out. Once the mesh currents are known, the branch currents can be readily obtained. The advantages of this method is that it usually reduces the number of equations to solve a network problem.

## Node Analysis:-

In this method, one of the nodes is taken as the reference node. The potentials of all the points in the circuit are measured wrt this reference node. In the figure A, B, C & D are four nodes and node D has been taken as the reference node. A glance at the circuit shows that voltage at nodes A & C wrt reference node D are known. These are  $E_1 = 120V$  &  $E_2 = 65V$  respectively. The only potential of the node B wrt D is unknown. If this potential  $V_B$  can be found, each branch current can be determined because the voltage across each resistor will then be known.

Hence Nodal analysis essentially aims at choosing a reference node in the network and then finding the unknown voltage at the nodes wrt reference node.



The voltage  $V_B$  can be found by applying Kirchhoff's current law at point B.

$$I_1 + I_2 = I_3$$

Now, mesh ABCDA, the voltage drop across  $R_1$  is

$$E_1 - V_B$$

$$I_1 = \frac{E_1 - V_B}{R_1}$$

In mesh CBDC, the voltage drop across  $R_3$  is  $E_2 - V_B$

$$I_2 = \frac{E_2 - V_B}{R_3}$$

Also current  $I_3 = \frac{V_B}{R_2}$ . Putting the values of  $I_1, I_2, I_3$  in eqn-1 we get

$$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_2}$$

All quantities except  $V_B$  are known. Hence  $V_B$  can be found out. Once  $V_B$  is known, all branch current can be calculated. It may be seen that nodal analysis required only one equation for determining the branch current in this circuit. However, Kirchhoff's solution would have needed two equations.

### Sources transformation Method:-

A given independent voltage source with a series resistance can be converted into a current into a current source with parallel resistance & vice versa. This type of source is converted to the other type is that to simplify the analysis of the circuit. For example, meth anglish when the circuit contains all the voltage source & nodal contained all current sources.

## Unilateral Elements :-

The elements in which conduction of current in one direction is called as unilateral elements.

Ex:- Diode, triode, etc.

## Bilateral Elements :-

The elements in which conduction of current in both direction is called as bidirectional elements.

Ex:- Resistor; Inductor, capacitor, etc.

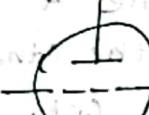
## Linear Circuit :-

A linear circuit is one whose parameters don't change with voltage & current.

## Non-Linear Circuit :-

A linear circuit is one whose parameter change with voltage & current.

Diode



Trilode



Resistor

Capacitor

Inductor.