LECTURE NOTE ON

ENGINEERING MECHANICS

1ST/2ND SEMESTER (ALL BRANCHES)

PREPARED BY:

KEDARNATH JENA

LECT. IN MECHANICAL

GP JAJPUR

FUNDAMENTALS OF ENGINEERING MECHANICS

Definitions of Mechanics –

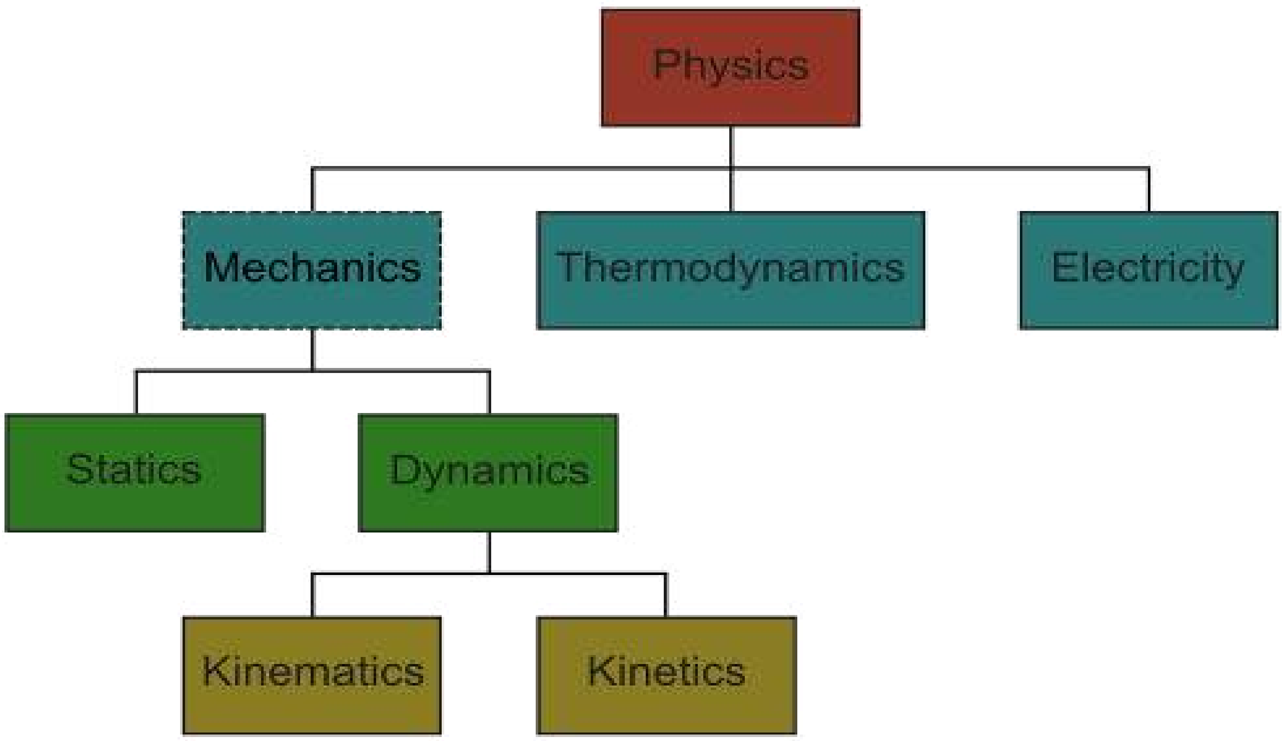
1. A branch of physical science that deals with energy and forces and their effect on bodies.

2. the practical application of **mechanics** to the design, construction, or operation of machines or tools

Definitions of enginnering Mechanics

The subject engineering mechanics is the branch of applied science which deals with the laws and principles of mechanics, along with their applications to engineering problems .

Sub division of Engg. Mechanics



1. Particle: A particle is defined as an object that has a mass but no size.
2. Body: A body is defined as the matter limited in all directions. It has a finite volume and finite mass.
3. Rigid Body: A body in which the particles do not change their relative positions under the action of any external force is called as Rigid Body. No body is perfectly rigid.
4. Deformable Body: A body in which the particles change their position under the action of any external force is called as Deformable body.
5. Mass: Mass of the body is the quantity of matter contained by the body.
6. Weight: The force with which the earth attracts any body to itself is called the weight of the body.

$$\boxed{W = m \cdot g}$$

7. Space: The unlimited universe in which all the materials are located is known as space. It is a three dimensional region.
8. Statics: It is the branch of engineering mechanics which deals with the study of bodies at rest under the action of forces.
9. Dynamics: It is the branch of engineering mechanics which deals with the study of bodies in motion.
10. Kinetics: This branch of dynamics is the study of the behaviour of bodies in motion without considering the forces which causing the motion.
11. Kinematics: The kinematics studies the behaviour of bodies in motion by considering the forces which causing the motion.
12. Force: It is the agent which changes or tends to change the state of rest or motion of a body.

Force

Defination –

Force is an external agent capable of changing the state of rest or motion of a particular body. It has a magnitude and a direction. The direction towards which the force is applied is known as the direction of the force and the application of force is the point where force is applied.

The Force can be measured using a spring balance. The SI unit of force is Newton(N).

Common symbols:	$\vec{F}$, F
SI unit:	Newton
In SI base units:	$kg \cdot m/s^2$

Other units:	dyne, poundal, pound-force, kip, kilo pond
Derivations from other quantities:	F = m a
Dimension:	LMT^{-2}

Classification of force system according to plane & line of action

System of Forces

When two, or more than two, forces act on a body, they are called to form a *system of forces*. Following systems of forces are important from the subject point of view :

1. *Coplaner forces.* The forces, whose lines of action lie on the same plane, are known as coplaner forces.
2. *Collinear forces.* The forces, whose lines of action lie on the same line, are known as collinear forces.
3. *Concurrent forces.* The forces, which meet at one point, are known as concurrent forces. The concurrent forces may or may not be collinear.
4. *Coplaner concurrent forces.* The forces, which meet at one point and their lines of action also lie on the same plane, are known as coplaner concurrent forces.
5. *Coplaner non-concurrent forces.* The forces which do not meet at one point, but their lines of action lie on the same plane, are known as coplaner non-concurrent forces.
6. *Non-coplaner concurrent forces.* The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplaner concurrent forces.
7. *Non-coplaner non-concurrent forces.* The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplaner non-concurrent forces.

Effects of a Force

A force may produce the following effects in a body, on which it acts :

1. It may change the motion of the body, *i.e.* if a body is at rest, the force may set the body in motion, and if the body is already in motion, the force may accelerate it.
2. It may retard the motion of a body.
3. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium. We shall study this effect in chapter 5 of this book.
4. It may give rise to the internal stresses in the body, on which it acts. We shall study this effect in chapters 12 and 13 of this book.

Characteristics of a Force

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force :

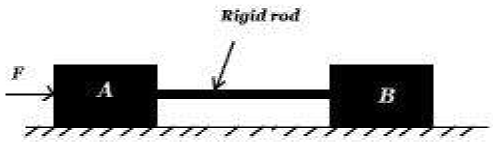
1. Magnitude of the force (*i.e.*, 10 kgf, 20 tf, 50 N, 15 kN, etc.)
2. The direction of the line, along which the force acts (*i.e.* along OX, OY or at 30° North or East etc.). It is also known as line of action of the force.
3. Nature of the force (*i.e.*, whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.
4. The point at which (or through which) the force acts on the body.

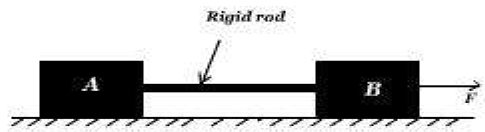
Principle of transmissibility

The state of rest or of motion of a rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the applied forces. In the following animation, two rigid blocks A and B are joined by a rigid rod. If the system is moving on a frictionless surface, the acceleration of the system in both the cases is given by,

Acceleration=Applied force/total mass

It is independent of the point of application

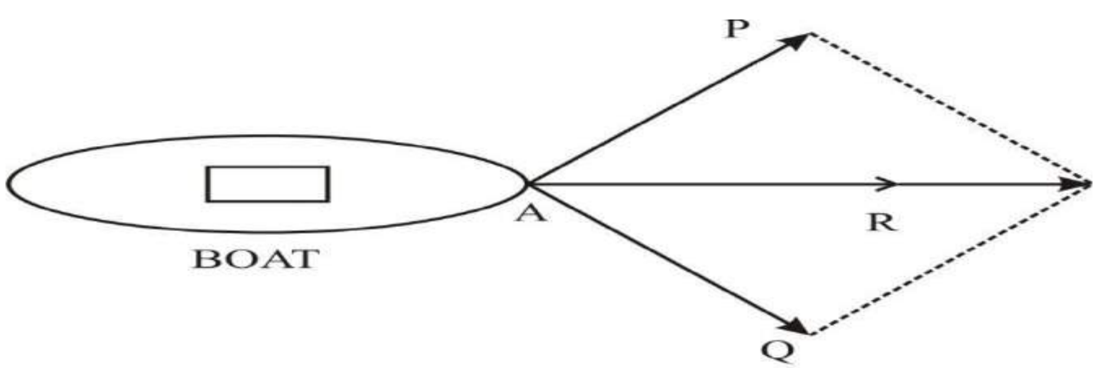




Principle of Superposition

This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces.

Consider two forces *P* and *Q* acting at *A* on a boat as shown in Fig.3.1. Let *R* be the resultant of these two forces *P* and *Q*. According to Newton's second law of motion, the boat will move in the direction of resultant force *R* with acceleration proportional to *R*. The same motion can be obtained when *P* and *Q* are applied simultaneously.



Principle of Superposition

Action & Reaction Forces

1. A force is a push or a pull that acts upon an object as a results of its interaction with another object.

2. Forces result from interactions but some forces result from *contact interactions* (normal, frictional, tensional, and applied forces are examples of contact forces) and other forces are the result of action-at-a-distance interactions (gravitational, electrical, and magnetic forces). According to Newton, whenever objects A and B interact with each other, they exert forces upon each other. When you sit in your chair, your body exerts a downward force on the chair and the chair exerts an upward force on your body. There are two forces resulting from this interaction - a force on the chair and a force on your body. These two forces are called *action* and *reaction* forces and are the subject of Newton's third law of motion. Formally stated, Newton's third law is:

For every action, there is an equal and opposite reaction.

The statement means that in every interaction, there is a pair of forces acting on the two interacting objects. The size of the forces on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object. Forces always come in pairs - equal and opposite action-reaction force pairs.

Concept of Free Body Diagram

Free-body Diagrams. To investigate the equilibrium of a constrained body, we shall always imagine that we remove the supports and replace them by the *reactions* which they exert on the body. Thus,

3.1. Free Body

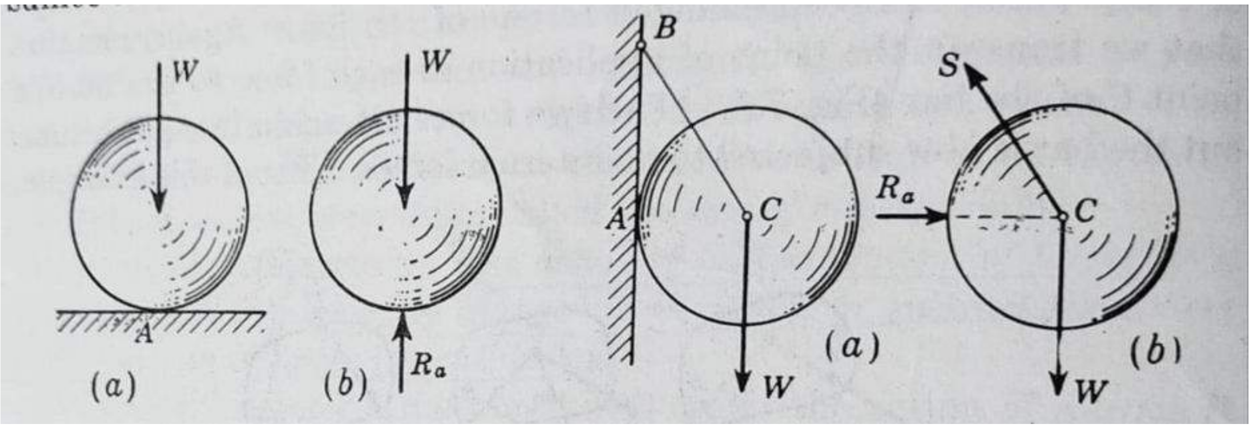
A body is said to be free body if it is isolated from all other connected members.

3.2. Free Body Diagram

Free body diagram of a body is the diagram drawn by showing all the external forces and reactions on the body and by removing the contact surfaces.

Steps to be followed in drawing a free body diagram

1. Isolate the body from all other bodies.
2. Indicate the external forces on the free body.
   (The weight of the body should also be included. It should be applied at the centre of gravity of the body.)
3. The magnitude and direction of the known external forces should be mentioned.
4. The reactions exterted by the supports on the body should be clearly indicated.
5. Clearly mark the dimensions in the free body diagram.



Resolution of a Force

The process of splitting up the given force into a number of components, without changing its effect on the body is called *resolution of a force*. A force is, generally, resolved along two mutually perpendicular directions.

In fact, the resolution of a force is the reverse action of the addition of the component vectors.

2·13. Principle of Resolution

It states, "*The algebraic sum of the resolved parts of a number of forces, in a given direction, is equal to the resolved part of their resultant in the same direction.*"

Proof

Now consider for simplicity, two forces P and Q; which are represented in magnitude and direction by the two adjacent sides OA and OB of a parallelogram $OACB$ as shown in Fig. 2·2.

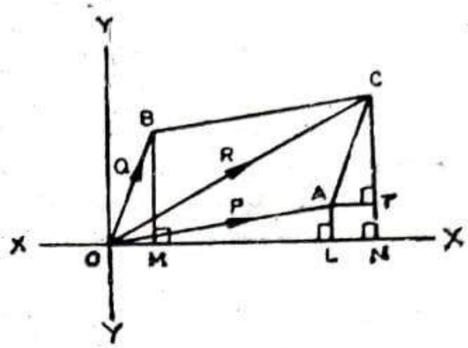


Fig. 2·2 Principle of resolution.

We know that the resultant (R) of these two forces P and Q will be represented, in magnitude and direction, by the diagonal OC of the parallelogram.

Let OX be the given direction, in which the forces are to be resolved. Now draw AL, BM, and CN perpendiculars from the points A, B and C on OX. Similarly, draw AT perpendicular from the point A on CN.

In the two triangles OBM and ACT, the two sides OB and AC are parallel and equal in magnitude. Moreover, the two sides OM and AT are also parallel.

$$\therefore \quad OM = AT = LN$$

Now from the geometry of the figure, we find that

$$ON = OL + LN = OL + OM \quad \ldots (\because \; LN = OM)$$

But ON is the resolved part of the resultant R, OL is the resolved part of the force P, and OM is the resolved part of the force Q.

Hence resolved part of R along OX

= Resolved part of P along OX

+ Resolved part of Q along OX

Note: We have considered, for the sake of simplicity only, the two forces P and Q. But this principle may be extended for any number of forces.

2·14. Method of Resolution for the Resultant Force

The resultant force, of a given system of forces, may be found out by the method of resolution as discussed below :

1. Resolve all the forces vertically and find the algebraic sum of all the vertical components (*i.e.*, ΣV).

2. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (*i.e.*, ΣH).
3. The resultant R of the given forces will be given by the equation :

$$R = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}$$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

Note : The value of the angle θ will vary depending upon the values of ΣV and ΣH as discussed below :

1. When ΣV is +ve, the resultant makes an angle between 0° and 180°. But when ΣV is −ve, the resultant makes an angle between 180° and 360°.
2. When ΣH is +ve, the resultant makes an angle between 0° and 90° and 270° to 360°. But when ΣH is −ve, the resultant makes an angle between 90° to 270°.

Example 2·3. *A triangle ABC has its sides AB = 40 mm along positive x-axis and sides BC = 30 along positive y-axis. Three forces of 40 kgf, 50 kgf and 30 kgf act along the sides AB, BC and CA respectively. Determine the resultant of such a system of forces.*

(*Osmania University, 1985*)

Solution.

The system of the given forces is shown in Fig. 2·3. From the geometry of the figure, we find that the triangle ABC is a right angled triangle in which the *side AC = 50 mm. Moreover,

$$\sin\theta = \frac{30}{50} = 0{\cdot}6$$

and $$\cos\theta = \frac{40}{50} = 0{\cdot}8$$

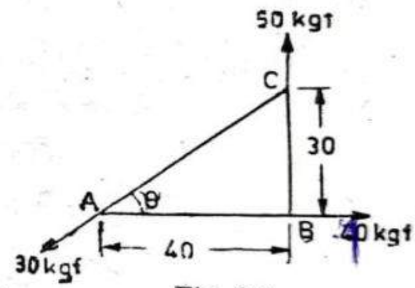


Fig. 2·3

Resolving all the forces horizontally (*i.e.* along AB)

$$\Sigma H = 40 - 30\cos\theta = 40 - 30\times 0{\cdot}8 = 16 \text{ kgf} \quad ...(i)$$

and now resolving all the forces vertically (*i.e.* along BC),

$$\Sigma V = 50 - 30\sin\theta = 50 - 30\times 0{\cdot}6 = 32 \text{ kgf} \quad ...(ii)$$

We know that the magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2} \text{ kgf}$$

$$= \mathbf{35{\cdot}8\ kgf} \quad \textbf{Ans.}$$

Example 2·4. *The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting on one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.* (*Cambridge University*)

Solution.

The system of the given forces is shown in Fig. 2·4.

Magnitude of the resulant force

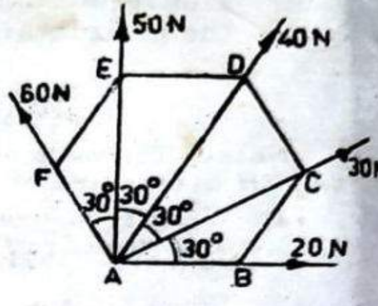


Fig. 2·4

Resolving all the forces horizontally (*i.e.*, along AB),

$$\Sigma H = 20 \cos 0° + 30 \cos 30° + 40 \cos 60° + 50 \cos 90° + 60 \cos 120° \text{ N}$$

$$= (20 \times 1) + (30 \times 0·866) + (40 \times 0·5) + (50 \times 0) + 60(-0·5) \text{ N}$$

$$= 36·0 \text{ N} \quad ...(i)$$

and now resolving the all forces vertically (*i.e.* at right angles to AB)

$$\Sigma V = 20 \sin 0° + 30 \sin 30° + 40 \sin 60° + 50 \sin 90° + 60 \sin 120° \text{ N}$$

$$= (20 \times 0) + (30 \times 0·5) + (40 \times 0·866) + (50 \times 1) + (60 \times 0·866) \text{ N}$$

$$= 151·6 \text{ N} \quad ...(ii)$$

We know that magnitude of the resulant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(36·0)^2 + (151·6)^2} \text{ N}$$

$$= 155·8 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant makes with the horizontal (*i.e.*, AB).

$$\therefore \quad \tan \theta = \frac{\Sigma V}{\Sigma H} = \frac{151·6}{36·0} = 4·211$$

or $\theta = 76° \, 39'$ **Ans.**

Resultant Force

If a number of forces, P, Q, R......etc. are acting simultaneously on a particle, it is possible to find out a single force which could replace them *i.e.* which would produce the same effect as produced by all the given forces. This single force is called *resultant force*, and the given forces P, Q, R......etc. are called component forces.

Composition of Forces

The process of finding out the resultant force of a number of given forces is called *composition of forces* or compounding of forces.

Methods for the Resultant Force

Though there are many methods for finding out the resultant force of a number of given forces, yet the following are important from the subject point of view :

1. Analytical method, 2. Graphical method.

Analytical Method for Resultant Force

The resultant force, of a given system of forces, may be found out analytically by the following methods :

1. Parallelogram law of forces, 2. Method of resolution.

Parallelogram Law of Forces

It states "*If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram ; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection.*" Mathematically, resultant force,

$$R = \sqrt{P^2+Q^2+2PQ \cos \theta}$$

and

$$\tan \alpha = \frac{Q \sin \theta}{P+Q \cos \theta}$$

where P and Q = Forces whose resultant is required to be found out,

θ = Angle between the forces P and Q, and

α = Angle which the resultant force makes with one of the forces (say P).

Note. If the angle (α) which the resultant force makes with the other force Q, then

$$\tan \alpha = \frac{P \sin \theta}{Q+P \cos \theta}$$

Cor.

1. If $\theta = 0$ *i.e.*, when the forces act along the same line, then

$R = P+Q$ ...(*since* $\cos 0° = 1$)

2. If $\theta = 90°$ *i.e.*, when the forces act at right angle, then

$R = \sqrt{P^2+Q^2}$ ...(*since* $\cos 90° = 0$)

3. If $\theta = 180°$ *i.e.*, when the forces act along the same straight line but in *opposite direction* then

$R = P-Q$ ...(*since* $\cos 180° = -1$)

In this case, the resultant force will act in the direction of the greater force.

4. If the two forces are equal *i.e.* when $P = Q$

then $R = \sqrt{P^2+P^2+2P^2 \cos \theta} = \sqrt{2P^2 (1+\cos \theta)}$

$$= \sqrt{2P^2 \times 2 \cos^2 \frac{\theta}{2}} \quad \cdots\left(\because \ 1+\cos \theta = 2 \cos^2 \frac{\theta}{2}\right)$$

$$= \sqrt{4P^2 \cos^2 \frac{\theta}{2}} = 2P \cos \frac{\theta}{2}$$

Example 2·1. *Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.*

Solution

Given : $P = 40$ N ;

$\angle AOC = 120$;

$\angle BOC = 90°$

$\therefore$ $\angle AOB$,

$\alpha = 120° - 90°$

$= 30°$

Let Q = Smaller force.

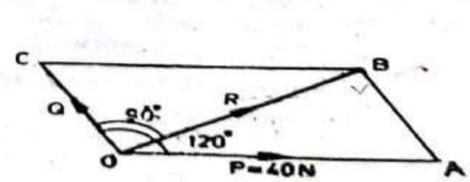


Fig. 2·1

We know that

$$\tan \alpha = \frac{Q \sin \theta}{P+Q \cos \theta}$$

$$\tan 30° = \frac{Q \sin 120°}{40+Q \cos 120°} = \frac{Q \sin 60°}{40+Q\,(-\cos 60°)}$$

$$0{\cdot}577 = \frac{Q \times 0{\cdot}866}{40-Q \times 0{\cdot}5} = \frac{0{\cdot}866\, Q}{40-0{\cdot}5\, Q}$$

$$40-0{\cdot}5\, Q = \frac{0{\cdot}866\, Q}{0{\cdot}577} = 1{\cdot}5\, Q$$

$\therefore \qquad 2Q = 40 \quad$ or $\quad Q = 20$ N **Ans.**

Example 2·2. *Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60°, their resultant is $\sqrt{13}$ N.* (*Bihar University, 1986*)

Solution

Let $\qquad P$ and Q = Two given forces.

First of all, consider the two forces acting at right angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{P^2+Q^2}$$

or $\qquad 10 = P^2+Q^2 \qquad$ ...(Squaring both sides)

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{P^2+Q^2+2PQ \cos 60°}$$

$\therefore \qquad 13 = P^2+Q^2+2PQ \times 0{\cdot}5 \qquad$ ...(Squaring both sides)

$\qquad = 10+PQ \qquad$ ...(Substituting $P^2+Q^2 = 10$)

or $\qquad PQ = 13-10 = 3$

We know that $(P+Q)^2 = P^2+Q^2+2PQ = 10+6 = 16$

$\therefore \qquad P+Q = \sqrt{16} = 4 \qquad$ ...(*i*)

Similarly $\qquad (P-Q)^2 = P^2+Q^2-2PQ = 10-6 = 4$

$\therefore \qquad P-Q = \sqrt{4} = 2 \qquad$ ...(*ii*)

Solving equations (*i*) and (*ii*),

$P = 3$ N and $Q = 1$ N **Ans.**

General Laws for the Resultant Force

The resultant force, of a given system of forces, may also be found out by the following general laws :

1. Triangle law of forces. 2. Polygon law of forces.

Triangle Law of Forces

It states, "*If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.*"

Polygon Law of Forces

It is an extension of Triangle Law of Forces for more than two forces, which states, *"If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order ; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."*

Graphical (Vector) Method for the Resultant Force

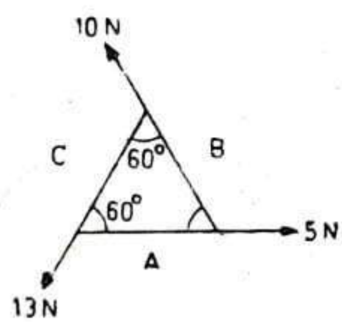
This is another name given to the method of finding out, graphically, magnitude and direction of the resultant force by the polygon law of forces. It is done as discussed below :

1. *Construction of space diagram (position diagram).* It means the construction of a diagram showing the various forces (or loads) alongwith their magnitude and lines of action.
2. *Use of Bow's notations.* All the forces in the space diagram are named by using the Bow's notations. It is a convenient method in which every force (or load) is named by two capital letters, placed on its either side in the space diagram.
3. *Construction of vector diagram (force diagram).* It means the construction of a diagram starting from a convenient point and then go on adding all the forces vectorially one by one (keeping in view the directions of all the forces) to some suitable scale.

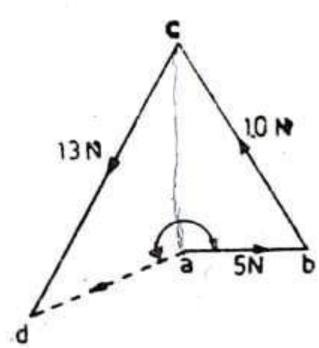
Now the closing side of the polygon, taken in opposite order, will give the magnitude of the resultant force (to the scale) and its direction.

Example 2·7. *A particle is acted upon by three forces equal to 5 N, 10 N and 13 N, along the three sides of an equilateral triangle, taken in order. Find graphically the magnitude and direction of the resultant forces.* (*Madurai University, 1985*)

Solution.



(a) Space diagram



(c) Vector diagram

Fig. 2·7

First of all, draw the space diagram for the given system of forces (acting along the sides of an equilateral triangle) and name the forces according to Bow's notations as shown in Fig. 2·7 (a). The 5 N force is named as *AB*, 10 N force as *BC* and 13 N force as *CD*.

Now draw the vector diagram for the given system of forces as shown in Fig. 2·7 (b) and as discussed below :

1. Select some suitable point *a* and draw *ab* equal to 5 N to some suitable scale and parallel to the force *AB* of the space diagram.
2. Through *b*, draw *bc* equal to 10 N to the scale and parallel to the force *BC* of the space diagram.
3. Similarly, through *c*, draw *cd* equal to 13 N to the scale and parallel to the force *CD* of the space diagram.
4. Join *ad*, which gives the magnitude as well as direction of the resultant force.
5. By measurement, we find the magnitude of the resultant force is equal to 7 N and acting at an angle of 200° with *ab*. **Ans.**

Example 2·8. *The following forces act at a point :*

(*i*) *20 N inclined at 30° towards North of East.*

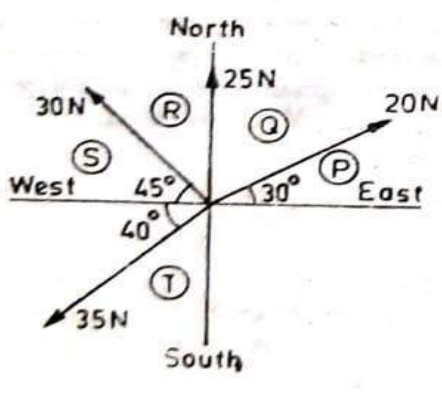
(*ii*) *25 N towards North.*

(*iii*) *30 N towards North West, and*

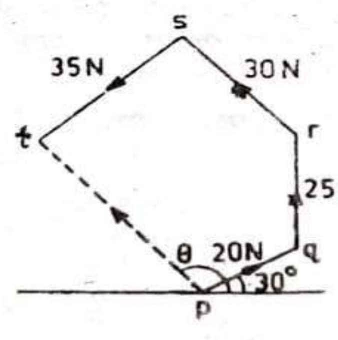
(*iv*) *35 N inclined at 40° towards South of West.*

Find the magnitude and direction of the resultant force.
(*Jiwaji University, 1986*)

***Solution**



(*a*) Space diagram



(*c*) Vector diagram

Fig. 2·8

First of all, draw the space diagram for the given system of forces (acting at point *O*) and name the forces according to Bow's notations as shown in Fig. 2·8 (*a*). The 20 N force is named as *PQ*, the 25 N force as *QR*, 30 N force as *RS* and 35 N force as *ST*.

Now draw the vector diagram for the given system of forces as shown in Fig. 2·8 (*b*) and as discussed below :

1. Select some suitable point *p* and draw *pq* equal to 20 N to some suitable scale and parallel to the force *PQ*.
2. Through *q*, draw *qr* equal to 25 N to the scale and parallel to the force *QR* of the space diagram.
3. Now through *r*, draw *rs* equal to 30 N to the scale and parallel to the force *RS* of the space diagram.
4. Similarly, through *s*, draw *st* equal to 35 N to the scale and parallel to the force *ST* of the space diagram.
5. Join *pt*, which gives the magnitude as well as direction of the resultant force.
6. By measurement, we find that the magnitude of the resultant force is equal to 45·6 N and acting at an angle of 132° with the horizontal *i.e.* East-West line. **Ans.**

2·19. Relation Between Mass and Weight

The term 'mass' is defined as the matter contained in a body, whereas the term 'weight' is defined as the force with which a body is attracted towards the centre of the earth. From the above mentioned two definitions, it is clear that the units of mass are kg, tonnes etc; whereas the units of weight are N, kN and kgf etc.

It will be interesting to know that there is an important relation between the mass and weight of a body, which will be discussed in detail in chapter 23 of this book. But for the time being, it may be taken as

$$P = m.g = 9 \cdot 8\ m \qquad ...(g = 9 \cdot 8\ )$$

where P = Weight of the body in newtons,

m = Mass of the body in kg, and

g = Gravitational acceleration whose value is taken as $9 \cdot 8\ m/sec^2$.

Example 2·9. *A machine shaft BC 1·5 m long and of mass 100 kg is supported by two ropes AB and CD as shown in Fig. 2·9 given below :*

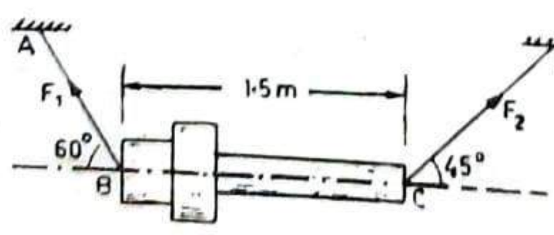


Fig. 2·9

Calculate the tensions F_1 and F_2 in the rope AB and CD.

(*London University*)

Solution. Given : Mass of shaft $= 100$ kg

We know that weight of the mass

$$= m.g = 100 \times 9{\cdot}8 = 980 \text{ N}$$

Resolving the forces horizontally (*i.e.* along *BC*) and equating the same,

$$F_1 \cos 60° = F_2 \cos 45°$$

$$\therefore \quad F_1 = \frac{\cos 45°}{\cos 60°} \times F_2 = \frac{0{\cdot}707}{0{\cdot}5} \times F_2 = 1{\cdot}414\, F_2 \quad ...(i)$$

and now resolving the forces vertically,

$$F_1 \sin 60° + F_2 \sin 45° = 980$$

$$(1{\cdot}414\, F_1)\, 0{\cdot}866 + F_2 \times 0{\cdot}707 = 980$$

$$1{\cdot}93\, F_2 = 980$$

$$\therefore \quad F_2 = 980/1{\cdot}93 = 507{\cdot}8 \text{ N} \quad \textbf{Ans.}$$

and $$F_1 = 1{\cdot}414 \times 507{\cdot}8 = 718 \text{ N} \quad \textbf{Ans.}$$

Moment of a Force

It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required, and the line of action of the force. Mathematically, moment,

$$M = P \times l$$

where P = Force acting on the body, and

l = Perpendicular distance between the point about which the moment is required and the line of action of the force.

Graphical Representation of Moment

Consider a force P represented, in magnitude and direction, by the line AB. Let O be a point, about which the moment of this force is required to be found out, as shown in Fig. 3·1.

From O, draw OC perpendicular to AB. Join OA and OB.

Now moment of the force P about O

$$= P \times OC = AB \times OC$$

But $AB \times OC$ is equal to twice the area of the triangle ABO.

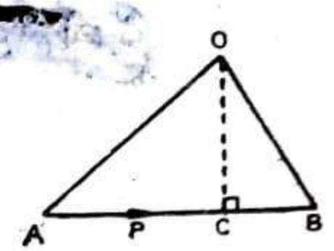


Fig. 3.1
Representation of moment

Thus the moment of a force, about any point, is geometrically equal to twice the area of the triangle, whose base is the line representing the force and whose vertex is the point, about which the moment is taken.

Units of Moment

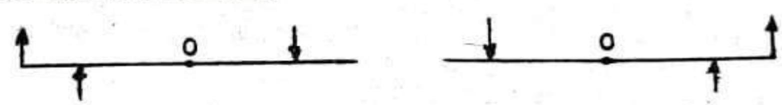
Since the moment, of a force, is the product of force and distance, therefore the units of the moment will depend upon the units of force and distance. Thus, if the force is in Newton and the distance is in metres, therefore the units of moment will be Newton-metre (briefly written as N-m). Similarly, the units of moment may be kN-m (*i.e.* kN × m), N-mm (*i.e.* N × mm) kgf-m (kgf × m) etc

Types of Moments

Broadly speaking, the moments are of the following two types :

1. Clockwise moments. 2. Anticlockwise moments.

Clockwise Moment



(*a*) Clockwise moments (*b*) Anticlockwise moments

Fig. 3·2

It is the moment of a force, whose effect is to turn or rotate the body, in the *same* direction in which the hands of a clock move, as shown in Fig. 3·2 (*a*).

Anticlockwise Moment

It is the moment of a force, whose effect is to turn or rotate the bady, in the *opposite* direction in which the hands of a clock move, as shown in Fig. 3·2 (*b*).

Note. The general convention is to take clockwise moment as possitive and anticlockwise moment as negative.

Varignon's Principle of Moments (or Law of Moments)

It states, "*If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.*"

Example 3·1. *A force of 15 N is applied perpendicular to the edge of a door 0·8 m wide as shown in Fig. 3·4 (a). Find the moment of the force about the hinge.*

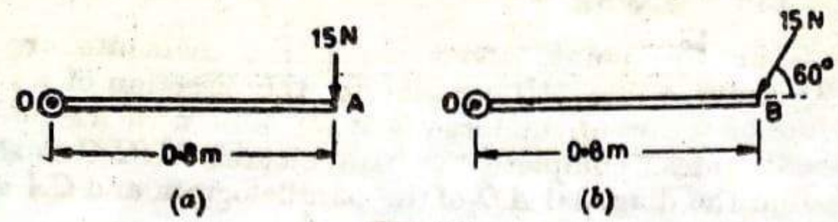


Fig. 3·4

If this force is applied at an angle of 60° to the edge of the same door, as shown in Fig. 3·4 (b), find the moment of this force.

(Gujarat University, 1984)

Solution. Given : $P = 15$ N ; $l = 0{\cdot}8$ m

Moment when the force acts perpendicular to the door

We know that the moment of the force about the hinge,

$$= P \times l = 15 \times 0{\cdot}8 = 12{\cdot}0 \text{ N-m} \qquad \textbf{Ans.}$$

Moment when the force acts at an angle of 60° to the door

This part of the example may be solved either by finding out the perpendicular distance between the hinge and the line of action of the force as shown in Fig. 3·5 (a) or by finding out the vertical component of the force as shown in Fig. 3·4 (b).

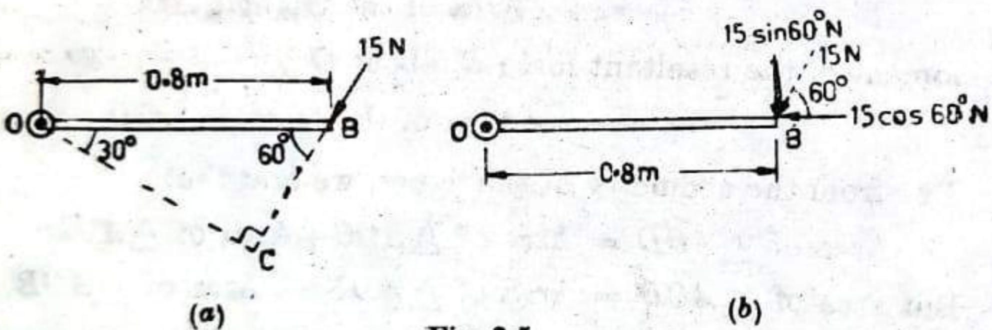


Fig. 3·5

From the geometry of Fig. 3·5 (a), we find that the perpendicular distance between the line of action of the force and hinge,

$$OC = OB \sin 60° = 0{\cdot}8 \times 0{\cdot}866 = 0{\cdot}693 \text{ m}$$

$$\therefore \quad \text{Moment} = 15 \times 0{\cdot}693 = 10{\cdot}4 \text{ N} \qquad \textbf{Ans.}$$

In the second case, we know that the vertical component of the force

$$= 15 \sin 60° = 15 \times 0{\cdot}866 = 13{\cdot}0 \text{ N}$$

$$\therefore \quad \text{Moment} = 13 \times 0{\cdot}8 = 10{\cdot}4 \text{ N} \qquad \textbf{Ans.}$$

Example 3·2. *A uniform plank ABC of weight 30 N and 2 m long is supported at one end A and at a point B 1·4 m from A as shown in Fig. 3·6.*

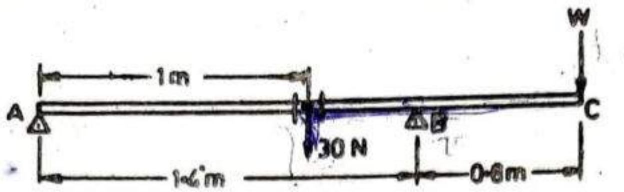


Fig. 3·6

Find the maximum weight W, that can be placed at C, so that the plank does not topple. (*Patna University, 1986*)

Solution. Given : $W = 30$ N ; Length $ABC = 2$ m

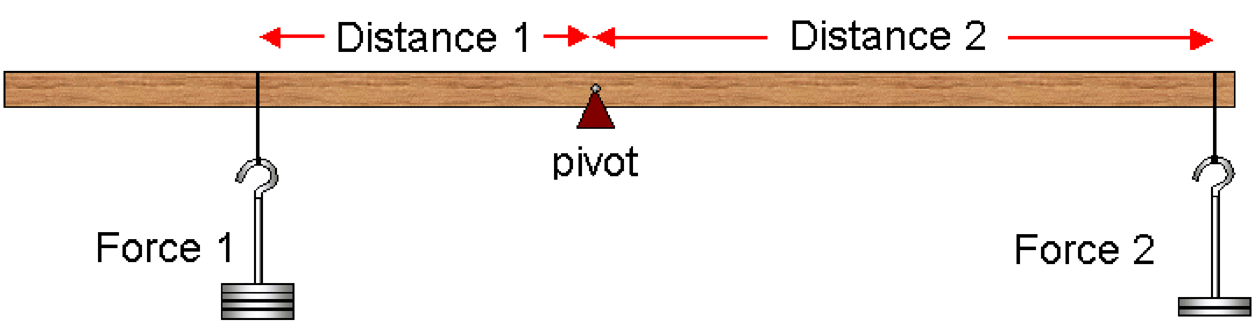
We know that weight of the plank (30 N) will act at its mid-point, as it is of uniform section. This point is at a distance of 1 m from A or 0·4 m from B.

We also know that if the plank is not to topple, then the reaction at A should be zero for the maximum weight at C. Now taking moments about B and equating the same,

$$30 \times 0{\cdot}4 = W \times 0{\cdot}6$$

$$\therefore \quad W = \frac{30 \times 0{\cdot}4}{0{\cdot}6} = 20 \text{ N}$$

Law of moments



When an object is balanced (in equilibrium) the sum of the clockwise moments is equal to the sum of the anticlockwise moments.

Force 1 x its distance from pivot = Force 2 x distance from the pivot

$$F_1 d_1 = F_2 d_2$$

COUPLE

Definition – Couple, in mechanics, pair of equal parallel forces that are opposite in direction. The only effect of a couple is to produce or prevent the turning of a body.

- The turning effect, or moment, of a couple is measured by the product of the magnitude of either force and the perpendicular distance between the action lines of the forces.

Arm of a Couple

The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known as *arm of the couple* as shown in Fig. 4·12.

Moment of a Couple

The moment of a couple is the product of the force (*i.e.* one of the forces of the two equal and opposite parallel forces) and the arm of the couple. Mathematically :

Fig 4·12. Couple

Moment of a couple $= P \times a$

where P = Force, and

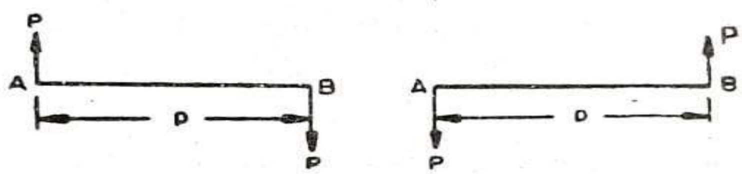
a = Arm of the couple.

Classification of Couples

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which they act :

1. Clockwise couple, and 2. Anticlockwise couple.

Clockwise Couple



(*a*) Clockwise couple (*b*) Anticlockwise couple

Fig. 4·13

A couple, whose tendency is to rotate the body, on which it acts, in a *clockwise direction*, is known as a clockwise couple as shown in Fig. 4·13 (*a*). Such a couple is also called *positive* couple.

Anticlockwise Couple

A couple, whose tendency is to rotate the body, on which it acts, in an *anticlockwise direction*, is known as an anticlockwise couple as shown in Fig. 4·13 (*b*). Such a couple is also called a *negative* couple.

Characteristics of a Couple

A couple (whether clockwise or anticlockwise) has the following characteristics :

1. The algebraic sum of the forces, constituting the couple, is zero.

2. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.

3. A couple cannot be balanced by a single force, but can be balanced only by a couple ; but of opposite sense.

4. Any number of coplaner couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Example 4·6. *A square ABCD has forces acting along its sides as shown in Fig. 4·14. Find the values of P and Q, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 1 m.* *(Allahabad University, 1985)*

Solution. Given : Length of square = 1 m

Values of P and Q

We know that if the system reduces to a couple, the resultant force in horizontal and vertical directions is zero. Therefore resolving the forces horizontally,

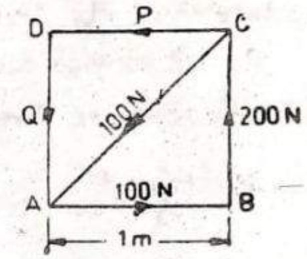


Fig. 4·14

$$100 - 100 \cos 45° - P = 0$$

$$\therefore \quad P = 100 - 100 \cos 45° \text{ N}$$
$$= 100 - 100 \times 0{\cdot}70 \text{ N}$$
$$= 29{\cdot}3 \text{ N} \quad \textbf{Ans.}$$

Now resolving the forces vertically,

$$200 - 100 \sin 45° - Q = 0$$

$$\therefore \quad Q = 200 - 100 \times 0{\cdot}707 = 129{\cdot}3 \text{ N} \quad \textbf{Ans.}$$

Magnitude of the Couple

We know that moment of the couple is equal to the algebraic sum of the moments about any corner. Therefore moment of the couple (taking moments about *A*)

$$= (-200 \times 1) + (-P \times 1) = -200 - 29{\cdot}3 \times 1 \text{ N-m}$$

$$= -229{\cdot}3 \text{ N-m} \quad \textbf{Ans.}$$ ...(Minus sign due to anticlockwise)

CHAPTER-02 EQUILIBRIUM OF FORCES

2.1 If a system of forces acting simulteneously on a body produces no change in the state of rest or the state of motion of the body, the system of forces is said to be in equill^m.

A system of forces can be in equill^m under two situations.

↳* If the resultant of a number of forces acting at a point is zero.

↳ When the resultant of a system of forces applied on a particle has a non-zero value, then the particle will remain at rest by applying a force equal in magnitude but opposite in direc^n of the resultant.

Principles of Equilibrium

Two - force principle

When a body is acted upon by two, equal opposite collinear forces, the resultant force is zero. The system of forces is said to be in equilibrium.

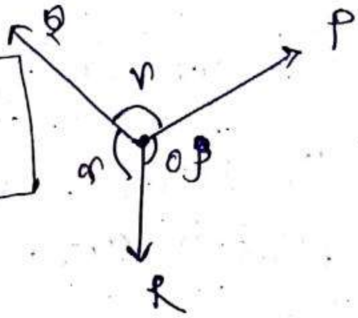
Three Force principle

Three non-parallel forces will be in equili^m when they lie in one plane, intercect at one point and their free vectors form a closed triangle.

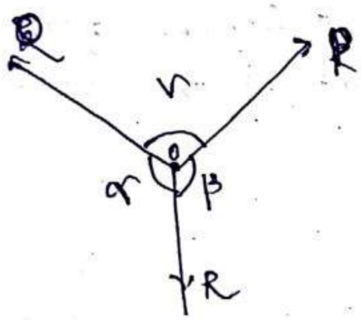
2.2 Lami's Theorem

If three coplanner concurrent forces are acting on a body kept in equilibrium, then each force is proportional to the sine angle between other two forces and the const. of proportionality is the same.

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} = k = \text{const}$$



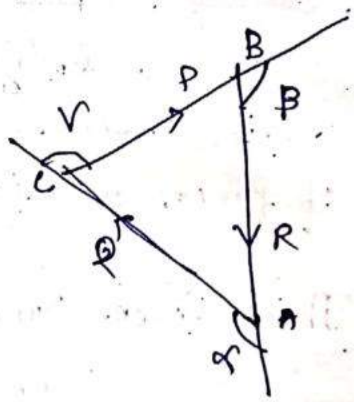
proof



Let force P, Q, R acting at point O.

since P, Q, R are in equilibrium the triangle of forces should be a closed one. (Vector diagram)

Draw a line AB ∥ to force R.
From end A draw a line ∥ to Q. name it AC. From 'C' draw a line ∥ to P. It will intersect the line AB at B.



$\angle A = \pi - \alpha$

$\angle B = \pi - \beta$

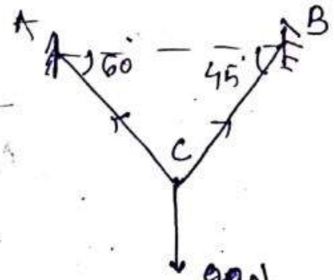
$\angle C = \pi - \gamma$

Applying sine rule to the ΔABC.

$$\frac{P}{\sin(\pi-\alpha)} = \frac{Q}{\sin(\pi-\beta)} = \frac{R}{\sin(\pi-\gamma)}$$

$$\Rightarrow \boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

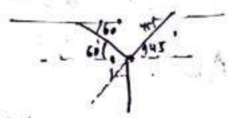
Q) An electric lamp weighing 20N is suspended from a point C supported by 2 wire AC & BC. The point A, B are at same level. AC makes an angle 60° and BC makes 45° to horizontal as shown in fig. Determine the tension in the string AC & BC.



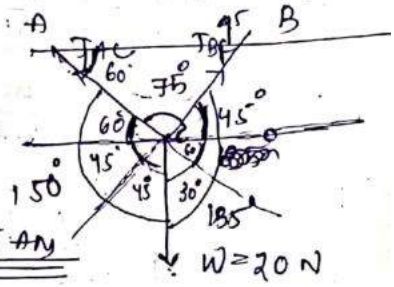
Sol^n W at C = 20

T_{AC} - tension in AC

T_{BC} = " " BC.



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

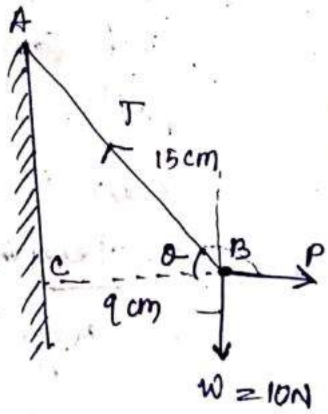


$$\Rightarrow \frac{20}{\sin 75^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{T_{AC}}{\sin 135^\circ}$$

$$T_{AC} = \frac{20 \times \sin 135}{\sin 75} = \frac{14.14}{\sin 75} = 14.95 \text{ Ans}$$

$$T_{BC} = \frac{20 \times \sin 150}{\sin 75^\circ} = \frac{10}{\sin 75^\circ} = \frac{10}{.965} = 10.35$$

Q) Body weighing 10N is suspended from a fixed point by a string 15cm long & is kept at rest by a horizontal force P at a distance of 9 cm from the vertical line drawn through the point of suspension. What are the tension of the string & the value of P?



Sol^n

Let tension T developed in the string AB. The point B is in equil^m, under the three forces 10, T_{AB} & P.

Let ∠ABC = θ

Applying Lami's theorem

$$\frac{P}{\sin(90+\theta)} = \frac{T_{AB}}{\sin 90} = \frac{10}{\sin(180-\theta)}$$

$$\frac{P}{\cos\theta} = \frac{T}{1} = \frac{10}{\sin\theta}$$

From $\triangle ABC$

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow AC^2 = AB^2 - BC^2$$

$$= 15^2 - 9^2$$

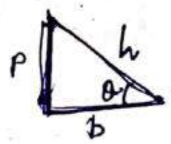
$$= 225 - 81$$

$$AC = \sqrt{144}$$

$$= 12 \text{ Ans}$$

$$\sin\theta = \frac{AC}{AB} = \frac{12}{15} = 0.8$$

$$\cos\theta = \frac{BC}{AB} = \frac{9}{15} = 0.6$$



$\sin\theta = p/h$

$\cos\theta = b/h$

$\tan\theta = \frac{p}{b}$

$$\frac{T}{1} = \frac{P}{0.6} = \frac{10}{0.8}$$

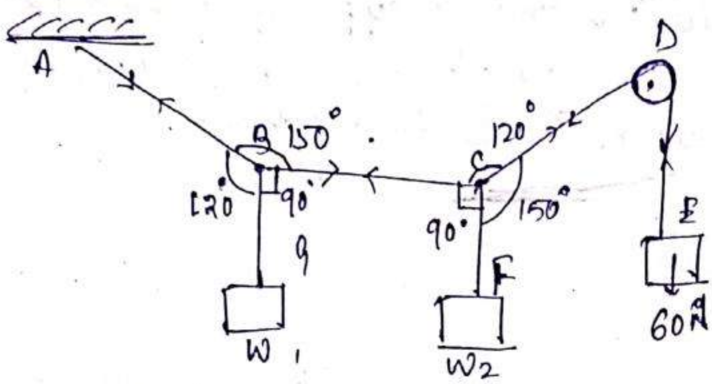
$$\Rightarrow P = \frac{10 \times 0.6}{0.8} = \frac{60}{.8} = 75 N \text{ Ans}$$

$$\Rightarrow T = \frac{10}{0.8} = 12.5 N \text{ Ans}$$

2) A fine light string ABCDE with one end A fixed, has weights W_1 & W_2 attached to it at B and C. The string passes round a smooth pulley D carrying wt 80N at free end E as shown in fig. If the position of equⁿ, BC is horizontal with AB & CD makes an angle 150° & 120° with BC. Find

i) Tension in portion AB, BC, DE.

ii) Magnitude of W_1 & W_2



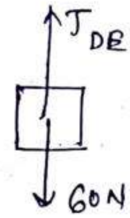
T_{AB} = tension in AB

T_{BC} = " " BC

T_{CD} = " " CD

pulley is smooth no friction $T_{CD} = T_{DE}$

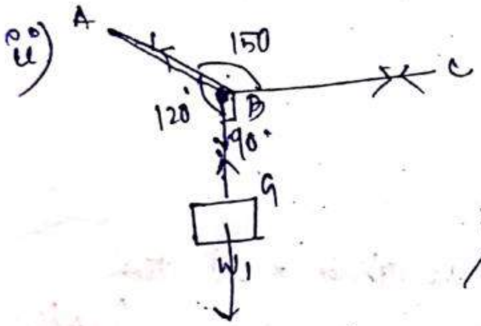
$T_{DE} = 60N = T_{CD}$



Apply lamis theorem at C & B.

known from (i)

(ii)

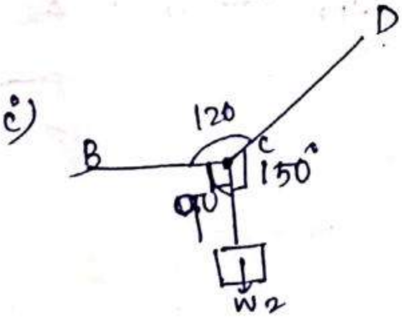


$$\frac{T_{AB}}{\sin 90} = \frac{T_{BC}}{\sin 120} = \frac{W_1}{\sin 150}$$

$$\Rightarrow T_{AB} = \frac{T_{BC} \times \sin 90}{\sin 120} = \frac{30 \times 1}{\sin 120} = 34.63N$$

$$\Rightarrow W_1 = \frac{T_{BC} \times \sin 150}{\sin 120} = \frac{30 \times .5}{.866} = 17.32N$$

(i)

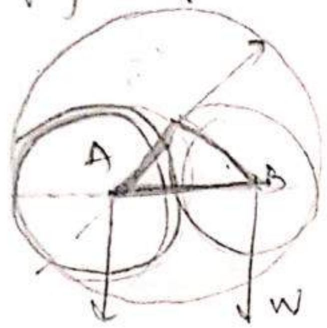


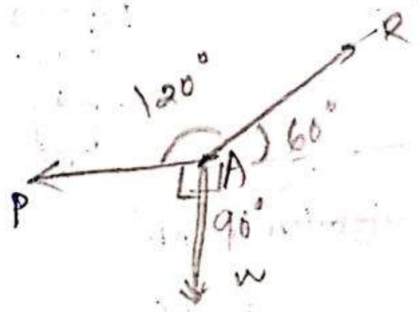
$$\frac{T_{CD}}{\sin 90°} = \frac{T_{BC}}{\sin 150} = \frac{W_2}{\sin 120°}$$

$$\Rightarrow T_{BC} = \frac{T_{CD} \times \sin 150}{\sin 90} = \frac{60 \times .5}{1} = 30N$$

$$\Rightarrow W_2 = \frac{T_{CD} \times \sin 120}{\sin 90°} = 51.96\ N$$

Q) Two equal and heavy spheres of 40 mm radius are in equillm with in a cup of radius 120 mm. Show that the reacn betn the cup & one sphere is double of that betn the two spheres. As shown in the fig





$$\frac{R}{\sin 90°} = \frac{W}{\sin 120} = \frac{P}{\sin 150°}$$

$$\Rightarrow R = \frac{W}{\sqrt{3}/2} = \frac{P}{1/2}$$

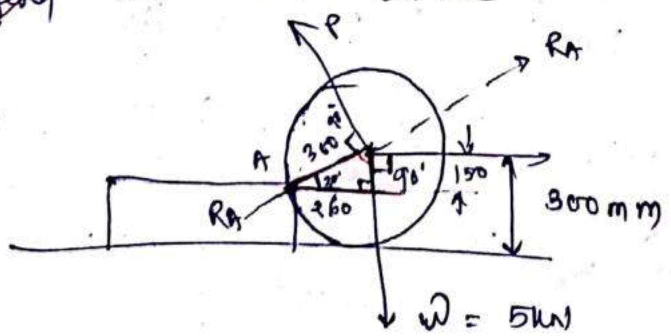
$$\Rightarrow R = \frac{P}{1/2}$$

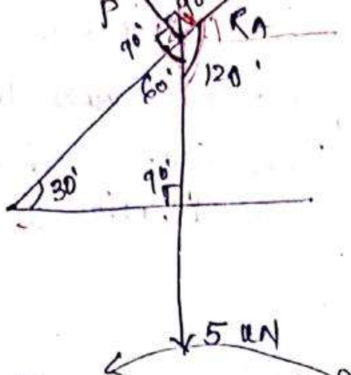
$$\Rightarrow R = 2P$$ ✓ Ans

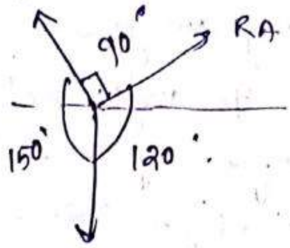
Q) A uniform wheel 600 mm dia weighing 5 kN rest against a rigid rectangular block of 150mm height as shown in the fig. Find the minim force reqd. to turn the wheel over the corner A & find the reactn on the block.

2015 (W)

2016







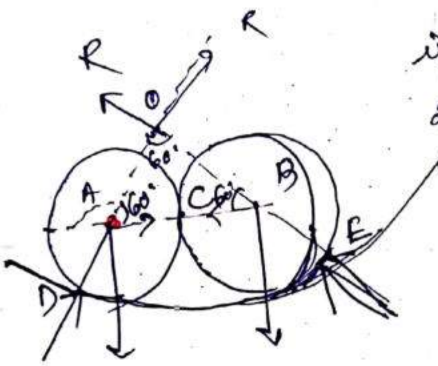
$$\frac{P}{\sin 120} = \frac{R_A}{\sin 150} = \frac{5000}{\sin 90°}$$

$\Rightarrow P = 4330\,N = 4.33\,kN$

$R_A = 2500\,N$

$= 2.5\,kN$

→ Two spheres with center A & B, lying in equilm in cup with center 'O', let the 2 sphere contact at pt C. and sphere A with cup D & sphere B with cup E.



$R \rightarrow$ reaⁿ at D & E

$P \rightarrow$ reaⁿ at C.

From geometry. $OD = 120$ mm $AD = 40$ mm so $AO = 120-40 = 80$.

similarly $OB = 80$. $AB = AC + CB = 40+40 = 80$

OAB becomes equilateral Δ.

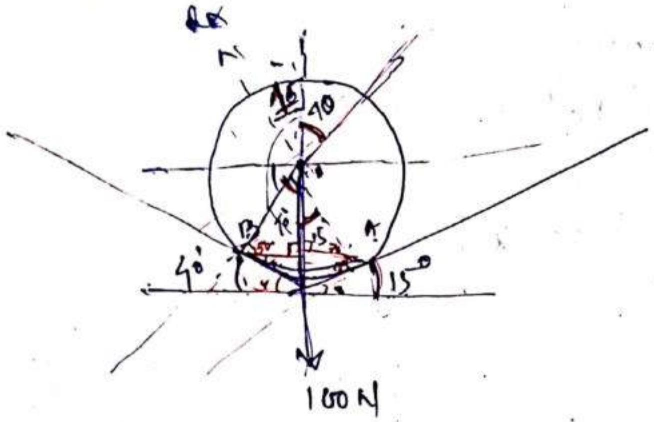
$$\frac{R}{\sin 90} = \frac{W}{\sin 120} = \frac{P}{\sin 150}$$

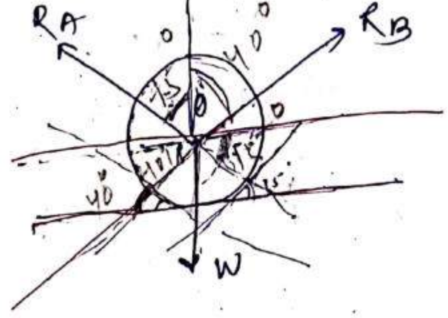
$\Rightarrow R = \frac{W}{\sqrt{3}/2} = \frac{P}{1/2}$

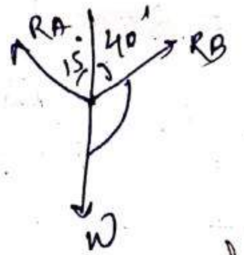
$\Rightarrow R = P/\frac{1}{2}$

$\Rightarrow R = 2P$

2) A smooth circular cylinder of radius 1.5 meter is laying in triangular groove. One side of which makes 15° angle & other 40° angle, with horizontal. Find the reactions at the surface of contact. If there is no friction & the cylinder weighs 100N.







$R_A \rightarrow$ Reaction of A

$R_B \rightarrow$ Reaction of B

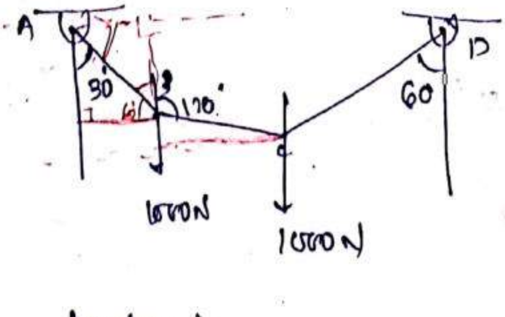
from friction form

$$\frac{R_A}{\sin(180-40)} = \frac{R_B}{\sin(180-15)} = \frac{100}{\sin(15+45')}$$

$R_A = 78.5N$

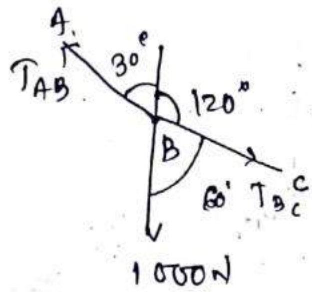
$R_B = 31.6N$

2) A string ABCD attached to fixed points A & D has two equal weights of 1000N attached to B & C. The weights rest with the portions AB & CD inclined angle as shown in fig.



Find the tension in AB, BC & CD

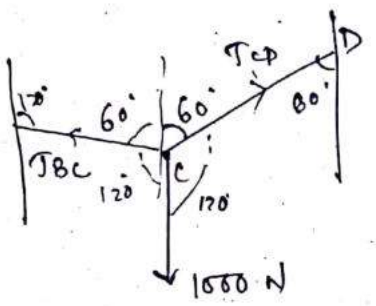
Sol^n Free body diagram.



$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin(180-30)} = \frac{1000}{\sin 150°}$$

$\Rightarrow T_{AB} = 1732\,N$

$\Rightarrow T_{BC} = 1000\,N$

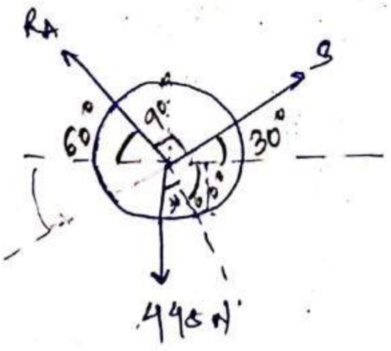


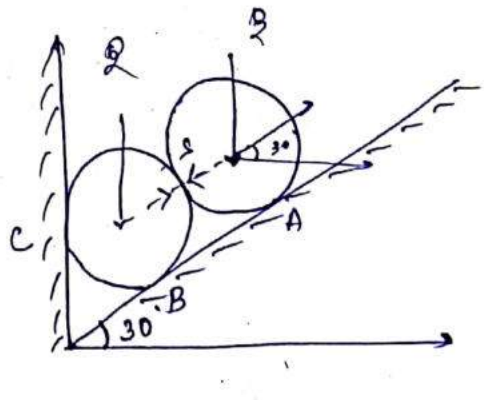
$$\frac{T_{BC}}{\sin 120°} = \frac{T_{CD}}{\sin 120°} = \frac{1000}{\sin 120°}$$

$T_{CD} = 1000\,N$ Ans

Q) Two identical rollers each of weight $Q = 445\,N$ are supported by an inclined plane and a vertical wall as shown in the fig. Assuming smooth surface, Find the reactions induced at pt A, B, C

Sol^n



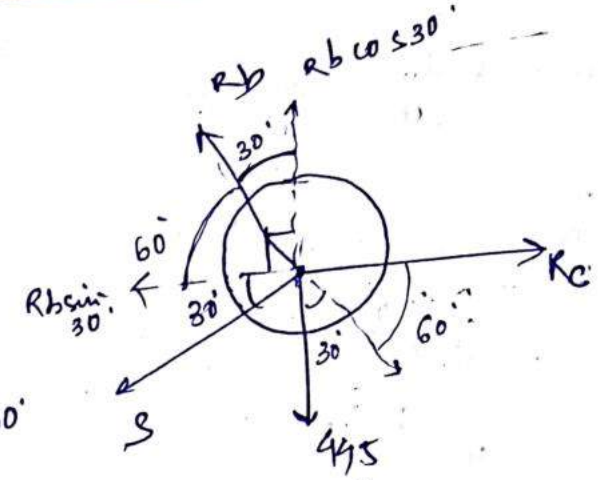


$$\frac{R_a}{\sin 120} = \frac{S}{\sin 150} = \frac{445}{\sin 90°}$$

$\Rightarrow R_a = 385.38\,N$ $S = 225.5\,N$

Resolving vertically

$\Sigma y = 0$



$Rb \cos 30 = 445 + S \sin 30°$

$\Rightarrow$ Rb = ( ) N

Resolving horizontally

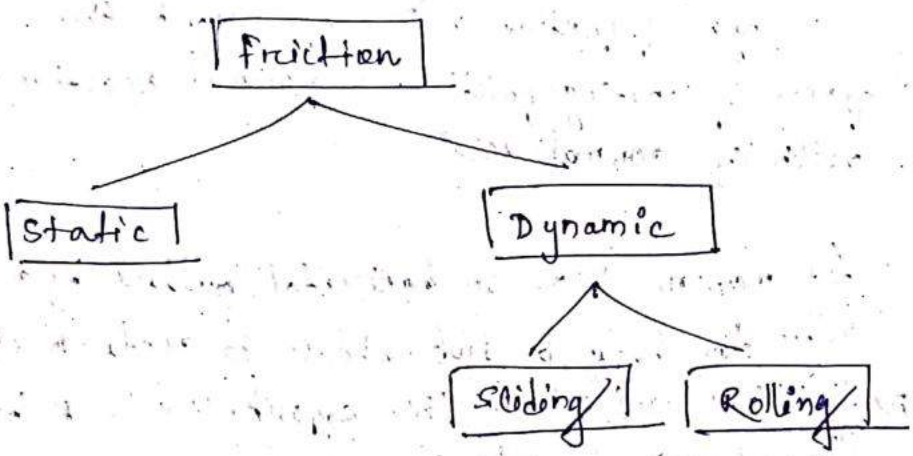
$\Sigma x = 0$

$Rb \sin 30° + S \cos 30° = Rc$

$\Rightarrow$ Rc = ( ) N

CHAPTER → 03 < FRICTION >

3.1 When a body slides or tends to slide over anothere surface an opposing force, called as force of friction. It acts tangent to the surface and oppositer to the direction the body is moving or tends to move,



↳ **Static Friction**

It is experienced by a body when it is at rest or when the body is tends to move.

↳ **Sliding Friction**

It is experienied when a body slids onere arothere body.

↳ **Rolling Friction**

It is experienced when a body rolls over anothere body.

Limiting Friction

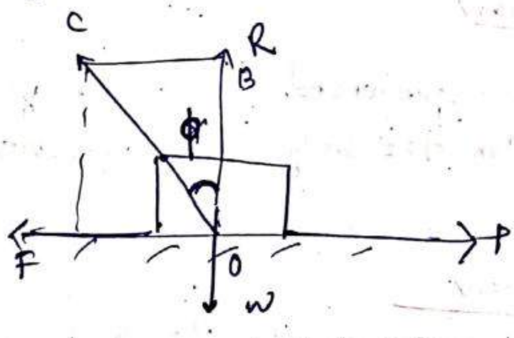
This is the maximum value of frictional force which comes in to play, when a body just begins to slide overe another body, known as limiting friction.

If the applied force is less than the limiting friction, the body remains at rest & the friction is called static friction, which may have any value betn. zero to limiting friction.

Angle of friction

Angle of friction is the angle which the resultant of force of limiting friction & normal reaction makes with the normal reaⁿ.

- Let mass m kept on horizontal, pulled by a force p. When the body is just about to slide a limiting (F) friction will act on the opposite side. R be the normal reaⁿ of wt. w.



Let OC is the resultant betⁿ R & F, makes an angle ϕ with R.

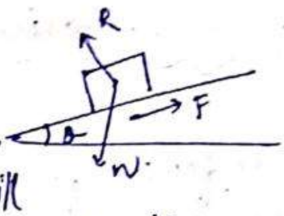
$\Delta OBC \quad \tan\phi = \frac{BC}{BO} = \frac{F}{R}$

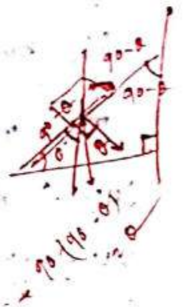
Coefficient of friction

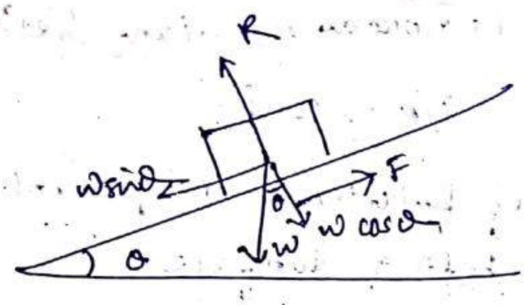
It is the ratio of friction to the normal reaction betⁿ 2 bodies denoted by μ

$$\mu = \frac{F}{R} = \tan\phi \rightarrow \boxed{F = \mu R}$$

Angle of repose

consider the block of weight W resting on an inclined plane which makes an angle θ with horizontal. 
When θ is very small the block will rest on the plane. If θ increases gradually, a stage is reached at which the block will starts to slide. That angle is called as angle of repose.





$\Sigma V = 0$

$R = W\cos\theta$ —— ①

$\Sigma H = 0 \quad F = W\sin\theta$ —— ②

$\frac{W \sin\theta}{W \cos\theta} = \frac{F}{R}$

$\Rightarrow \boxed{\tan\theta = \frac{F}{R}}$

$\therefore \tan\varphi = \tan\theta$

$\Rightarrow \varphi = \theta$

Angle of friction = Angle of repose.

Laws of friction

↳ Laws of static friction

→ The force of friction always act opposite in the direc^n^ of applied force.

→ The magnitude of force of friction is exactly equal to the applied force, which tend to move the body.

→ The magnitude of the limiting friction bears a const ratio to normal reaction bet^n^ the two surface.

$$F/R = \text{const.}$$

→ The force of friction is independent of the area of contact bet^n^ 2 surface.

→ The force of friction depends upon the surface roughness.

↳ Laws of Dynamic friction

→ The force of friction always act in a direction opposite in which the body is moving.

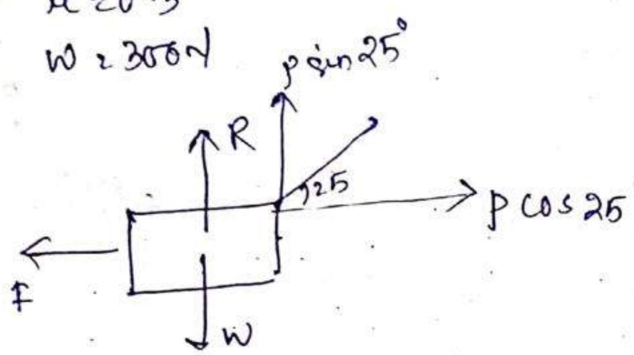
→ For moderate speed the force of friction remains const, but it decreases with increase of the speed.

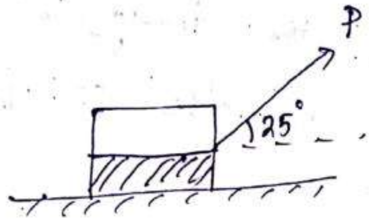
Q) A body of weight 300N is lying on a rough horizontal plane having a co-efficient of friction 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.

Soln

$\mu = 0.3$

$W = 300N$





$\Sigma H = 0 \Rightarrow p\cos 25^\circ = F \Rightarrow F = 0.9063p$

$\Sigma V = 0 \Rightarrow R = W - p\sin 25^\circ$

We know that $F = \mu R$

$\Rightarrow 0.9063p = \mu [W - p \times .4226]$

$\Rightarrow 0.9063p = 0.3[300 - .4226p]$

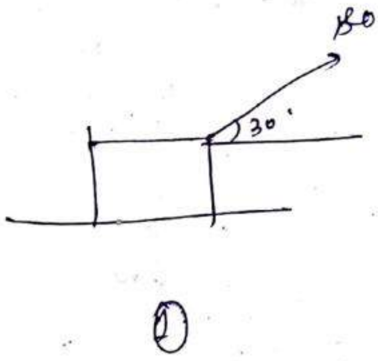
$\Rightarrow 0.9063p = 90 - .1268p$

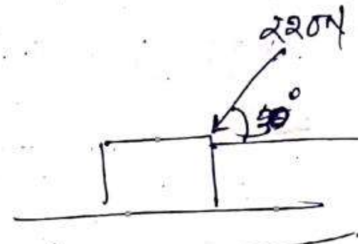
$\Rightarrow p = 87.1$ N. Ans

21/15

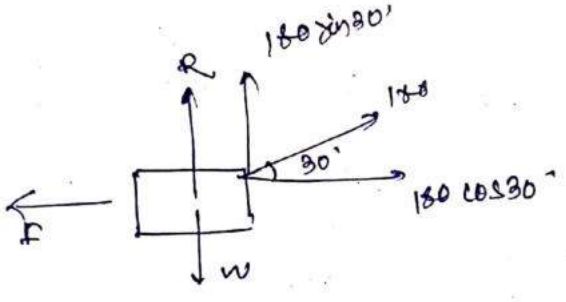
Q) A body resting on a rough horizontal plane requires a pull of 180N inclined at 30°, to the plane to move it. It was found that a push of 220N inclined at 30° to the plane just m[oves] the body. Determine the weight of the body and the co-efficient of friction.

Sol)





FBD of fig 1



$\Sigma H = 0$

$F_1 = 180 \cos 30°$ N

$\Sigma V = 0$

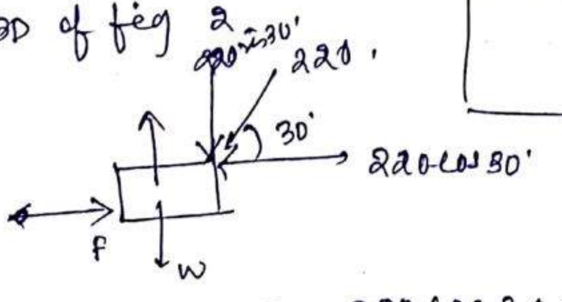
$R = W - 180 \sin 30°$

$\Rightarrow R = W - 90$

$F_1 = \mu R$

$\Rightarrow 155.88 = \mu (W - 90)$ —— (1)

FBD of fig 2



$\Sigma H = 0$

$F_2 = 220 \cos 30°$

$\Rightarrow F_2 = 190.52$ N

$\Sigma V = 0$

$R = W + 220 \sin 30°$

$\Rightarrow R = W + 110$

$F = \mu R$

$\Rightarrow 190.52 = \mu (W + 110)$ —— (2)

~~Adding~~ equ^n (1) & (2)

Substracting

$155.88 = \mu W - 90\mu$

$- \quad 190.52 = \mu W + 110\mu$

$(-) \qquad (-) \quad (-)$

$+34.64 = +200\mu$

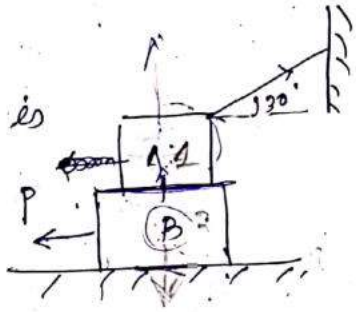
$\Rightarrow \mu = 0.1732$ Ans

puting value of μ in eqn ①

we get $155.88 = 0.1732(W - 90)$

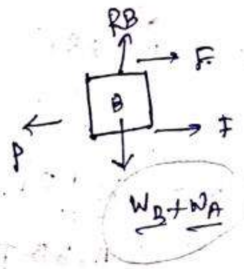
$W = 991.68N$

2) if co-efficient betn the 2 blocks is 0.3. Find force p req to move the block.

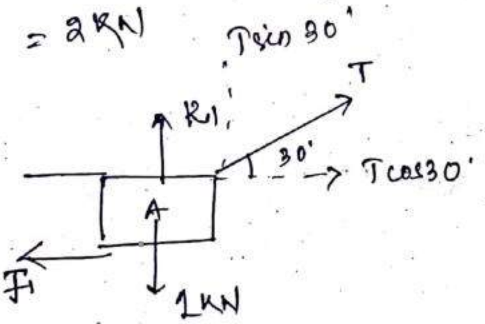


$W_A = 1kN$

$W_B = 2kN$



soln



$R_1 + T\sin 30° = 1kN$ (vertically)

$\Rightarrow T\sin 30° = 1 - R_1$ ——①

Horizontally

$T\cos 30° = F_1$

$\Rightarrow T\cos 30° = \mu R_1$ ——①

$\Rightarrow T\cos 30° = 0.3R_1$

Dividing equn ① & ②

$$\frac{T\sin 30°}{T\cos 30°} = \frac{1-R_1}{0.3R_1} \Rightarrow \tan 30° = \frac{1-R_1}{0.3R_1}$$

$\Rightarrow 0.5774 = \frac{1-R_1}{0.3R_1}$

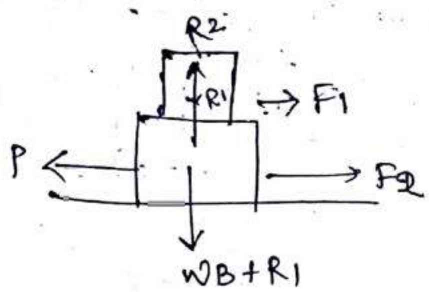
$\Rightarrow 0.5774 \times 0.3R_1 = 1-R_1$

$\Rightarrow 0.173R_1 = 1-R_1$

$\Rightarrow R_1 = 0.85\ kN$

$F = \mu R_1 = 0.3 \times 0.85$

$= .255\ kN$



$R_2 = 2+R_1$

$= 0.85+2 = 2.85\ kN$

$F_2 = \mu R_2$

$= 0.03 \times 2.85 = .855\ kN$

$P = F_1 + F_2$

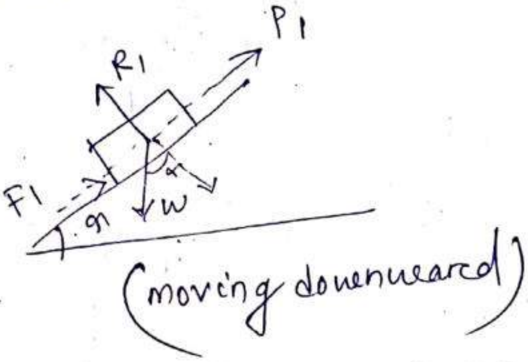
$= .255 + .855$

$= 1.11\ kN$

3.2 Equill^m of a body on Rough Inclined plane

Consider a body laying on a rough inclined plane subjected to force P. as shown in fig

1. Minimum force (P_1) which will keep the body in equill^m when it is sliding down ward.



$F_1 = \mu R_1$

Net horizontal force.

$P_1 = W\sin\alpha - F_1$

$\Rightarrow P_1 = W\sin\alpha - \mu R_1$ —— ①

Net vertical force.

$W\cos\alpha = R_1$ —— ②

putting value of R_1 in equ ① we get

$P_1 = W\sin\alpha - \mu(W\cos\alpha)$

$= W(\sin\alpha - \mu \times \cos\alpha)$

$= W(\sin\alpha - \tan\phi \times \cos\alpha)$ $(\because \mu = \tan\phi$

$= W\left(\sin\alpha - \frac{\sin\phi}{\cos\phi} \times \cos\alpha\right)$ $\because \left(\tan\phi = \frac{\sin\phi}{\cos\phi}\right)$

$\Rightarrow P_1\cos\phi = W(\sin\alpha \times \cos\phi - \sin\phi \times \cos\alpha)$

$\Rightarrow P_1\cos\phi = W\sin(\alpha - \phi)$

$$\Rightarrow P_1 = \frac{W\sin(\alpha - \phi)}{\cos\phi}$$

2. Minimum force (P_1) which will keep the body in equ^m when moving upward.

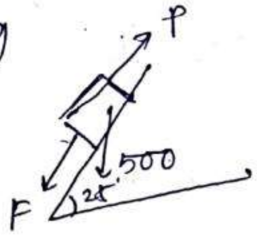
$P_1 = W\sin\alpha + F_1$ —— ①

$R_1 = W\cos\alpha$

$$P_1 = \frac{W\sin(\alpha + \phi)}{\cos\phi}$$

Q) A body of wt 500 N is lying on a rough plane inclined at an angle of 25°. supported by horizontal force P as shown in fig

Soln Determine P for both upward & downward motion.



$$P_1 = \frac{W \sin(\alpha - \varphi)}{\cos \varphi} = 46.4 \text{ N}$$

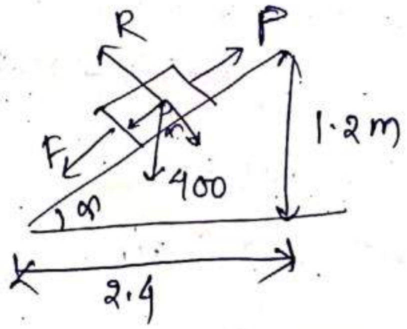
$$P_2 = \frac{W \sin(\alpha + \varphi)}{\cos \varphi} = 376.2 \text{ N}$$

Q) An inclined plane as shown in fig is used to unload a body of wt 400N. from a height 1.2 m. $\mu = 0.3$. (State weather it is necessary to push the body down the plane or hold it back from sliding down. What minim force is req. parallel for this purpose) Find (P) —

Soln $\tan \alpha = \frac{1.2}{2.4} = 0.5$

$\alpha = 26.5°$

& normal reacn



$R = W \cos \alpha$

$= 400 \times \cos 26.5°$

$= 357.9 \text{ N}$

$F = \mu R$

$W \sin \alpha + \mu R = P$

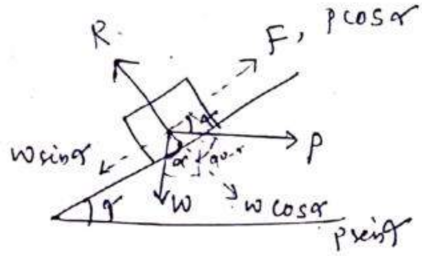
$\Rightarrow P = 400 \times \sin 26.5 + 0.3 \times 357.9$

$=$

Equillibrium of a body on a rough inclined plane subjected to a force acting horizontally

Consider a body lying on a rough inclined plane subjected to a force acting horizontally.

1. Minimum force (P_1) which will keep the body in equil^m, when it is at the point of sliding downward.



$F = \mu R$

$\Sigma H = 0$

$P\cos\alpha + F = W\sin\alpha$

$\Rightarrow P\cos\alpha = W\sin\alpha - F$

$\Rightarrow P\cos\alpha = W\sin\alpha - \mu R$ — ① ($\because F = \mu R$)

$\Sigma V = 0$

$R = W\cos\alpha + P\sin\alpha$ — ②

putting the value of R in eq^n ①

$P_1\cos\alpha = W\sin\alpha - \mu(W\cos\alpha + P_1\sin\alpha)$

$\Rightarrow P_1\cos\alpha + \mu P_1\sin\alpha = W\sin\alpha - \mu W\cos\alpha$

$\Rightarrow P_1(\cos\alpha + \mu\sin\alpha) = W(\sin\alpha - \mu\cos\alpha)$

put $\mu = \tan\phi$

$$\Rightarrow P_1 = W\frac{(\sin\alpha - \mu\cos\alpha)}{\cos\alpha + \mu\sin\alpha}$$

$$= \frac{W(\sin\alpha - \tan\phi.\cos\alpha)}{(\cos\alpha + \tan\phi\cdot\sin\alpha)}$$

$$= W\frac{\left(\sin\alpha - \frac{\sin\phi}{\cos\phi}.\cos\alpha\right)}{\left(\cos\alpha + \frac{\sin\phi}{\cos\phi}\cdot\sin\alpha\right)}$$

$$= W\left(\frac{\sin\alpha\cdot\cos\phi - \sin\phi\cdot\cos\alpha}{(\cos\alpha\cdot\cos\phi + \sin\phi\cdot\sin\alpha)}\right)$$

$$P_1 = W \frac{\sin(\alpha - \phi)}{\cos(\alpha - \phi)}$$

~~$\Rightarrow P_1 = W \tan\alpha\phi$~~

$$\Rightarrow \boxed{P_1 = W \tan(\alpha - \phi)}$$

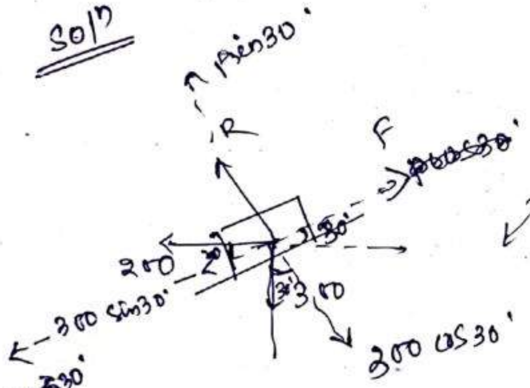
ii) Maximum force (P_1), when the body is moving upward.

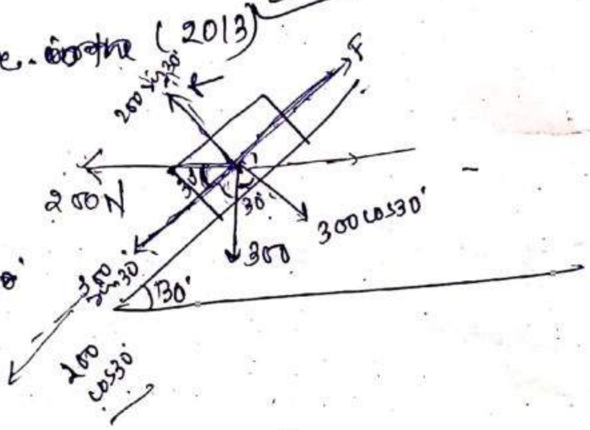
$$P_1 = W \frac{\sin(\alpha + \phi)}{\cos(\alpha + \phi)}$$

$$\Rightarrow \boxed{P_1 = W \tan(\alpha + \phi)}$$

Q) Find the total force. (2013) e.e. ?

Sol^n





$\Sigma V = 0$

$300 \cos 30° = R + 200 \sin 30°$

$\Rightarrow R = 300 \cos 30° - 200 \sin 30°$

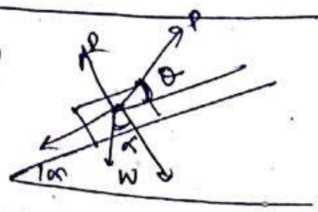
$= (\quad)$

$\Sigma H = 0.$

$200 \cos 30° + 300 \sin 30° = F$

$\Rightarrow \mu R = 200 \cos 30° + 300 \sin 30°$

Minimum force (P_1), keep the body in equi^m when sliding downward



$$P_1 = W \frac{\sin(\alpha - \phi)}{\cos(\theta + \phi)}$$

$$P_{max} = P_2 = \frac{W \sin(\alpha + \phi)}{\cos(\theta - \phi)}$$

$$P_1 = W \frac{\sin(\alpha-\varphi)}{\cos(\alpha-\varphi)}$$

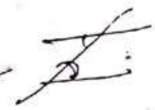
~~$\Rightarrow P_1 = ...$~~

$$\Rightarrow \boxed{P_1 = W \tan(\alpha-\varphi)}$$

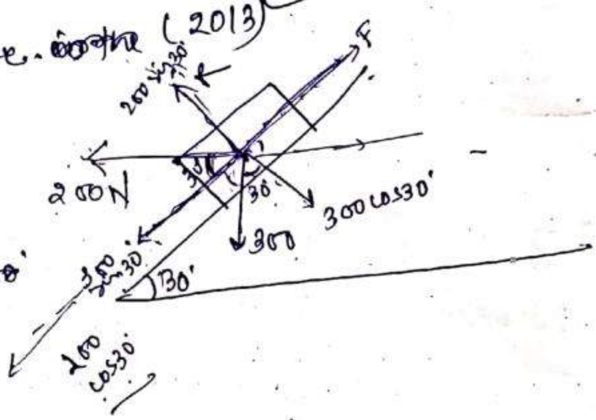
ii) Maximum force (P_1), when the body is moving upward.

$$P_1 = W \frac{\sin(\alpha+\varphi)}{\cos(\alpha+\varphi)}$$

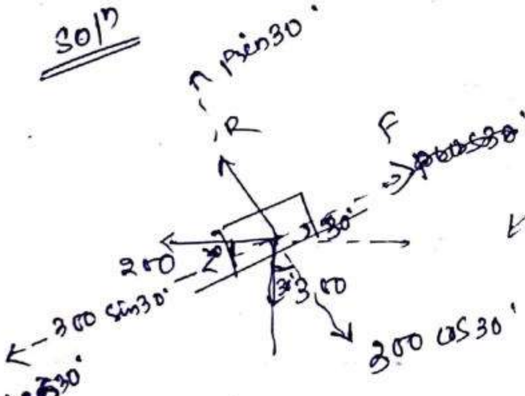
$$\Rightarrow \boxed{P_1 = W \tan(\alpha+\varphi)}$$



2) Find the total force ~~on the~~ (2013) μ ?



Sol^n



$\Sigma V = 0$

$300 \cos 30° = R + 200 \sin 30°$

$\Rightarrow R = 300 \cos 30° - 200 \sin 30°$

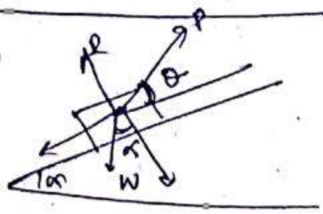
$= (\quad)$

$\Sigma H = 0$

$200 \cos 30° + 300 \sin 30° = F$

$\Rightarrow \mu R = 200 \cos 30° + 300 \sin 30°$

Minimum force (P_1), keep the body in equi^{llm} when sliding downward



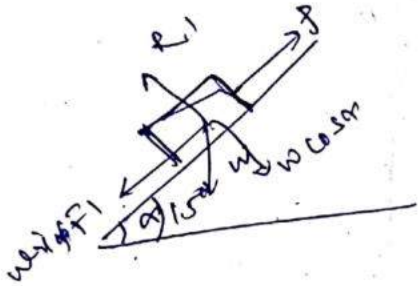
$$P_1 = W \frac{\sin(\alpha-\varphi)}{\cos(\theta+\varphi)}$$

$$P_{max} = P_2 = \frac{W \sin(\alpha+\varphi)}{\cos(\theta-\varphi)}$$

2) An effort of 200N is required just to move certain body up an inclined plane at an angle 15° the forces acting ∥ to plane. If angle of inclin is 20°, then the effort req is found to be 230N. Find weight of the body & μ.

given — $P_1 = 200N$ $P_2 = 230N$

$\alpha = 15°$ $\alpha = 20°$



$\Sigma F_H = 0$

$F_1 + W\sin\alpha_1 = 200$

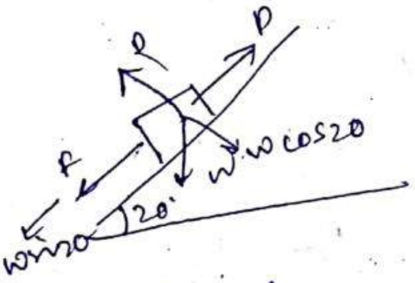
$\Rightarrow \mu R_1 + W\sin 15 = 200$

$\Rightarrow \mu W\cos\alpha + W\sin 15 = 200$

$\Rightarrow W(\mu\cos\alpha + \sin\alpha) = 200$ — ①

$\Sigma F_V = 0$

$R_1 = W\cos\alpha$



$\Sigma F_H = 0$

$P = W\sin 20 + F$

$\Rightarrow \mu R + W\sin 20 = 230$

$\Rightarrow \mu W\cos 20 + W\sin 20 = 230$

$\Rightarrow W(\mu\cos 20 + \sin 20) = 230$ — ②

$\Sigma F_V = 0$

$R_2 = W\cos 20$

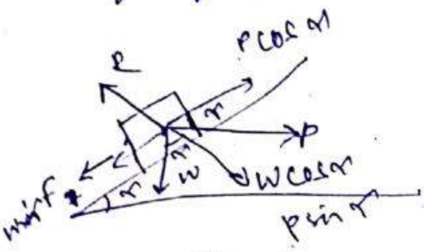
$$\frac{eq^n ②}{eq^n ①} = \frac{\mu\cos 20 + \sin 20}{\mu\cos 15 + \sin 15} = \frac{230}{200}$$

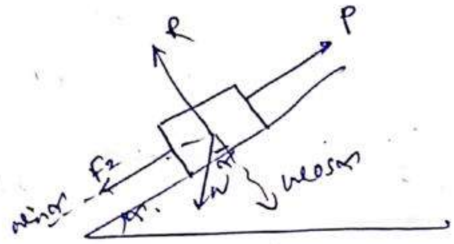
$\Rightarrow \mu = 0.259$

$eq^n ① \rightarrow W(0.259\cos 15 + \sin 15) = 200$

$\Rightarrow W = 392N$ Ans

Q) A load of 1.5kN resting on an inclined rough plane, can be moved up the plane by a force of 2kN applied horizontally & by a force of 1.25 kN applied ∥ to the plane. Find angle of inclination & μ.





$P = W\tan(\alpha + \varphi)$

$\Rightarrow 2 = 1.5\tan(\alpha + \varphi)$

$\Rightarrow \alpha + \varphi = 53.1^\circ$

$P = W\dfrac{\sin(\alpha+\varphi)}{\cos\varphi}$

$\Rightarrow 1.25 = 1.5\dfrac{\sin(53.1)}{\cos\varphi}$

$\Rightarrow \cos\varphi = .96$

$\Rightarrow \varphi = 16.3^\circ$

$\alpha = 53.1 - 16.3^\circ$

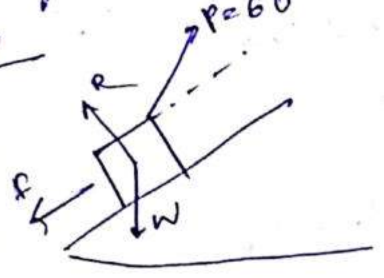
$= 36.8^\circ$

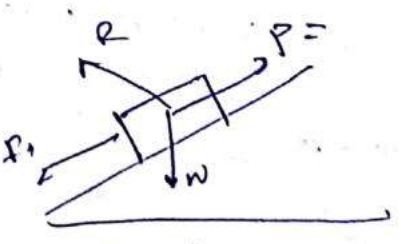
$\mu = \tan\varphi = \tan \times 16.3^\circ$

$= .292$

Q) Find the force req. to move a load 300N up a rough plane the force being ∥ to the plane. The inclination of the plane is such that when the same load is kept on a perfectly smooth plane inclined at same angle, a force 60N applied at an inclination of 30° to the plane, keep the same load in equilm. μ = 0.3.

Sol)





For smooth $\mu = 0 \therefore \varphi = 0$

$P = W\dfrac{\sin(\alpha+\varphi)}{\cos(\theta-\varphi)} \Rightarrow 60 = \dfrac{300\sin\alpha}{\cos 30^\circ} \Rightarrow \alpha = 10^\circ$

For Rough

$P = W\dfrac{\sin(\alpha+\varphi)}{\cos\varphi} \Rightarrow P = 140.7N$

$\mu = 0.3$

$\tan\varphi = 0.3$

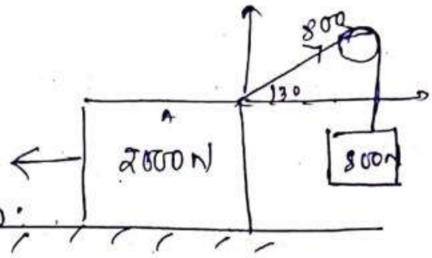
$\varphi = \tan^{-1} 0.3$

$= 16.7$

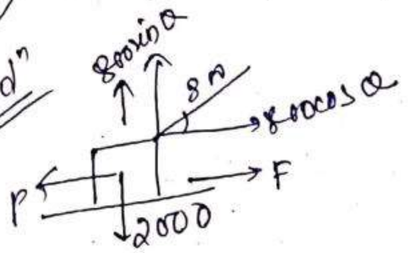
29) (2014) $\mu = 0.35$

Determine value of P.

Consider the pulley is frictionless.



Sol^n



$P = F + 800 \cos 30°$ ⇒ $P = \mu R_n + 800 \cos 30°$

$2000 = R_n + 800 \sin 30°$

⇒ $R_n = 2000 - 800 \sin 30°$

⇒ putting value of R_n.

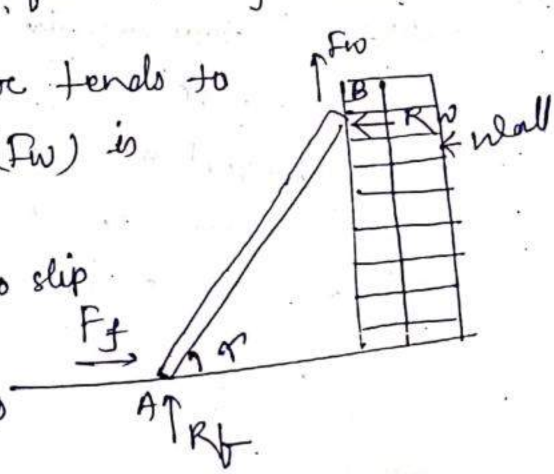
$P = \mu \times (2000 - 800 \sin 30°) + 800 \cos 30°$

$= (1252.82)$ ✓

Application of friction

3.3 LADER FRICTION

A ladder is a device for climbing on walls.



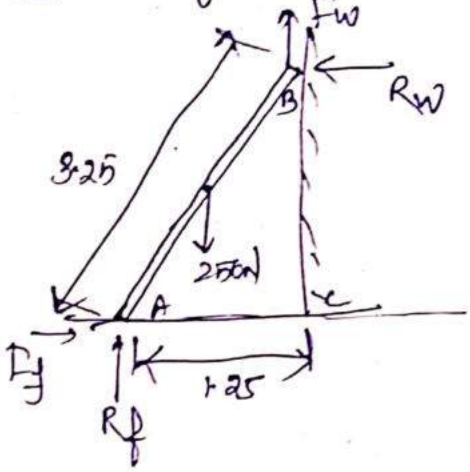
- As upper end of the ladder tends to slip downward, friction (Fw) is upward.
- As the lower end tries to slip away from wall, so friction (Ff) is towards the wall.
- Since the system is in equil^m, therefore the algebraic sum of horizontal & vertical components of the forces must also be equal to zero.

2) 2014 A uniform ladder of length 3.25m and weighing 250 N placed against a smooth vertical wall. It's lower end 1.15 m from the wall. The co-efficient of friction bet^n ladder & floor is 0.3. Determine ~~the~~ ~~total~~ frictional force acting on ladder at point of contact bet^n ladder & floor.

Soln

$\Sigma V = 0$

$R_f = 250 N$



from geometry

$BC^2 = \sqrt{AB^2 - AC^2}$

$= 3.0 m$

Taking moment about 0.

$R_f \times 1.25 - 250 \times \left(\frac{1.25}{2}\right) = F_f \times 3$

$\Rightarrow F_f = 521. N$

2015

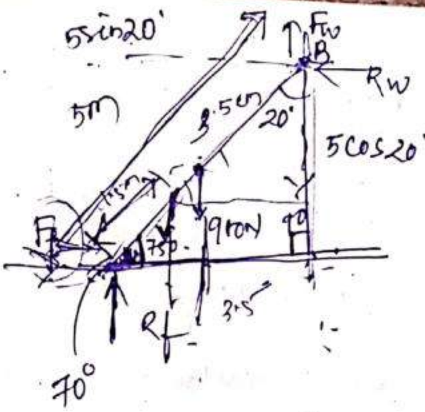
2) A ladder 5 meter long rest on a horizontal ground and leans against a smooth vertical wall at an angle 70° with horizontal. The weight of ladder is 900N and acts at it's middle. The ladder is at the point of sliding, when a man weighing 750N stands on ~~a~~ the ladder 1.5m from bottom. calculate μ..

Sol $l = 5m$

$\alpha = 70^\circ$

$W_1 = 900 N$

$W_2 = 750 N$



$R_f = 900 + 750 = 1650 N$

$F_f = \mu_f \times R_f = \mu_f \times 1650 N$

Taking moment about B — $R_f \times 5 \cos 70 - 900 \times 2.5 \cos 70 - 750 \times 3.5 \cos 70 = F_f \times 5 \sin 70$.

$R_f \times 5 \sin 20^\circ - 900 \times 2.5 \sin 20 - 750 \times 3.5 \sin 20^\circ = F_f \times 5 \cos 20^\circ$

put the value of F_f

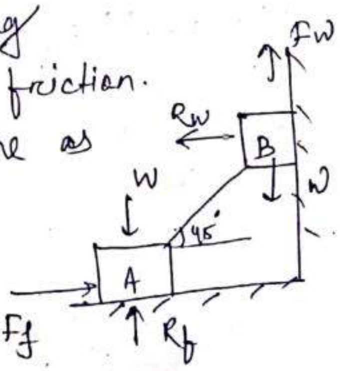
$R_f \times 5 \sin 20 - 900 \times 2.5 \sin 20 - 750 \times 3.5 \sin 20 = \mu_f \times 1650 \times 5 \cos 20^\circ$

$\Rightarrow 1650 \times 5 \sin 20' = (\mu_f \times 1650 \times 5 \cos 20') + 975$

$= 4533 \mu_f + 975$

$\Rightarrow \mu_f = 0.15$ Ans

2) Two identical blocks of weight W are supported by a rod inclined at 45° with horizontal, as shown in fig. If both the blocks are limitting equilibrium, Find the coefficient of friction (μ). assuming it to be same as floor aswellas at wall.



Sol^n Resolving forces vertically.

$$F_w + R_f = 2W$$

$$\Rightarrow \mu R_w + R_f = 2W \quad \text{——} (1)$$

Now resolving the forces horizontally.

$$R_w = F_f$$

$$\Rightarrow R_w = \mu R_f \quad \text{——} (2)$$

Substituting R_w in equ^n (1).

$$\mu(\mu R_f) + R_f = 2W$$

$$\Rightarrow \mu^2 R_f + R_f = 2W$$

$$\Rightarrow R_f = \frac{2W}{(1+\mu^2)} \quad \text{——} (3)$$

putting values of R_f in equ^n (2)

$$R_w = \mu \times \frac{2W}{\mu^2+1}$$

Taking moment of the forces about block A

$$R_w \times l \cos 45^\circ + F_w \times l \cos 45^\circ = W \times l \cos 45^\circ.$$

$$R_w + F_w = W$$

$$\Rightarrow R_w + \mu R_w = W$$

$$\Rightarrow R_w(1+\mu) = W$$

putting value of R_w $\quad \frac{\mu \times 2W}{\mu^2+1}(1+\mu) = W$

$$\Rightarrow 2\mu(1+\mu) = \mu^2+1$$

$$\Rightarrow 2\mu + 2\mu^2 = \mu^2+1$$

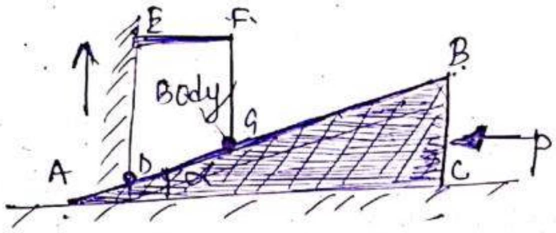
$$\Rightarrow \mu^2 + 2\mu - 1 = 0$$

$$\mu = \frac{-2 \pm \sqrt{(2)^2+4}}{2} = 0.414 \text{ Ans}$$

Scanned by CamScanner

WEDGE FRICTION

A wedge is usually, of a triangular in cross-section & is, generally, used for slight adjustments in the position of a body i.e for tightening fits or keys for shafts. Sometimes, a wedge is also used for lifting heavy weight. It is made of up wood or metal.



Wedge ABC, used to lift the body DEFG.

↑ effect

← P (effort)

W = weight of the body DEFG

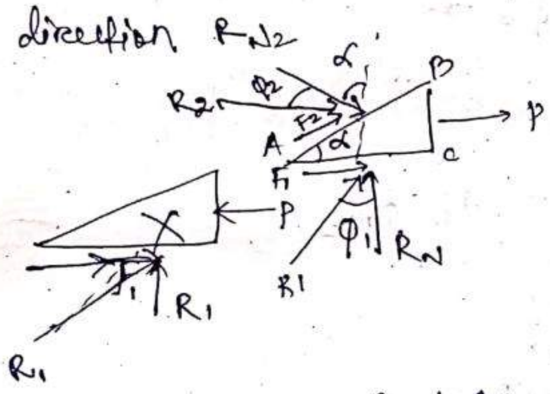
P = Force req. to lift the body

μ = co-efficient of friction = tanφ

due to horizontal movement we get vertical.

W_{wedge} → Not considered.

When force p is applied in, the body will lift in upward direction

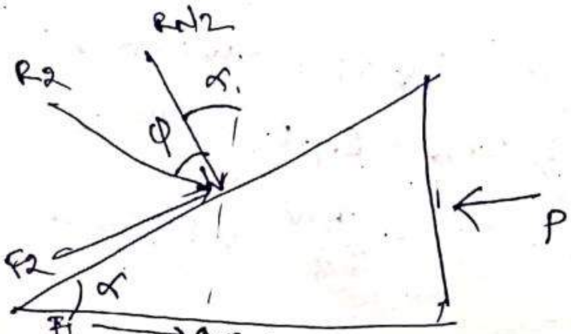


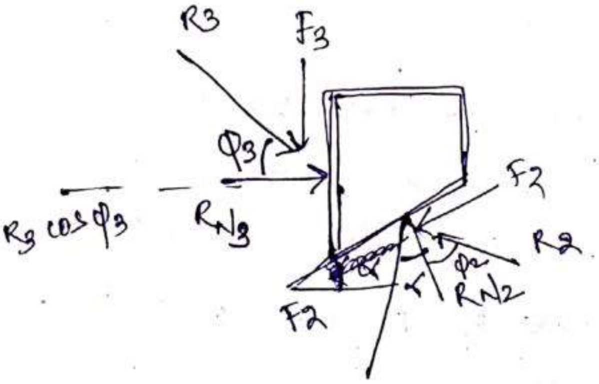
R_1 → resultant of frictional force & normal reaction betn floor & wedge. F_1 & R_N

φ_1 & φ_2 → angle of friction.

R_{N_2} → normal reaction at AC, & frictional force F_2.

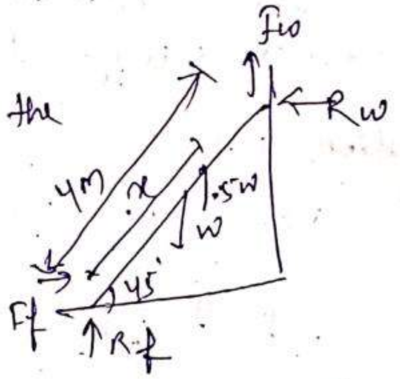
The resultant of both is R_2 making an angle φ_2.





2) A uniform ladder of 4m length rests against a vertical wall with which it makes an angle of 45°. The co-effi of friction betⁿ ladder & wall 0.4 & that betⁿ ladder & floor 0.5. If a man whose weight is one-half of that ladder ascends it. how high it will be when the ladder slips?

Solⁿ x → distance betⁿ A & the man.



weight of man = $\frac{W}{2}$
$= .5W$

$F_f = \mu_f R_f = 0.5 R_f$

$F_w = \mu_w R_w = 0.4 R_w$

$R_w = F_f = 0.5 R_f$

$R_f = 2R_w$

Resolving vertically $R_f + F_w = W + 0.5W$

$\Rightarrow 2R_w + 0.4 R_w = 1.5W$

$\Rightarrow R_w = \frac{1.5W}{2.4} = 0.625W$

$Fw = .4 \times .625W$

$= 0.25W$

Taking moment about A.

$(W \times 2\cos 45° + .5W \times x\cos 45°)$

$= Rw \times 4\sin 45° + Fw \times 4\cos 45°$

put value of Rw & Fw

$x = 3.0\,m.$

CHAPTER → 04 Centre of Gravity

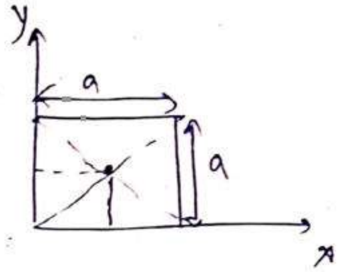
Centre of gravity can be defined as a point through which the whole weight of the body acts, irrespect of it's position. It may be noted that every body has one and only one centre of gravity.

4.1 Centroid

The plane figures like triangle, rectangle, circle etc have only area, but no mass. The centre of area of such fig is known as centroid.

Centroid of basic geometrical figures

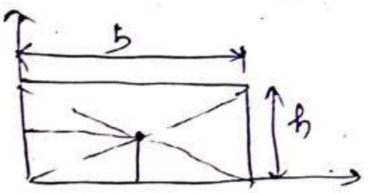
1)



$\bar{x} = a/2$

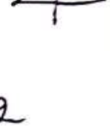
$\bar{y} = a/2$

2)



$\bar{x} = b/2$

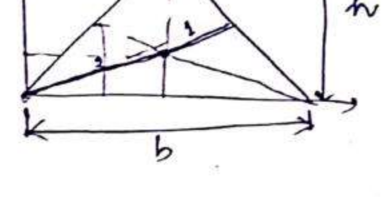
$\bar{y} = h/2$



$x = \frac{2b}{3}$

$y = h/3$

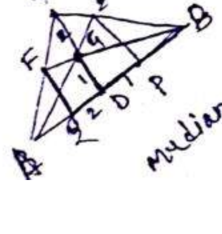
3)



(2:1)

$\bar{x} = b/2$

$\bar{y} = h/3$



median divided into 2:1 ratio.

Draw a line FQ

BFQ & DAGD.

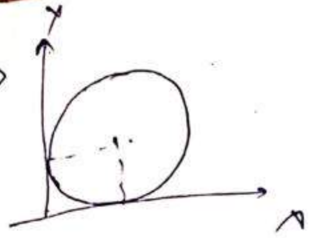
$\frac{AQ}{D} = \frac{AF}{AC}$

D:DB = 1:2

DQ BF & D DBG

$FG/GB = \frac{QD}{DB} = \frac{1}{2}$

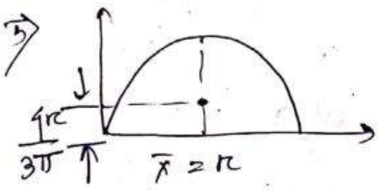
4)



$\bar{x} = r$

$\bar{y} = r$

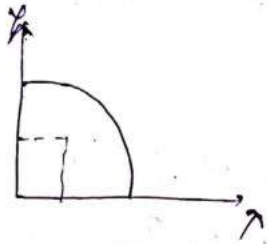
5)



$\bar{x} = r$

$\bar{y} = 4r/3\pi$

6)

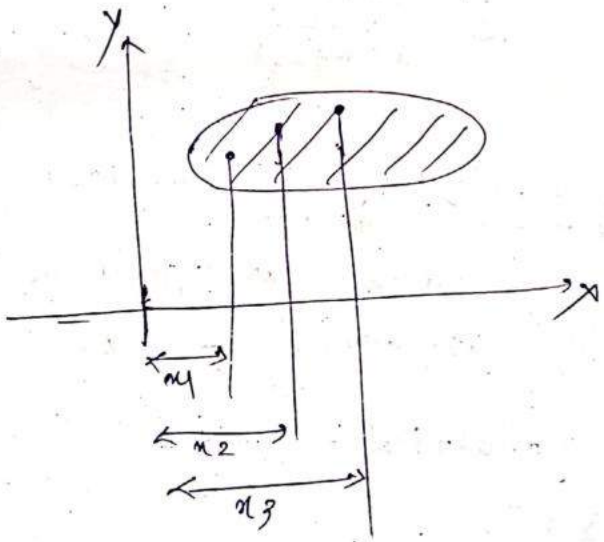


$\bar{x} = 4r/3\pi$

$\bar{y} = 4r/3\pi$

Where $\bar{x}$ & $\bar{y}$ is the co-ordinates of centroid gra...

Centre of gravity by Moments



Consider a body of mass M whose centre of gravity is required to be found out. Let it is divided into small masses m_1, m_2, m_3 .... & the co-ordinates are (x_1, y_1) (x_2, y_2) & (x_3, y_3)

$$M\bar{x} = m_1 x_1 + m_2 x_2 + m_3 x_3 \ldots$$

$$\bar{x} = \frac{\Sigma m x}{M}$$

$$\bar{y} = \frac{\Sigma my}{M}$$

$M = m_1 + m_2 + m_3 + \ldots$

Axis of Reference

The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference, called as axis of reference. from where $\bar{x}$ & $\bar{y}$ is calculated.

Centre of gravity of plane figure

The plane geometrical sections such as T, I, L sections only have area but no mass. For these the centroid & centre of gravity is same.

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}$$

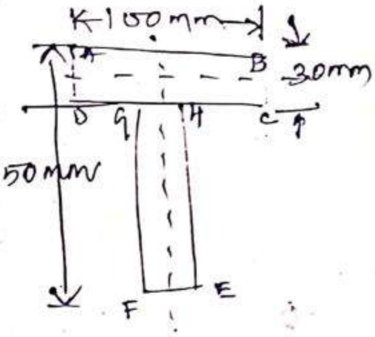
Centre of gravity of Symmetrical Sections

- If the given section is symmetrical about X-X axis then we have to find $\bar{X}$.

- If it is symmetrical to Y-Y axis then we have to find $\bar{x}$ & $\bar{y}$.

2) Find the centre of gravity of 100 mm × 150 mm × 30 mm of T-section.

Sol:- This section of is symmetrical about Y-Y axis.



Split the section in 2 section.

ABCD ; EFGH

For rectangle ABCD.

$$a_1 = 100 \times 30 = 3000 \, mm^2$$

$$y_1 = \left(150 - \frac{30}{2}\right) = 135 \, mm$$

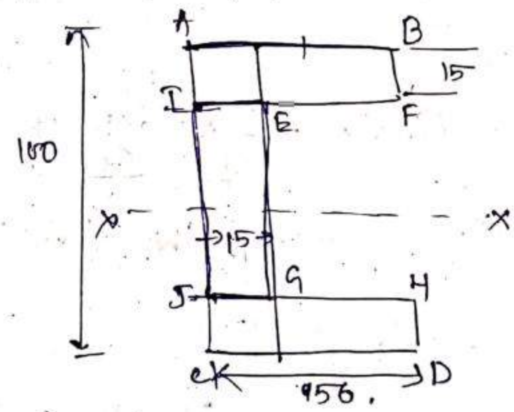
rectangle EFGH $a_2 = (150-30) \times 30 = 120 \times 30 = 3600 \, mm^2$

$$y_2 = 120/2 = 60 \, mm.$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{3000 \times 135 + 3600 \times 60}{3000 + 3600}$$

$$= 94.1 \, mm.$$

2) Symmetrical about X-X axis.



1) Rectangle ABIF.

$a_1 = 15 \times 50 = 750 \, mm^2$

$x_1 = 50/2 = 25 \, mm$

2) Rectangle CDHJ

$a_2 = 50 \times 15 = 750 \, mm^2$

$x_2 = 50/2 = 25 \, mm.$

3) Rectangle IEJG

$a_3 = 15 \times (100 - 30)$

$= 1050 \, mm^2$

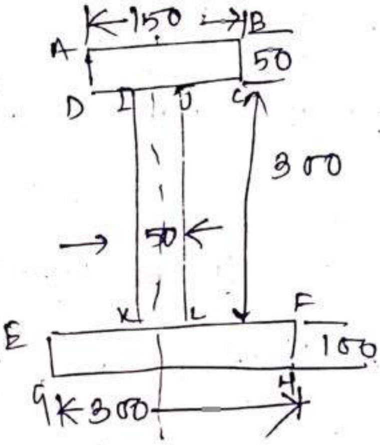
$x_3 = 15/2 = 7.5 \, mm$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{750 \times 25 + 750 \times 25 + (1050 \times 7.5)}{750 + 1050 + 750}$$

$$= 17.8mm$$

Q2)



$a_1 = 150 \times 50$

$y_1 = 100 + 300 + \frac{50}{2}$

$= 400 + 25 = 425mm$

$a_2 = 300 \times 100$

$y_2 = 100/2 = 50mm$

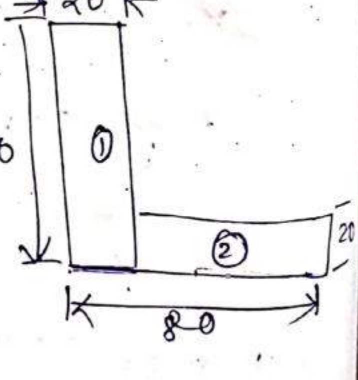
$a_3 = 300 \times 50$

$y_3 = 100 + \frac{300}{2} = 250mm$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

Center of gravity of unsymmetrical section

Q2) Find C.G of the given L section



Rectangle ①

$a_1 = 20 \times 100 = 2000 mm^2$

$y_1 = 100/2 = 50mm$

$x_1 = 20/2 = 10mm$

Rectangle ② $a_2 = (80-20) \times 20 = 1200 mm^2$

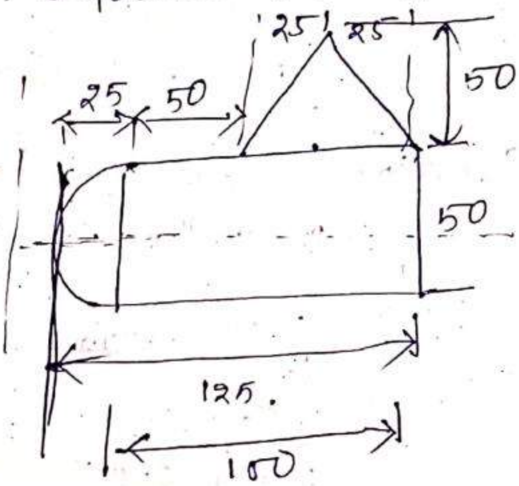
$y_2 = 20/2 = 10mm$ $x_2 = 20 + \frac{(80-20)}{2}$

$= 50 mm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1+a_2} = 25mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = 35mm$$

3) A uniform lamina is shown in fig. Determine the C.G of the lamina.



a) for the rectangle:

$a_1 = 100 \times 50 = 5000\ mm^2$

$x_1 = 25 + 100/2 = 75mm$

$y_1 = 50/2 = 25mm$

for Semicircle: $a_2 = \frac{\pi r^2}{2} = \pi/2\,(25)^2 = 982mm^2$

$x_2 = 25 - 4r/3\pi = 14.4\ mm$

$y_2 = 50/2 = 25mm$

for Δ. $a_3 = \frac{1}{2} \times b \times h = \frac{1}{2}\, 50 \times 50 = 1250mm^2$

$x_3 = 25 + 50 + 25 = 100mm$

$y_3 = 50 + 50/3 = 66.7mm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1+a_2+a_3} = 71.1\ mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1+a_2+a_3} = 32.2mm$$

4.2 MOMENT OF INERTIA

Moment of force = F × ⊥ distance. (1st moment of force.

If F × ⊥r distance × ⊥r distance (2nd moment of force) or moment, moment of force)

M.M.O.F / Second moment of force.

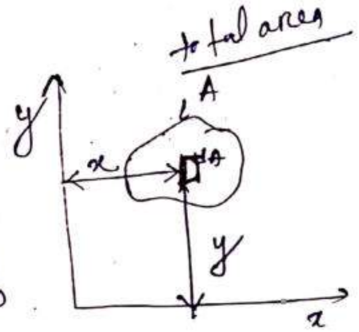
Sometime Area & mass can be found out by above methode.

→ also known as Moment of inertia.

{ M.M.O.A
{ M.M.O.M

$I_{yy} = \Sigma dA \cdot x$ (M.I about yy)

$= \Sigma dA \cdot x \cdot x$



$$I_{yy} = \Sigma dA \cdot x^2$$ — MI about yy axis

$$I_{yy} = \int dA \cdot x^2$$

$$I_{xx} = \int dA \cdot y^2$$ — M.I about xx axis

Moment of inertia = Force × (perpendicular distance)²

unit = N m²

Moment of inertia of a rectangular section.

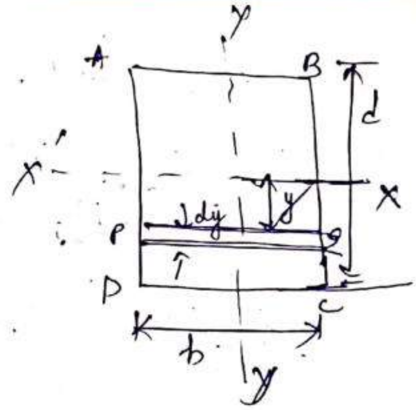
Consider a rectangular section ABCD.

b → Width of the section

d → depth of the section

Consider a small strip PQ of thickness dy ∥ to X-X axis, at a distance y from the centre axis.

Area of small strip = $dA = b \times dy$



M.O.I of strip about x-x axis

$= \text{Area} \times y^2$

$= dA \cdot y^2$

$= b \times dy \cdot y^2$

$$I_{x-x} = \int_{-d/2}^{d/2} dA \cdot y^2$$

$$= \int_{-d/2}^{d/2} b \cdot dy \cdot y^2$$

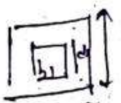
$$= b \int_{-d/2}^{d/2} y^2 \cdot dy = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right]$$

$$= b\left[\frac{d^3/8 - (-d^3/8)}{3}\right]$$

$$= b\left[\frac{2d^3}{8 \times 3}\right]$$

$$I_{xx} = bd^3/12.$$

for hollow  d

$$I_{xx} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$

$$I_{yy} = db^3/12 - d_1 b_1^3/12$$

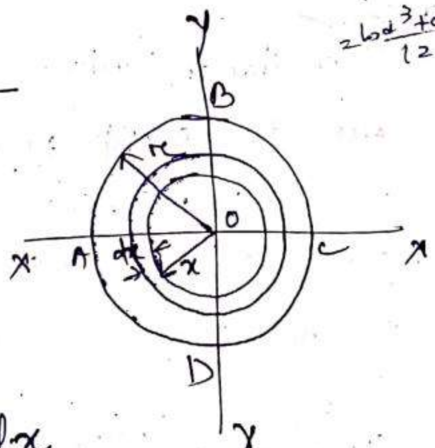
Similarly $I_{yy} = \frac{db^3}{12}$.

I_p $\frac{bd^3}{12} + \frac{db^3}{12}$

$= \frac{bd^3 + db^3}{12}$

M.I of a circular section

- Consider a circle ABCD with centre O.
- Consider a ring of radius x and thickness dx.



area of the ring $da = 2\pi x \cdot dx$

M.O.I about XX axis = area × distance^2

or YY axis $= 2\pi x \cdot dx \times x^2$

$= 2\pi x^3\, dx.$

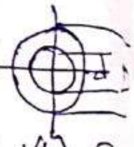
Now M.I about the central axis let it be I_{xx}.

$$I_{zz} = \int_0^r 2\pi r^3 \cdot dx = 2\pi \int_0^r x^3\, dx$$

$$= 2\pi \left[\frac{x^4}{4}\right]_0^r$$

$$= \frac{\pi}{2} \times r^4 = \frac{\pi}{32} d^4 \quad (r = d/2)$$

$$\therefore I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{\pi}{64} d^4$$



for hallow $I_{xx} = \frac{\pi}{64}(D^4 - d^4) = I_{yy}$

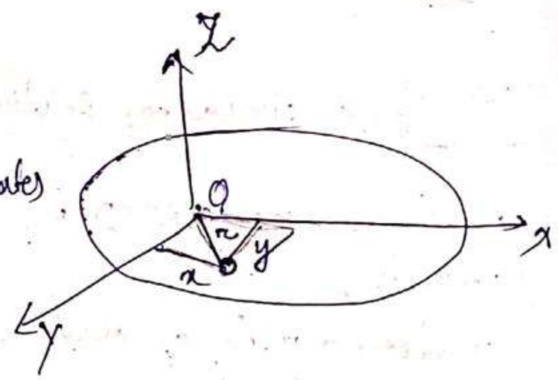
Theorem of perpendicular Axis

It states that if I_{xx} & I_{yy} be the moment of inertia of a plane section about 2 perpendicular axis meeting at O, the moment of inertia about I_{zz} about the zz axis perpendicular to the plane and passing through intersection of x-x & y-y is given by

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Proof

Consider a lamina (p) of area da having co-ordinates x & y along ox & oy axis as shown in fig.



Consider a plane OZ ⊥ to OX & OY. Let r be the distance of lamina p from zz axis. $OP = r$

from geometry $r^2 = x^2 + y^2$

M.I about XX $\quad I_{xx} = da \cdot y^2$

YY $\quad I_{yy} = da \cdot x^2$

$$I_{zz} = da \cdot r^2$$

$$= da(x^2 + y^2)$$

$$= da\,x^2 + da \cdot y^2$$

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Theorem of parallel axes

It states that if the M.I of a plane area about an axis through it's centre of gravity is denoted by I_G, then moment of inertia of the area about any other axis AB, parallel to the 1st, and at a distance h from the C.G is given by

$$\boxed{I_{AB} = I_G + ah^2}$$

$I_{AB} \rightarrow$ M.I of the area about axis AB.

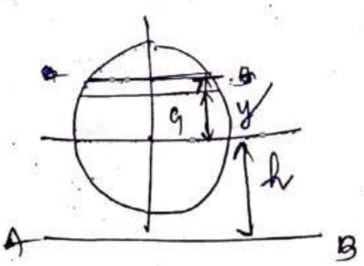
$I_G \rightarrow$ M.I - - - about C.G

$a \rightarrow$ area of section

$h \rightarrow$ distance betn C.G & secn AB.

proof

consider a strip of a circle, where M.I required to be found out



let δa = area of strip

y = distance of strip from C.G.

h = distance of C.G from axis AB

M.I of whole section about an axis passing through C.G $= \delta a \cdot y^2$

$I_G = \Sigma \delta a \cdot y^2$ M.I of whole secn passing through C.G.

M.I of section about AB

$I_{AB} = \sum \delta a (h+y)^2$

$= \sum \delta a (h^2+y^2+2hy)$

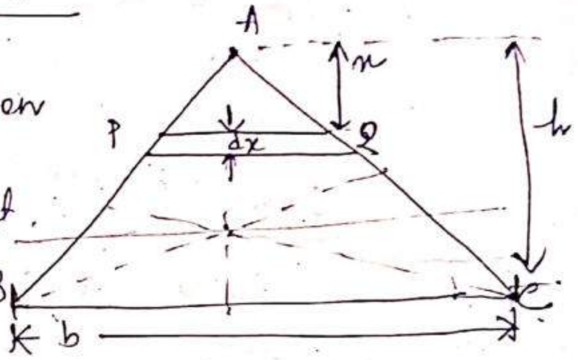
$= (\sum h^2 \delta a) + (\sum y^2 \cdot \delta a) + (\sum 2hy\, \delta a)$ → 0

$$\boxed{I_{AB} = ah^2 + I_G}$$

$\sum h^2 \delta a = ah^2$ sum of moments

$\sum y^2 \delta a = I_G$

M.I of a triangular section



Consider a triangular section ABC whose M.I is required to be found out.

b → base

h → height

(BC = base = b)

Consider a small secⁿ PQ of thickness dx at a distance from vertex A.

for △APQ, △ABC

$\frac{PQ}{BC} = \frac{x}{h}$

$\Rightarrow PQ = \frac{BC \cdot x}{h} = \frac{b \cdot x}{h}$

Small area of PQ $= \frac{b \cdot x}{h} \times dx$

M.I of strip about BC = Area × (distance)^2

$= \frac{bx}{h} \cdot dx \times (h-x)^2$

$= \frac{bx}{h} \cdot (h-x)^2 \cdot dx$

M.I of whole △ can be found out by integrating the above from 0 to h

$$I_{BC} = \int_0^h \frac{bx}{h}(h-x)^2\,dx$$

$$= \frac{b}{h}\int_0^h x\cdot(h^2+x^2-2hx)\,dx$$

$$= \frac{b}{h}\int_0^h (xh^2+x^3-2hx^2)\,dx$$

$$= \frac{b}{h}\left[\frac{x^2h^2}{2}+\frac{x^4}{4}-\frac{2hx^3}{3}\right]_0^h$$

$$= \frac{b}{h}\left[\frac{h^4}{2}+\frac{h^4}{4}-\frac{2h^4}{3}\right] = \frac{b}{h}\left[\frac{2h^4+h^4}{4}-\frac{2h^4}{3}\right]$$

~~$= \frac{b}{h}\left[h^4-\frac{2h^4}{3}\right]$~~

~~$= \frac{b}{h}\left[\frac{3h^4-2h^4}{3}\right] = \frac{b}{h}\left[\frac{h^4}{3}\right] = \frac{bh^3}{3}$~~

$$= \frac{b}{h}\left[\frac{3h^4}{4}-\frac{2h^4}{3}\right] = \frac{b}{h}\left[\frac{9h^4-8h^4}{12}\right] = \frac{bh^3}{12}$$

M.I. of triangular section through axis of the centre of gravity, parallel to X-axis

$$I_G = I_{BC} - ad^2 \qquad d = h/3$$

$$= \frac{bh^3}{12} - \frac{bh}{2}\times\left(\frac{h}{3}\right)^2$$

$$I_{BC} = I_G + ah^2$$

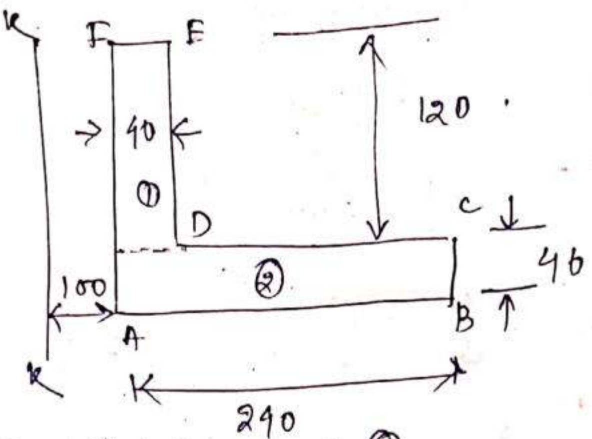
$$\boxed{I_G = \frac{bh^3}{36}}$$

Moment of Inertia of a Composite Section.

Steps

- 1st split up the given section into plane areas.
- Find M.I of these areas about their respective C.G.
- Apply parallel axis theorem.
- Obtain the M.I.

2) Find M.I about axis KK



Split up the secⁿ into ① & ②.

for secⁿ ①. I_{G1} = M.I about c.G about the axis K-K.

$$I_{G1} = \frac{db^3}{12} = \frac{120 \times 40^3}{12} = 640 \times 10^3 \text{ mm}^4$$

$h_1 = 100 + \frac{40}{2} = 120$ mm. (distance betⁿ c.G of secⁿ ① & axis K-K)

M.I of secⁿ ① axis K-K.

$$I_{KK} = I_{G1} + a_1 h_1^2$$
$$= [(640 \times 10^3) + (120 \times 40) \times (120)^2]$$
$$= \underline{69.76 \times 10^6 \text{ mm}^4}$$

Similarly M.I of section ② above it's c.G & parallel to axis K-K.

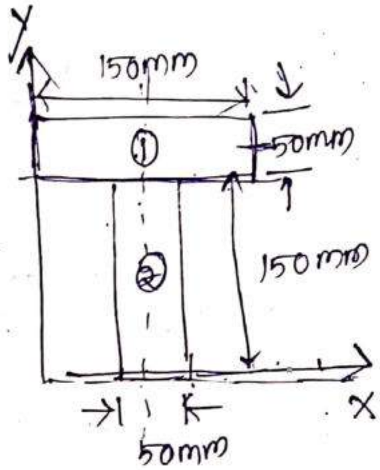
$$I_{G2} = \frac{db^3}{12} = 46.08 \times 10^6 \text{ mm}^4$$

$$h_2 = 100 + \frac{240}{2} = 220 \text{ mm}$$

$$I_{G2} + a_2 h_2^2$$
$$= [(46.08 \times 10^6) + (240 \times 40) \times (220)^2]$$
$$= \underline{510.72 \times 10^6 \text{ mm}^4}$$

$$I_{KK} = 69.76 \times 10^6 + 510.72 \times 10^6$$
$$= 580.48 \times 10^6 \text{ mm}^4$$

Q) Find the M.I of a T-section with as 150 mm × 50 mm and web 150 mm × 50 mm about x-x & y-y axis through the centre of gravity of the section.



Soln Rectangle ①

$a_1 = 150 \times 50 = 7500\ mm^2$

$y_1 = 150 + \frac{50}{2} = 175\ mm$

Rectangle ②

$a_2 = 150 \times 50 = 7500\ mm^2$

$y_2 = \frac{150}{2} = 75\ mm$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500} = 125\ mm$

M.I of ① about X-X axis

$I_{G1} = \frac{db^3}{12} = \frac{150 \times 50^3}{12} = 1.5625 \times 10^6\ mm^4$

$h_1 = \overbrace{150 + \frac{50}{2}}^{y_1} - \underbrace{125}_{\bar{y}} = 50\ mm$

y → distance from C.G

M.I about x-x axis $I_{G1} + a_1 h_1^2$

$= 1.5625 \times 10^6 + 7500 \times (50)^2$

$= 20.3125 \times 10^6\ mm^4$

Similarly M.I of ② about X-X axis

$I_{G2} = \frac{bd^3}{12} = \frac{50 \times (150)^3}{12} = 14.06 \times 10^6\ mm^4$

$h_2 = 125 - \frac{150}{2} = 50\ mm$

M.I about X-X axis $I_{G2} + a_2 h_2^2$

$= 14.06 \times 10^6 + 7500 \times 50^2$

$= 32.8125 \times 10^6\ mm^4$

$I_{XX} = 20.3125 \times 10^6 + 32.8125 \times 10^6$

$= 53.125 \times 10^6\ mm^4$ Ans

Moment about yy axis of ...

$$I_{G_1} = \frac{db^3}{12} = \frac{50 \times 150^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

$$I_{G_2} = \frac{db^3}{12} = \frac{150 \times 50^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

From y axis the distance is zero.

M.I about Y-Y axis ①.

$$I_{G_1} + a_1 h_1^2 \,(\rightarrow 0) = 14.0625 \times 10^6 \text{ mm}^4$$

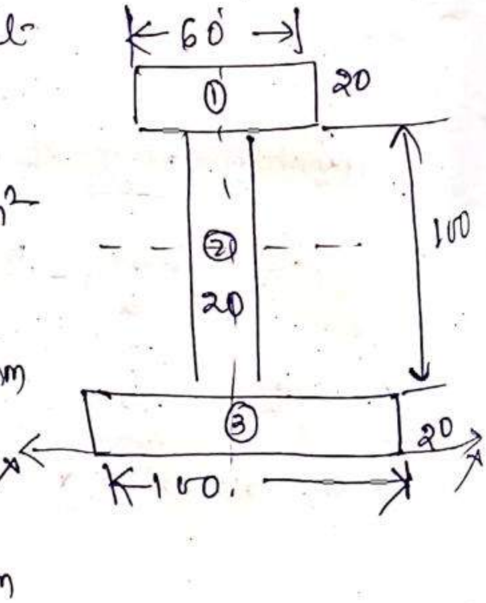
M.I about Y-Y axis ②

$$I_{G_2} + a_1 h_1^2 \,(\rightarrow 0) = 1.5625 \times 10^6 \text{ mm}^4$$

$$I_{YY} = 14.0625 \times 10^6 + 1.5625 \times 10^6$$
$$= 15.625 \times 10^6 \text{ mm}^4 \quad \text{Ans}$$

2019
2) Find the M.I of the given section about horizontal axis passing through C.G. Find M.I about X-X axis

Soln This secn is symmetric about y axis. C.G. pass



Rect ① $a_1 = 60 \times 20 = 1200 \text{ mm}^2$

$x_1 = 60/2 = 30$

$y_1 = 120 + \frac{20}{2} = 130 \text{ mm}$

② $a_2 = 100 \times 20 = 2000$

$y_2 = 20 + \frac{100}{2} = 70 \text{ mm}$

③ $a_3 = 100 \times 20 = 2000$

$y_3 = 20/2 = 10 \text{ mm}$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = 60.8 \text{ mm}$$

$I_{G_1} = \frac{bd^3}{12} = \frac{60 \times 20^3}{12} = 40 \times 10^3 \, mm^4$

$h_1 = y_1 - \bar{y} = 130 - 60.8 = 69.2 mm$

M.I of rectangle ① about X-X

$I_{G_1} + a_1 h_1^2 = 40 \times 10^3 + [1200 \times (69.2)^2]$

$= 5786 \times 10^3 mm^4$

for ②

$I_{G_2} = \frac{bd^3}{12} = \frac{20 \times 100^3}{12} = 1666.7 \times 10^3 mm^4$

$h_2 = y_2 - \bar{y} = 70 - 60.8 = 9.2 mm$

$I_{xx(2)} = I_{G_2} + a_2 h_2^2 = 1836 \times 10^3 mm^4$

for ③ $I_{G_3} = \frac{100 \times 20^3}{12} = 66.7 \times 10^3 mm^4$

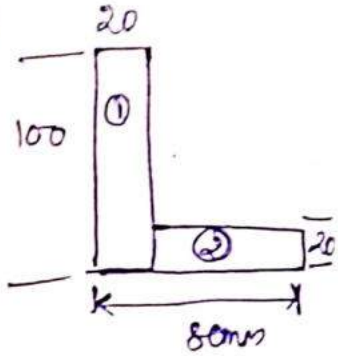
$h_3 = \bar{y} - y_3 = 60.8 - 10 = 50.8 mm$

$I_{xx(3)} = I_{G_3} + a_3 h_3^2 = 5228 \times 10^3 mm^4$

$I_{xx} = (5786 \times 10^3) + (1836 \times 10^3) + (5228 \times 10^3)$

$= 12850 \times 10^3 mm^4$

(2018-2D) Find the M.I about the centroidal X-X & Y-Y axis of the angle section.



Sol^n This section is not symmetrical about x any axis.

Rectangle (1)

$a_1 = 100 \times 20 = 2000\ mm^2$

$y_1 = 100/2 = 50\ mm$

(2) $a_2 = 80 \times 20 = 1600\ mm^2$

$y_2 = \frac{20}{2} = 10\ mm$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{2000 \times 50 + 1600 \times 10}{2000 + 1600} = 35\ mm$

M.I of ① about X-X axis.

$I_{G_1} = \frac{bd^3}{12} = \frac{20 \times (100)^3}{12} = 1.667 \times 10^6\ mm^4$

$h_1 = y_1 - \bar{y} = 50 - 35 = 15\ mm$

$I_{xx(1)} = I_{G_1} + a_1 h_1^2 = 1.667 \times 10^6 + 2000 \times (15)^2$

$= 2.117 \times 10^6\ mm^4$

M.I of ② about X-X-axis

$I_{G_2} = \frac{bd^3}{12} = \frac{60 \times 20^3}{12} = 0.04 \times 10^6\ mm^4$

$h_2 = \bar{y} - \bar{y}_2 = 35 - 10 = 25\ mm$

$I_{xx(2)} = I_{G_1} + a_2 h_2^2 = 0.79 \times 10^6\ mm^4$

A

$I_{x-x} = I_{xx(1)} + I_{xx(2)} = 2.407 \times 10^6 \ mm^4$

M.I about y axis

$x_1 = 20/2 = 10 \ mm$

$x_2 = 20 + 60/2 = 50 \ mm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = 25mm.$$

M.I of ① about Y-Y axis

$$I_{G1} = \frac{db^3}{12} = \frac{100 \times 20^3}{12} = 0.06 \times 10^6 \ mm^4$$

$h_1 = \bar{x} - x_1 = 25 - 10 = 15mm$

$I_{yy(1)} = I_{G1} + a_1h_1^2 = 0.06 \times 10^6 + 2000 \times 15^2$

$= 0.517 \times 10^6 \ mm^4$

M.I of ② Y-Y

$$I_{G2} = \frac{db^3}{12} = \frac{20 \times 80^3}{12} = 0.36 \times 10^6 \ mm^4$$

$h_2 = x_2 - \bar{x} = 50 - 25 = 25mm$

$I_{yy(2)} = I_{G2} + a_2h_2^2 = 1.11 \times 10^6 \ mm^4$

$I_{yy} = I_{yy(1)} + I_{yy(2)}$

$= 1.627 \times 10^6 \ mm^4$

CHAPTER 5 SIMPLE MACHINES

GEAR TRAIN OR TRAIN OF WHEELS

Two or more gears are made to mesh with each other, so as to operate as a single system, to transmit power from one shaft to another. Such a combination is called gear train or train of wheels. Following are the two types of train of wheels depending upon the arrangement of wheels:

1. Simple gear train.
2. Compound gear train.

1. SIMPLE GEAR TRAIN

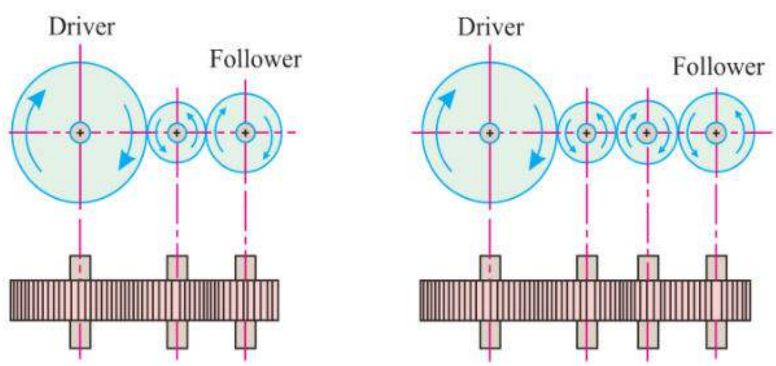
Sometimes the distance between the two wheels is great. The motion from one wheel to another, in such a case, may be transmitted by either of the following two methods :

1. By providing a large sized wheel, or

2. By providing intermediate wheels,

Providing large wheel is veryinconvenient and uneconomical; whereas providing intermediate wheels is very convenient and economical.

It may be noted that when the number of intermediate wheels is odd, the motion of both thewheels (i.e., driver and follower) is same. But, if the number of intermediatewheels is even, the motion of the follower is the opposite direction of the driver.



Simple gear train

Now consider a simple train of wheels with one intermediate wheel.

Let N_1 = Speed of the driver

T_1 = No. of teeth on the driver,

N_2, T_2 = Corresponding values for the intermediate wheel, and

N_3, T_3 = Corresponding values for the follower.

Since the driver gears with the intermediate wheel, therefore

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \qquad \text{.......... (i)}$$

Similarly, as the intermediate wheel gears with the follower, therefore

$$\frac{N_3}{N_2} = \frac{T_2}{T_3} \qquad \text{..........(ii)}$$

Multiplying equation (ii) by (i),

$$\frac{N_3}{N_2} \times \frac{N_2}{N_1} = \frac{T_2}{T_3} \times \frac{T_1}{T_2}$$

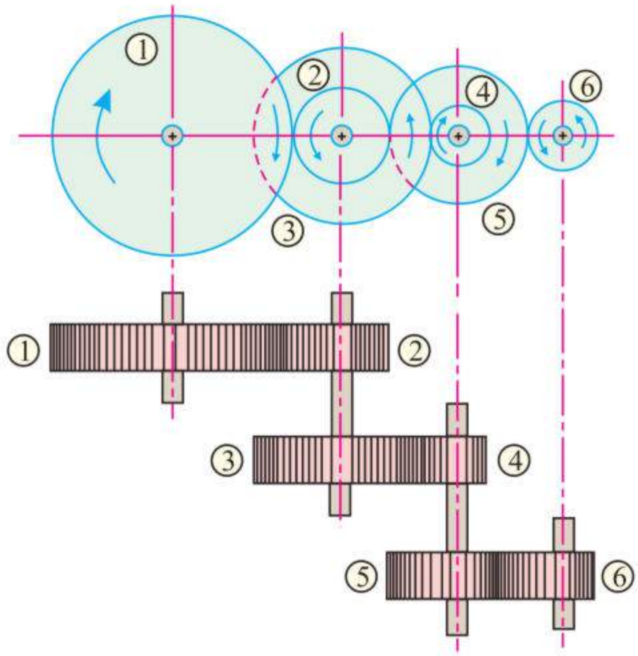
Or $$\frac{N_3}{N_1} = \frac{T_1}{T_3}$$

$$\therefore \frac{\text{Speed of the follower}}{\text{Speed of the driver}} = \frac{\text{No. of teeth on the driver}}{\text{No. of teeth on the follower}}$$

Similarly, it can be proved that the above equation also holds good, even if there are anynumber of intermediate wheels. It is thus obvious, that the velocity ratio, in a simple train of wheels, is independent of the intermediate wheels. These intermediate wheels are also called idle wheels, as they do not effect the velocity ratio of the system.

2. *COMPOUND GEAR TRAIN*

We have seen that the idle wheels, in a simple train of wheels, do not affect the velocity ratio of the system. But these wheels are useful in bridging over the space between the driver and the follower. But whenever the distance between the driver and follower has to be bridged over by intermediate wheels and at the same time a great (or much less) velocity ratio is required then the advantageof intermediate wheels in intensified by providing compound wheels on intermediate shafts. In thiscase, each intermediate shaft has two wheels rigidly fixed to it, so that they may have the samespeed. One of these two wheels gears with the driver and the other with the follower attached to thenext shaft.



Compound gear train

Let N_1 = Speed of the driver 1

T_1 = No. of teeth on the driver 1,

Similarly, N_2, N_3, ...N_6 = Speed of the respective wheels.

T_2, T_3, ...T_6 = No. of teeth on the respective wheels.

Since the wheel 1 gears with the wheel 2, therefore

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \qquad \ldots\ldots(i)$$

Similarly, $$\frac{N_4}{N_3} = \frac{T_3}{T_4} \qquad \ldots\ldots\ldots(ii)$$

And $$\frac{N_6}{N_5} = \frac{T_5}{T_6} \qquad \ldots\ldots\ldots(iii)$$

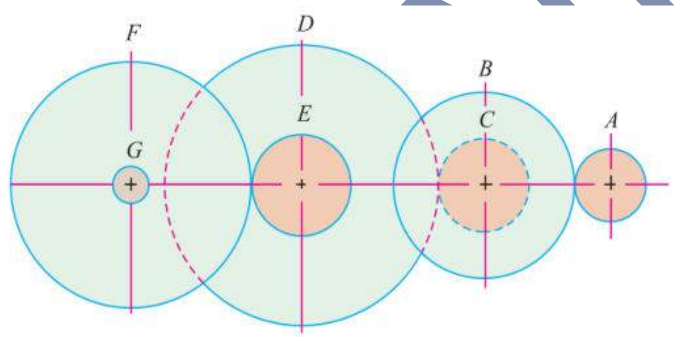
Multiplying equation (i), (ii) and (iii) we get

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\frac{N_6}{N_1} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6} \quad (\because N_2 = N_3 \text{ and } N_4 = N_5)$$

$$= \frac{\text{Product of teeth on the drivers}}{\text{Product of teeth on the followers}}$$

Example : The gearing of a machine tools is shown in Fig.



The motor shaft is connected to A and rotates at 975 r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft G. What is the speed of F ? The number of teeth on each wheel is as given below :

Gear	A	B	C	D	E	F
No. of teeth	20	50	25	75	26	65

Solution: Given:

Speed of the gear wheel A (N_A) = 975 r.p.m.;
No. of teeth on wheel A (T_A)= 20;
No. of teeth on wheel B (T_B) = 50;
No. of teeth on wheel C (T_C) = 25;
No. of teeth on wheel D(T_D) = 75;
No. of teeth on wheel E (T_E) = 26
and no. of teeth on wheel F (T_F) = 65.

Let N_F = Speed of the shaft F.

We know that $$\frac{N_F}{N_A} = \frac{T_A \times T_C \times T_E}{T_B \times T_D \times T_F}$$

$$\because \quad \frac{N_F}{975} = \frac{20 \times 25 \times 26}{50 \times 75 \times 65} = \frac{4}{75}$$

$$N_F = 975 \times \frac{4}{75} = 52 \text{ r.p.m.}$$

LIFTING MACHINE

It is a device, which enables us to lift a heavy load (W) by applying a comparatively smaller effort (P).

MECHANICAL ADVANTAGE

The mechanical advantage (briefly written as M.A.) is the ratio of weight lifted (W) to the effort applied (P) and is always expressed in pure number.

Mathematically, mechanical advantage,

$$\text{M.A.} = \frac{W}{P}$$

INPUT OF A MACHINE

The input of a machine is the work done on the machine. In a lifting machine, it is measured by the product of effort and the distance through which it has moved.

$$= P \times y$$

OUTPUT OF A MACHINE

The output of a machine is the actual work done by the machine. In a lifting machine, it is measured by the product of the weight lifted and the distance through which it has been lifted.

$$= W \times x$$

EFFICIENCY OF A MACHINE

It is the ratio of output to the input of a machine and is generally expressed as a percentage.

Mathematically, efficiency,

$$\eta = \frac{\text{Output}}{\mathit{Input}} \times 100$$

IDEAL MACHINE

If the efficiency of a machine is 100% i.e., if the output is equal to the input, the machine is called as a perfect or an ideal machine.

IDEAL LOAD (W_i):

The load lifted by an ideal machine is known as ideal load.

The load lifted by an ideal machine is always greater than that of a normal/actual machine.

$$W_i > W$$

In case of ideal machine friction is zero

So M.A =V.R

$$\frac{Wi}{P} = V.R$$

$$\Rightarrow \quad W_i = V.R \times P$$

Load lost due to friction : frictional load – Actual load

$$W_f = W_i - W$$

$$= (P \times V.R) - W$$

IDEAL EFFORT (P_i) :

The effort applied to lift the load in an ideal machine is known as ideal effort.

The ideal effort is always less than that of actual effort.

$$P_i < P$$

In case of ideal machine friction is zero

So M.A =V.R

$$\frac{W}{Pi} = V.R$$

$$\Rightarrow \quad P_i = \frac{W}{V.R}$$

Effort lost due to friction : Actual effort – frictional effort

$$P_f = P - P_i$$

$$= P - \frac{W}{V.R}$$

Example : In a certain machine, an effort of 100 N is just able to lift a load of 840 N, Calculate efficiency and friction both on effort and load side, if the velocity ratio of the machine is 10.

Solution.

Given: Effort (P) = 100 N ; Load (W) = 840 N and velocity ratio (V.R.) = 10.

We know that M.A.

$$\frac{W}{P} = \frac{840}{100} = 8.4$$

And efficiency, $\frac{M.A}{V.R} = \frac{8.4}{10} \times 100 = 84\%$

Friction of the machine in terms of effort,

$$P_f = P - \frac{W}{V.R} = 100 - \frac{840}{10} = 16 \text{ N}$$

and friction of the machine in terms of load,

$$W_f = (P \times V.R.) - W = (100 \times 10) - 840 = 160 \text{ N}$$

VELOCITY RATIO

The velocity ratio (briefly written as V.R.) is the ratio of distance moved by the effort (y) to the distance moved by the load (x) and is always expressed in pure number.

Mathematically, velocity ratio,

$$V.R. = \frac{y}{x}$$

RELATION BETWEEN EFFICIENCY, MECHANICAL ADVANTAGE AND

VELOCITY RATIO OF A LIFTING MACHINE

It is an important relation of a lifting machine, which throws light on its mechanism.

Now consider a lifting machine, whose efficiency is required to be found out.

Let W = Load lifted by the machine,

P = Effort required to lift the load,

y = Distance moved by the effort, in lifting the load, and

x = Distance moved by the load.

We know that M.A. $= \frac{W}{P}$ and V.R. $= \frac{y}{x}$

We also know that input of a machine

= Effort applied × Distance through which the effort has moved

$= P \times y$ ...(i)

and output of a machine = Load lifted × Distance through which the load has been lifted

$= W \times x$ ...(ii)

$\therefore$ Efficiency, $\eta = \frac{\text{Output}}{\text{Input}} = \frac{W \times x}{P \times y} = \frac{M.A}{V.R}$

Note. It may be seen from the above relation that the values of M.A. and V.R. are equal only in case of a machine whose efficiency is 100%. But in actual practice, it is not possible.

Example: In a certain weight lifting machine, a weight of 1 kN is lifted by an effort of 25 N. While the weight moves up by 100 mm, the point of application of effort moves by 8 m.Find mechanical advantage, velocity ratio and efficiency of the machine.

Solution.

Given: Weight (W) = 1 KN = 1000 N ;

Effort (P) = 25 N ;

Distance through which the weight is moved (x) = 100 mm = 0.1 m and distance through which effort is moved (y) = 8 m.

Mechanical advantage of the machine.

We know that mechanical advantage of the machine

$$M.A = \frac{W}{P} = \frac{1000}{25} = 40$$

Velocity ratio of the machine of the machine

We know that velocity ratio of the machine

$$V.R = \frac{y}{x} = \frac{8}{0.1} = 80$$

Efficiency of the machine

We also know that efficiency of the machine,

$$\eta = \frac{M.A}{V.R} \times 100 = \frac{40}{80} \times 100 = 50\%$$

REVERSIBILITY OF A MACHINE

Sometimes, a machine is also capable of doing some work in the reversed direction, after the effort is removed. Such a machine is called a reversible machine and its action is known as reversibility of the machine.

CONDITION FOR THE REVERSIBILITY OF A MACHINE

Consider a reversible machine, whose condition for the reversibility is required to be found out.

Let W = Load lifted by the machine,

P = Effort required to lift the load,

y = Distance moved by the effort, and

x = Distance moved by the load.

We know that input of the machine

$= P \times y$ ...(i)

and output of the machine = W × x ...(ii)

We also know that machine friction

= Input – Output = (P × y) – (W × x) ...(iii)

A little consideration will show that in a reversible machine, the *output of the machine should be more than the machine friction, when the effort (P) is zero. i.e.,

$$W \times x > P \times y - W \times x$$

Or $$2\,W \times x > P \times y$$

or $$\frac{W \times x}{P \times y} > \frac{1}{2}$$

or $$\frac{\frac{W}{P}}{\frac{y}{x}} > \frac{1}{2}$$

or $$\frac{M.A}{V.R} > \frac{1}{2}$$

$$\therefore \eta > \frac{1}{2} = 50\%$$

Hence the condition for a machine, to be reversible, is that its efficiency should be more than 50%.

SELF-LOCKING MACHINE

Sometimes, a machine is not capable of doing any work in the reversed direction, after the effort is removed. Such a machine is called a non-reversible or self-locking machine. A little consideration will show, that the condition for a machine to be non-reversible or self-locking is that its efficiency should not be more than 50%.

Example: A certain weight lifting machine of velocity ratio 30 can lift a load of 1500N with the help of 125 N effort. Determine if the machine is reversible.

Solution. Given: Velocity ratio (V.R.) = 30;

Load (W) = 1500 N and

effort (P) = 125 N.

We know that

$$\text{M.A.} = \frac{W}{P} = \frac{1500}{125} = 12$$

and efficiency,

$$\eta = \frac{M.A}{V.R} \times 100 = \frac{12}{30} \times 100 = 40\%$$

Since efficiency of the machine is less than 50%, therefore the machine is non-reversible.

Example :In a lifting machine, whose velocity ratio is 50, an effort of 100 N is required to lift a load of 4 KN. Is the machine reversible ? If so, what effort should be applied, so that themachine is at the point of reversing ?

Solution.

Given: Velocity ratio (V.R.) = 50 ;

Effort (P) = 100 N

and load (W) = 4 KN = 4000 N.

Reversibility of the machine

We know that

$$\text{M.A.} = \frac{W}{P} = \frac{4000}{50} = 40$$

and efficiency,

$$\eta = \frac{M.A}{V.R} \times 100 = \frac{40}{50} \times 100 = 80\%$$

Since efficiency of the machine is more than 50%, therefore the machine is reversible.

Effort to be applied

A little consideration will show that the machine will be at the point of reversing, when its efficiency is 50% or 0.5.

Let P_1 = Effort required to lift a load of 4000 N when the machine is at the point of reversing.

We know that M.A. $= \frac{W}{P_1} = \frac{4000}{P_1}$

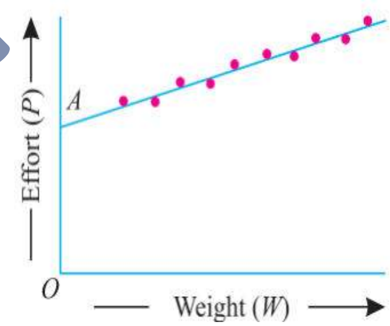
and efficiency, $0.5 = \frac{M.A}{V.R} = \frac{4000/P_1}{50} = \frac{80}{P_1}$

$P_1 = 0.5 \times 80 = 160$ N.

LAW OF A MACHINE

The term 'law of a machine' may be defined as relationship between the effort applied and the load lifted. Thus for any machine, if we record the various efforts required to raise the corresponding loads, and plot a graph between effort and load, we shall get a straight line AB as shown in Fig.

We also know that the intercept OA represents the amount of friction offered by the machine. Or in other words, this is the effort required by the machine to overcome the friction, before it can lift any load.



Mathematically,

the law of a lifting machine is given by the relation :

$P = mW + C$

where P = Effort applied to lift the load,

m = A constant (called coefficient of friction) which is equal to the slope of the line AB, Fig.

W = Load lifted, and

C = Another constant, which represents the machine friction, (i.e. OA).

Maximum M.A:

$$M.A_{max.} = \frac{1}{m}$$

Maximum Efficiency:

$$\eta_{max.} = \frac{1}{m \times V.R} \times 100$$

Example: What load can be lifted by an effort of 120 N, if the velocity ratio is 18 and efficiency of the machine at this load is 60% ?

Determine the law of the machine, if it is observed that an effort of 200 N is required to lift a load of 2600 N and find the effort required to run the machine at a load of 3.5 kN.

Solution.

Given:

Effort (P) = 120 N ;

Velocity ratio (V.R.) = 18 and

efficiency (η) = 60% = 0.6.

Load lifted by the machine

Let W = Load lifted by the machine.

Mechanical Advantage, $M.A = \frac{W}{P} = \frac{W}{120} = W/120$

And efficiency, $0.6 = \frac{M.A}{V.A} = \frac{W/120}{18} = \frac{W}{2160}$

or $W = 0.6 \times 2160 = 1296$ N

Law of machine

In the second case, P = 200 N and W = 2600 N

Substituting the two values of P and W in the law of the machine, i.e., P = m W + C,

$$120 = m \times 1296 + C \quad ...(i)$$

$$\text{and } 200 = m \times 2600 + C \quad ...(ii)$$

Subtracting equation (i) from (ii),

$$80 = 1304\ m$$

$$\text{or} \quad m = \frac{80}{1304} = 0.06$$

and now substituting the value of m in equation (ii)

$$200 = (0.06 \times 2600) + C = 156 + C$$

$$C = 200 - 156 = 44$$

Now substituting the value of m = 0.06 and C = 44 in the law of the machine,

$$P = 0.06\ W + 44$$

Effort required to run the machine at a load of 3.5 kN.

Substituting the value of W = 3.5 kN or 3500 N in the law of machine,

$$P = (0.06 \times 3500) + 44 = 254\ N$$

Problem : In a lifting machine, an effort of 40 N raised a load of 1 kN. If efficiency of the machine is 0.5, what is its velocity ratio ? If on this machine, an effort of 74 N raised a load of 2 kN, what is now the efficiency ? What will be the effort required to raise a load of 5 kN ?

Solution. Given:

When Effort (P) = 40 N;

Load (W) = 1 kN = 1000 N;

Efficiency (η) = 0.5;

When effort (P) = 74 N and

load (W) = 2 kN = 2000 N.

Velocity ratio when efficiency is 0.5.

$$\text{We know that M.A.} = \frac{W}{P} = \frac{1000}{40} = 25$$

$$\text{And Efficiency,} \ = 0.5 = \frac{M.A}{V.R} = \frac{25}{V.R}$$

$$\text{And Velocity ratio,} \quad V.R = \frac{25}{0.5} = 50$$

Efficiency when P is 74 N and W is 2000 N

$$\text{Mechanical Advantage,} \quad M.A = \frac{W}{P} = \frac{2000}{74} = 27$$

$$\text{And efficiency, } \eta = \frac{M.A}{V.R} \times 100 = \frac{27}{50} \times 100 = 0.54 \times 100 = 54\%$$

Effort required to raise a load of 5 KN or 5000 N

Substituting the two values of P and W in the law of the machine, i.e. P = mW + C

$$40 = m \times 1000 + C \qquad ...(i)$$

$$\text{And } 74 = m \times 2000 + C \qquad ...(ii)$$

Subtracting equation (i) from (ii),

$$34 = 1000\ m \text{ or } \quad m = \frac{34}{1000} = 0.034$$

and now substituting this value of m in equation (i),

$$40 = (0.034 \times 1000) + C = 34 + C$$

$$\therefore C = 40 - 34 = 6$$

Substituting these values of m = 0.034 and C = 6 in the law of machine,

$$P = 0.034\ W + 6 \qquad ...(iii)$$

∴ Effort required to raise a load of 5000 N,

$$P = (0.034 \times 5000) + 6 = 176 \text{ N}$$

SIMPLE WHEEL AND AXLE

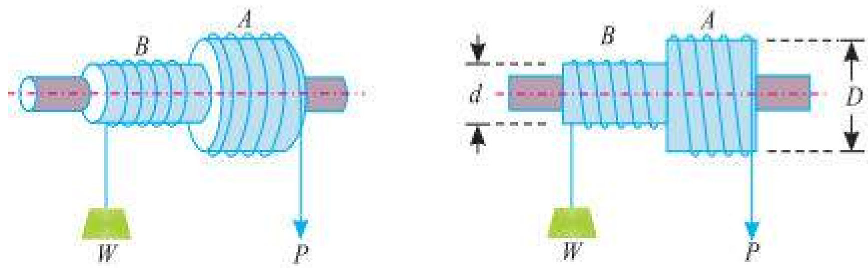


Fig. is shown a simple wheel and axle, in which the wheel A and axle B are keyed to the same shaft. The shaft is mounted on ball bearings, order to reduce the frictional resistance to a minimum. A string is wound round the axle B, which carries the load to be lifted. A second string is wound round the wheel A in the opposite direction to that of the string on B.

Let D = Diameter of effort wheel,

d = Diameter of the load axle,

W = Load lifted, and

P = Effort applied to lift the load.

One end of the string is fixed to the wheel, while the other is free and the effort is applied to this end. Since the two strings are wound in opposite directions, therefore a downward motion of the effort (P) will raise the load (W).

Since the wheel as well as the axle are keyed to the same shaft, therefore when the wheel rotates through one revolution, the axle will also rotate through one revolution.

We know that displacement of the effort in one revolution of effort wheel A,

$$= \pi D \qquad \text{........(i)}$$

and displacement of the load in one revolution

$$= \pi d \qquad \text{...(ii)}$$

$$\text{Velocity ratio, } V.R = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

$$\text{Mechanical advantage, } M.A. = \frac{\text{Load lifted}}{\text{Effort applied}} = \frac{W}{P}$$

Efficiency, $\eta = \frac{M.A.}{V.R.} \times 100$

Problem: A simple wheel and axle has wheel and axle of diameters of 300 mm and 30 mm respectively. What is the efficiency of the machine, if it can lift a load of 900 N by an effort of 100 N.

Solution.

Given:

Diameter of wheel (D) = 300 mm;

Diameter of axle (d) = 30 mm;

Load lifted by the machine (W) = 900 N and

effort applied to lift the load (P) = 100 N

We know that velocity ratio of the simple wheel and axle,

$$V.R = \frac{D}{d} = \frac{300}{30} = 10$$

$$\text{Mechanical advantage, } M.A. = \frac{W}{P} = \frac{900}{100} = 9$$

$$\therefore \text{Efficiency, } \eta = \frac{M.A.}{V.R.} \times 100 = \frac{9}{10} \times 100 = 90\%$$

Problem : A drum weighing 60 N and holding 420N of water is to be raised from a well by means of wheel and axle. The axle is 100 mm diameter and the wheel is 500 mm diameter. If a force of 120 N has to be applied to the wheel, find (i) mechanical advantage, (ii) velocity ratio and (iii) efficiency of the machine.

Solution.

Given: Total load to be lifted (W) = 60 + 420 = 480 N;

Diameter of the load axle (d) = 100 mm;

Diameter of effort wheel (D) = 500 mm and

effort (P) = 120 N.

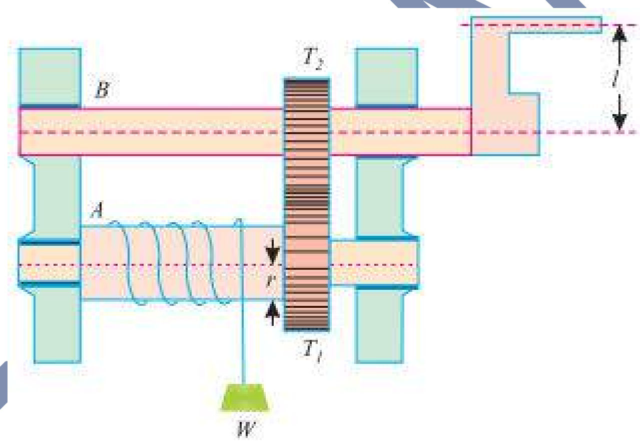
We know that mechanical advantage,

$$M.A. = \frac{W}{P} = \frac{480}{120} = 4$$

Velocity ratio, $$V.R. = \frac{D}{d} = \frac{500}{100} = 5$$

Efficiency of the machine, $$\eta = \frac{M.A.}{V.R.} \times 100 = \frac{4}{5} \times 100 = 80\%$$

SINGLE PURCHASE CRAB WINCH



In single purchase crab winch, a rope is fixed to the drum and is wound a few turns round it. The free end of the rope carries the load W. A toothed wheel A is rigidly mounted on the load drum. Another toothed wheel B, called pinion, is geared with the toothed wheel A as shown in Fig.

The effort is applied at the end of the handle to rotate it.

Let T_1 = No. of teeth on the main gear (or spur wheel) A,

T_2 = No. of teeth on the pinion B,

l = Length of the handle,

r = Radius of the load drum.

W = Load lifted, and

P = Effort applied to lift the load.

We know that distance moved by the effort in one revolution of the handle,

$= 2\pi l$ ...(i)

∴ No. of revolutions made by the pinion B = 1 and

no. of revolutions made by the wheel A = $\frac{T_2}{T_1}$

∴No. of revolutions made by the load drum = $\frac{T_2}{T_1}$

and distance moved by the load $= 2\pi r \times \frac{T_2}{T_1}$ ...(ii)

∴Velocity ratio V.R $= \frac{Distance\ moved\ by\ the\ effort}{Distance\ moved\ by\ the\ load}$

$$= \frac{2\pi l}{2\pi r \times \frac{T_2}{T_1}} = \frac{l}{r} \times \frac{T_1}{T_2} \quad .....(iii)$$

Mechanical advantage, M.A. = $\frac{W}{P}$

Efficiency, $\eta = \frac{M.A}{V.R} \times 100$

Example: In a single purchase crab winch, the number of teeth on pinion is 25 and that on the spur wheel 100. Radii of the drum and handle are 50 mm and 300 mm respectively. Find the efficiency of the machine, if an effort of 20 N can lift a load of 300 N.

Solution.

Given:

No. of teeth on pinion (T_2) = 25;

No. of teeth on the spur wheel (T_1) = 100;

Radius of drum (r) = 50 mm;

Radius of the handle or length of the handle (l) = 300 mm;

Effort (P) = 20 N and

load lifted (W) = 300 N.

Velocity ratio, V.R. $= \frac{l}{r} \times \frac{T_1}{T_2} = \frac{300}{50} \times \frac{100}{25} = 24$

Mechanical advantage, M.A. $= \frac{W}{P} = \frac{300}{20} = 15$

Efficiency, $\eta = \frac{M.A}{V.R} \times 100 = \frac{24}{15} \times 100 = 62.5$ %

Example:

A single purchase crab winch, has the following details:

Length of lever = 700 mm

Number of pinion teeth = 12

Number of spur gear teeth = 96

Diameter of load axle = 200 mm

It is observed that an effort of 60 N can lift a load of 1800 N and an effort of 120 N can lift

a load of 3960 N. What is the law of the machine ? Also find efficiency of the machine in both the cases.

Solution.

Given: Length of lever (l) = 700 mm;

No. of pinion teeth (T_2) = 12;

No. of spur gear teeth (T_1) = 96 and

Dia. of load axle = 200 mm or radius (r) = 200/2 = 100 mm.

(i) Law of the machine

When P_1 = 60 N, W_1 = 1800 N and when P_2 = 120 N, W_2 = 3960 N.

Substituting the values of P and W in the law of the machine i.e., P = mW + C

$$60 = (m \times 1800) + C \qquad \ldots(i)$$

and $120 = (m \times 3960) + C$ ...(ii)

Subtracting equation (i) from equation (ii)

$$60 = m \times 2160$$

or $$m = \frac{60}{2160} = \frac{1}{36}$$

Now substituting this value of m in equation (i),

$$60 = \left(\frac{1}{36} \times 1800\right) + C = 50 + C$$

or $$C = 60 - 50 = 10$$

and now substituting the value of m = 1/36 and C = 10 in the law of machine,

$$P = \frac{1}{36} \times W + 10$$

(ii) Efficiencies of the machine in both the cases

$$V.R. = \frac{l}{r} \times \frac{T_1}{T_2} = \frac{700}{100} \times \frac{96}{12} = 56$$

$$M.A. = \frac{W_1}{P_1} = \frac{1800}{60} = 30$$

Efficiency, $$\eta_1 = \frac{M.A}{V.R} \times 100 = \frac{30}{56} \times 100 = 53.6\ \%$$

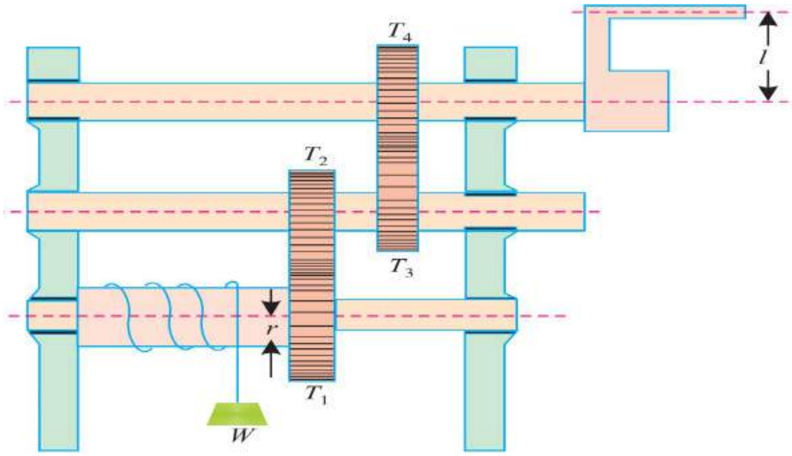
Similarly, mechanical advantage in the second case,

$$M.A. = \frac{W_2}{P_2} = \frac{3690}{120} = 33$$

Efficiency, $$\eta_2 = \frac{M.A}{V.R} \times 100 = \frac{33}{56} \times 100 = 58.9\ \%$$

DOUBLE PURCHASE CRAB WINCH

A double purchase crab winch is an improved form of a single purchase crab winch, in which the velocity ratio is intensified with the help of one more spur wheel and a pinion. In a double purchasecrab winch, there are two spur wheels of teeth T 1 and T 2 and T 3 as well as two pinions of teeth T 2 and T 4 .



The arrangement of spur wheels and pinions are such that the spur wheel with T 1 gears with the pinion of teeth T 2 . Similarly, the spur wheel with teeth T 3 gears with the pinion of the teeth T 4 , Theeffort is applied to a handle as shown in Fig.

Let T_1 and T_3 = No. of teeth of spur wheels,

T_2 and T_4 = No. of teeth of the pinions

l = Length of the handle,

r = Radius of the load drum,

W = Load lifted, and

P = Effort applied to lift the load, at the end of the handle.

We know that distance moved by the effort in one revolution of the handle,

$= 2\pi l$ ...(i)

∴ No. of revolutions made by the pinion 4 = 1and

no. of revolutions made by the wheel 3 $= \frac{T_4}{T_3}$

∴No. of revolutions made by the pinion 2 $= \frac{T_4}{T_3}$

and no. of revolutions made by the wheel 1 $= \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

∴ Distance moved by the load

$$= 2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3} \qquad \text{...(ii)}$$

Velocity ratio, $\quad V.R. = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$

$$= \frac{2\pi l}{2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}} = \frac{l}{r} \times \left(\frac{T_2}{T_1} \times \frac{T_4}{T_3}\right)$$

Mechanical advantage, M.A. $= \frac{W}{P}$

Efficiency, $\quad \eta = \frac{M.A}{V.R} \times 100$

Example: In a double purchase crab winch, teeth of pinions are 20 and 25 and that of spur wheels are 50 and 60. Length of the handle is 0.5 metre and radius of the load drum is 0.25 metre. If efficiency of the machine is 60%, find the effort required to lift a load of 720 N.

Solution.

Given:

No. of teeth of pinion (T_2) = 20 and (T_4) = 25;

No. of teeth of spur wheel (T_1) = 50 and (T_3) = 60;

Length of the handle (l) = 0.5 m;

Radius of the load drum (r) = 0.25m;

Efficiency (η) = 60% = 0.6 and

load to be lifted (W) = 720 N.

Let P = Effort required in newton to lift the load.

We know that velocity ratio, V.R.

$$= \frac{l}{r} \times \left(\frac{T_2}{T_1} \times \frac{T_4}{T_3}\right) = \frac{0.5}{0.25} \times \left(\frac{50}{20} \times \frac{60}{25}\right) = 12$$

Mechanical advantage, $\quad M.A. = \frac{W}{P} = \frac{720}{P}$

Efficiency, $\quad \eta = \frac{M.A}{V.R}$

$$0.6 = \frac{\frac{720}{P}}{12}$$

or $$P = \frac{60}{0.6} = 100 \text{ N}$$

Example: A double purchase crab used in a laboratory has the following dimensions :

Diameter of load drum = 160 mm

Length of handle = 360 mm

No. of teeth on pinions = 20 and 30

No. of teeth on spur wheels = 75 and 90

When tested, it was found that an effort of 90 N was required to lift a load of 1800 N and an effort of 135 N was required to lift a load of 3150 N.

Determine :

(a) Law of the machine,

(b) Probable effort to lift a load of 4500 N,

(c) Efficiency of the machine in the above case,

Solution.

Given:

Dia. of load drum = 160 mm or radius (r) = 100/2 = 80 mm;

Length of handle (l) = 360 mm;

No. of teeth on pinions (T_2) = 20 and (T_4) = 30

and no. of teeth on spur wheels (T_1) = 75 and (T_3) = 90.

When P = 90 N,

$W = 1800$ N

when $P = 135$ N,

$W = 3150$ N

(a) Law of the machine,

Substituting the values of P and W in the law of the machine, i.e., $P = m\,W + C$

$$90 = (m \times 1800) + C \qquad \text{...(i)}$$

$$\text{and } 135 = (m \times 3150) + C \qquad \text{...(ii)}$$

Subtracting equation (i) from equation (ii),

$$45 = m \times 1350$$

$$\text{or} \qquad m = \frac{45}{1350} = \frac{1}{30}$$

Now substituting this value of m in equation (i),

$$90 = \frac{1}{30} \times 1800 + C = 60 + C$$

$$\therefore \quad C = 90 - 60 = 30$$

Now substituting the value fo m and C in the law of the machine,

$$P = \frac{1}{30} \times W + C$$

(b) Effort to lift a load of 4500 N

Substituting the value of W equal to 4500 N in the law of the machine,

$$P = \left(\frac{1}{30} \times 4500\right) + 30 = 180 \text{ N}$$

(c) Efficiency of the machine in the above case

We know that velocity ratio.

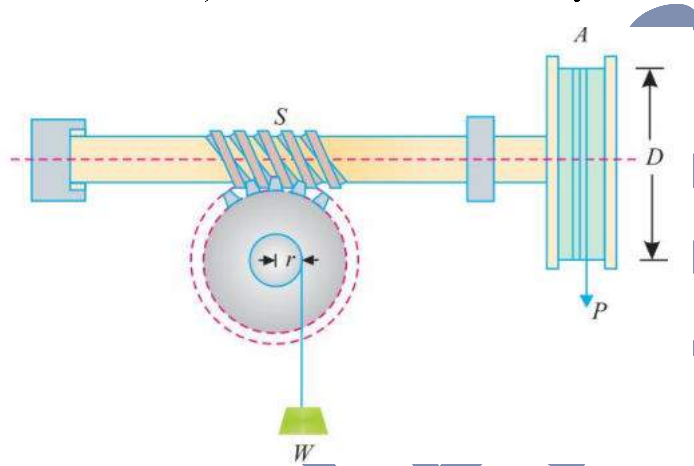
$$V.R. = \frac{l}{r} \times \left(\frac{T_2}{T_1} \times \frac{T_4}{T_3}\right) = \frac{360}{80} \times \left(\frac{75}{20} \times \frac{90}{30}\right) = 50.6$$

$$\text{Mechanical advantage,} \quad M.A. = \frac{W}{P} = \frac{4500}{180} = 25$$

Efficiency, $\eta = \frac{M.A}{V.R} \times 100 = \frac{25}{50.6} \times 100 = 49.4\ \%$

WORM AND WORM WHEEL

It consists of a square threaded screw, S (known as worm) and a toothed wheel (known asworm wheel) geared with each other, as shown in Fig. A wheel A is attached to the worm, overwhich passes a rope as shown in the figure. Sometimes, a handle is also fixed to the worm (instead ofthe wheel). A load drum is securely mounted on the worm wheel.



Let, D = Diameter of the effort wheel,

r = Radius of the load drum

W = Load lifted,

P = Effort applied to lift the load, and

T = No. of teeth on the worm wheel.

We know that distance moved by the effort in one revolution of the wheel (or handle)

$= \pi D$ ...(i)

If the worm is single-threaded (i.e., for one revolution of the wheel A, the screw S pushes theworm wheel through one teeth), then the load drum will move through

$= \frac{1}{T}$ revolution

and distance, through which the load will move

$$= \frac{2\pi r}{T} \qquad .....(ii)$$

Velocity ratio, $V.R. = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$

$$= \frac{\pi D}{\frac{2\pi r}{T}} = \frac{DT}{2r}$$

Mechanical advantage, $M.A. = \frac{W}{P}$

Efficiency, $\eta = \frac{M.A}{V.R} \times 100$

Notes : 1. If the worm is double-threaded i.e., for one revolution of wheel A, the screw S pushes theworm wheel through two teeths, then

$$V.R. = \frac{DT}{2 \times 2r} = \frac{DT}{4r}$$

2. In general, if the worm is n threaded, then

$$V.R. = \frac{DT}{2nr}$$

Example .A worm and worm wheel with 40 teeth on the worm wheel has effort wheel of 300 mm diameter and load drum of 100 mm diameter. Find the efficiency of the machine, if it can lift a load of 1800 N with an effort of 24 N.

Solution.

Given:

No. of teeth on the worm wheel (T) = 40 ;

Diameter of effort wheel = 300 mm

Diameter of load drum = 100 mm or radius (r) = 50 mm;

Load lifted (W) = 1800 N

and effort(P) = 24 N.

We know that velocity ratio of worm and worm wheel, Velocity ratio, V.R. $= \frac{DT}{2r} = \frac{300 \times 40}{2 \times 50} = 120$

Mechanical advantage, M.A. $= \frac{W}{P} = \frac{1800}{24} = 75$

Efficiency, $\eta = \frac{M.A}{V.R} \times 100 = \frac{75}{120} \times 100 = 62.5\ \%$

Example .In a double threaded worm and worm wheel, the number of teeth on the worm wheel is 60. The diameter of the effort wheel is 250 mm and that of the load drum is 100 mm.

Calculate the velocity ratio. If the efficiency of the machine is 50%, determine the effort required to lift a load of 300 N.

Solution.

Given :

No. of threads (n) = 2;

No. of teeth on the worm wheel (T) = 60;

Diameter of effort wheel = 250 mm;

Diameter of load drum = 100 mm or radius (r) = 50 mm;

Efficiency (η) = 50% = 0.5

and load to be lifted (W) = 300 N.

Velocity ratio of the machine

We know that velocity ratio of a worm and worm wheel,

$$V.R. = \frac{DT}{2nr} = \frac{250 \times 60}{2 \times 2 \times 50} = 75$$

Effort required to lift the load

Let P = Effort required to lift the load.

We also know that mechanical advantage, M.A. $= \frac{W}{P} = \frac{300}{P}$

Efficiency, $\eta = 0.5 = \frac{M.A}{V.R} = \frac{\frac{300}{P}}{75} = \frac{4}{P}$

or $P = \frac{4}{0.5} = 8$ N

SIMPLE SCREW JACK

It consists of a screw, fitted in a nut, which forms the body of the jack. The principle, on whicha screw jack works, is similar to that of an inclined plane.

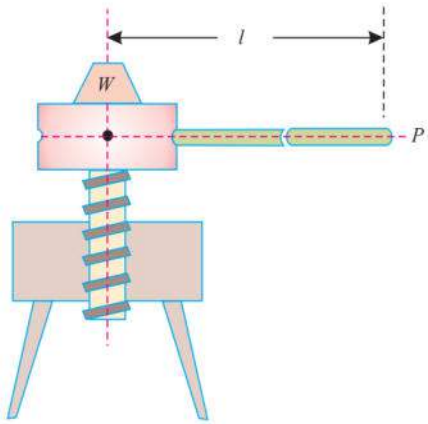


Fig. shows a simple screw jack, which is rotated by the application of an effort at the end of the lever, for lifting the load. Now consider a single threaded simple screw jack.

Let l = Length of the effort arm,

p = Pitch of the screw,

W = Load lifted, and

P = Effort applied to lift the load at the end of the lever.

We know that distance moved by the effort in one revolution of screw,

$= 2\pi l$ ...(i)

and distance moved by the load = p ...(ii)

Velocity ratio, $V.R. = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{2\pi l}{P}$

Mechanical advantage, $M.A. = \frac{W}{P}$

$$\text{Efficiency, } \eta = \frac{M.A}{V.R} \times 100$$

Note: The value of P i.e., the effort applied may also found out by the relation :

$$P = W \tan(\alpha + \varphi)$$

where W = Load lifted

$$\tan \alpha = \frac{P}{\pi d}$$

and $\tan \varphi = \mu$ = Coefficient of friction

Example: A screw jack has a thread of 10 mm pitch. What effort applied at the end ofa handle 400 mm long will be required to lift a load of 2 KN, if the efficiency at this load is 45%.

Solution.

Given:

Pitch of thread (p) = 10 mm;

Length of the handle (l) = 400 mm;

Load lifted (W) = 2 kN = 2000N

and efficiency (n) = 45% = 0.45.

Let P = Effort required to lift the load.

We know that velocity ratio

$$V.R. = \frac{2\pi l}{P} = \frac{2\pi \times 400}{10} = 251.3$$

And $$M.A. = \frac{W}{P} = \frac{2000}{P}$$

We also know that efficiency,

$$\eta = 0.45 = \frac{M.A}{V.R} = \frac{\frac{2000}{P}}{251.3} = \frac{7.96}{P}$$

or $$P = \frac{7.96}{0.45} = 17.7 \text{ N}$$

CHAPTER 6 DYNAMICS

Kinetics: It is the branch of dynamics which deals with the study of effects on the body when force applied on it with knowing the cause of force.

Kinematics: It is the branch of dynamics which deals with the study of effects on the body when force applied on it without knowing the causes of force.

PRINCIPLES OF DYNAMICS:

NEWTON'S LAWS OF MOTION

(a) **First Law of motion**: It states, "Everybody continues in its state of rest or of uniform motion in a straight line, unless compelled by some external force to change that state".

This law can also termed as law of inertia.

(b) **Second law of motion**: It states that,"The rate of change of momentum is directly proportional to the impressed force and takes place in the same direction in which the impressed force acts".

It relates to the rate of change of momentum and the external force.

Let, m = mass of the body

u = initial velocity of the body.

v = final velocity of the body

a = constant acceleration

t = time in seconds in which the velocity changes from u to v.

F = force that changes the velocity from u to v in t seconds.

For the body moving in straight line,

Initial momentum = mu

Final momentum = mv

Rate of change of momentum = $\frac{mv-mu}{t} = \frac{m(v-u)}{t}$ = ma $\quad\because \frac{v-u}{t} = a$

MECHANICS

According to Newtons Second law of Motion,

Rate of change of momentum ∝ impressed force

$\Rightarrow$ $F \propto ma$

$\Rightarrow$ $F = k \times ma$

Where k = a constant of proportionality

If a unit force is chosen to action a unit mass of 1kg to produce unit acceleration of $1m/s^2$ then , $F = ma$ = Mass x Acceleration

The SI unit of force is Newton, briefly written as N

A Newton may be defined as the force while acting upon a mass of 1kg, produces an acceleration of $1m/s^2$ in the direction of which it acts.

(c) **Third law of motion**: It states," To every action, there is always an equal and opposite reaction".

If a body exerts a force P on another body, the second body will exert the same force P on the first body in the opposite direction. The force exerted by first body is called action whereas the force exerted by the second body is called reaction.

MOTION OF PARTICLE ACTED UPON BY A CONSTANT FORCE

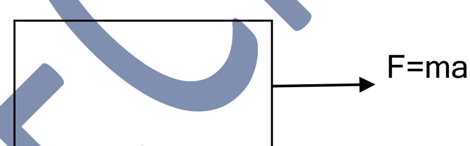


Fig6.1

The motion of a particle acted upon by a constant force is governed by Newton's second law of motion.

If a constant force, $F = m a$ is applied on a particle of mass m, then the particle will move with a uniform acceleration of a.

EQUATIONS OF MOTION

Let,

u = initial velocity of the body

v = final velocity of the body

s = distance travelled by the body

a = acceleration of the body

t = time taken by the body

∴The equations of motion are:

$v = u + at$ ....(1)

$S = ut + \frac{1}{2}at^2$ ....(2)

$V^2 - u^2 = 2as$ .... (3)

D'ALEMBERT'S PRINCIPLE

It states, "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics."

Let, P = resultant of number of forces acting on a body of mass m This resultant (P) will move the body with an acceleration(a) in its own direction.

We have, $P = ma$ (1)

The body will be at rest if a force equal to ma is applied in reverse direction. Hence, for dynamic equilibrium of the body, the sum of the resultant force and the reversed force will be equal to zero.

$P - ma = 0$ (2)

The force (-ma) is known as inertia force or reversed effective force. Equation 1 is an equation of dynamics where as equation2 is an equation of statics.

Equation 2 is known as the equation of dynamic equilibrium under the action of P. This principle is known as D" Alembert"s principle.

RECOIL OF GUN

According to Newtons third law of motion, when a bullet is fired from a gun, the opposite reaction of the bullet is known as the recoil of gun.

Let, M=mass of the gun

V = Velocity of the gun with which it recoils

m = mass of the bullet

v = velocity of the bullet after firing

Now, momentum of the bullet after firing= mv...................................... (1)

Momentum of the gun =MV... (2)

Equating equations (1) & (2) we get,

$$mv = MV$$

This relation is known as law of Conservation of Momentum.

WORK

When force acts on a body and the body undergoes some displacement, then work is said to be done.

The amount of work done is equal to the product of force and displacement in the direction of force.

Let, P =force acting on the body and

S = distance through which the body moves

Then work done, $W = P \times S$

Sometimes the force and displacement are not collinear.

In such a case, work done is expressed as the product of the component of the force in the direction of motion and the displacement.

Hence, work done $W = P \cos\theta \times s$

If θ =90, $\cos\theta = 0$ and there will be no work done i.e. if force and displacement are at right angles to each other, work done will be zero.

Similarly, work done against the force is taken as negative.

When the point of application of the force moves in the direction of motion of the body, work is said to be done by the force.

Work be done by the force is taken as +ve.

As work is the product of force and displacement, the units of work depend upon the units of force and displacement. Work is expressed in N-m or KN-m.

One Newton-meter is the work done by a force of 1N in moving the body through 1m. It is called Joule. 1 J = 1 N-m .

Similarly, 1 Kilo Newton-meter is the work done by a force of 1 KN in moving a body through 1m. It is also called kilojoules.

1KJ =1 KN-m

POWER

Power is defined as the rate of doing work.

In SI units, the unit of power is watt (briefly written as W) which is equal to 1 N-m/s or 1 J/s.

It is also expressed in Kilowatt (KW), which is equal to 10^3 W and Megawatt (MW) which is equal to10^6W. In case of engines, the following two terms are commonly used for power.

INDICATED POWER:

It is the actual power generated in the engine cylinder.

BRAKE POWER:

It is the amount of power available at the engine shaft.

EFFICIENCY OF AN MACHINE:

Efficiency of engine is expressed as the ratio of brake power to the indicated power. It is also called Mechanical efficiency of an engine.

Mechanical efficiency : $\frac{B.P}{I.P} \times 100$

ENERGY

Energy may be defined as the capacity for doing work.

Since energy of a machine is measured by the work it can do, therefore unit of energy is same as that of work.

In S.I system, energy is expressed in Joules or Kilo joules and N-m/ KN-m.

There are two types of mechanical energy.

There are two types of Energy

1. Potential Energy and 2. Kinetic Energy.

2. POTENTIALENERGY:

It is the energy possessed by a body by virtue of its position.

A body at some height above the ground level possesses potential energy.

If a body of mass (m) is raised to a height(h) above the ground level, the work done in raising the body is

= Weight of the body × distance through which it raised

= (mg) × h= mgh

This work (equal to mgh) is stored in the body as potential energy. The body, while coming down to its original level, can do work equal to mgh. Potential energy is zero when the body is on the earth.

3. KINETICENERGY:

It is the energy possessed by a body by virtue of its motion.

We can measure kinetic energy of a body by finding the work done by the body against external force to stop it.

Let, m = Mass of the body

u = initial velocity of the body.

P = External force applied on the body to bring it to rest.

a = Constant retardation of the body

S = distance travelled by the body before coming to rest

As the body comes to rest, its final velocity,

$v = 0$

Work done, W= Force × Distance = P × s ....(i)

Now substituting value of (P = m.a) in equation(i),

W= ma × s = mas ……………(ii)

We know that $v^2 - u^2 = -2as$ (minus sign due to retardation)

$$0 - u^2 = -2as$$

$$u^2 = 2as$$

$$as = \frac{u^2}{2}$$

Now substituting value of (as) in equation (ii) and replacing work done with kinetic energy,

$$K.E = \frac{mu^2}{2}$$

In most of cases, the initial velocity is taken as v (instead of u), therefore kinetic energy,

$$K.E = \frac{mv^2}{2}$$

MOMENTUM AND IMPULSE

MOMENTUM:

It is the product of mass and velocity of a body. It represents the energy of motion stored in a moving body.

If, m = mass of a moving body in kg

v=velocity of the body in m/sec,

∴ Momentum of the body=mv kg-m/sec

IMPULSE:

It is defined as the product of force and time during which the force acts on the body. Mathematically,

Impulse = Force × Time interval

Let force P act on a body for a time t.

According to the second law of motion,

Force = Mass × Acceleration.

$$P = ma$$

$$= m\left(\frac{v-u}{t}\right)$$

$$P \times t = m(v-u)$$

Hence, Impulse, I = P × t = mv–mu

where, v = Final velocity and u=Initial velocity

We have mu = momentum of the body at the beginning of motion and mv = momentum of the body after time t.

From equation (1), we see that change in linear momentum per unit time is directly proportional to the external force or applied force and takes place in the direction of force.

Hence, Impulse, $I = F \times t = mv - mu$

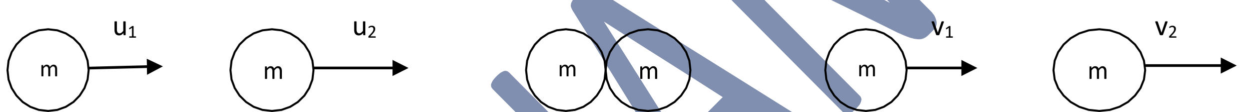
i.e. impulse is equal to change in momentum

Equation (2) is known as impulse–momentum relation.

LAW OF CONSERVATION OF LINEAR MOMENTUM

It states that "the total momentum of two bodies remains constant after their collision or any other mutual action. And no external forces action the bodies, the algebraic sum of their momentum along any direction is constant.

Momentum along a straight line is called linear momentum



If a body of mass m_1 moving with velocity u_1 collides with another body of mass m_2 moving with velocity u_2.

Let v_1 and v_2 be the velocities of the bodies after collision.

We have:

Total momentum before collision = $m_1u_1 + m_2u_2$

Total momentum after collision = $m_1v_1 + m_2v_2$

Now, according to the law of conservation of linear momentum,

$\therefore$ Momentum before collision = momentum after collision

$$\Rightarrow \quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

LAW OF CONSERVATION OF ENERGY

It states that " The energy can neither be created nor destroyed, though it can be transformed from one form into any of the forms, in which the energy can exist."

Suppose a body of mass„ m" is at a height 'h" dropped on the ground from A. Consider the ground level as the datum or reference level and other positions of B and C of the same body at various times of the fall.

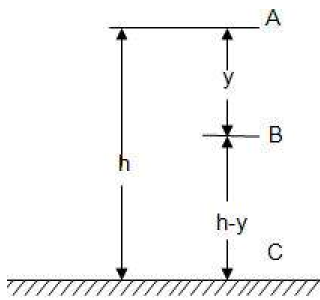


Fig 6.3

Total energy of the body at these points (A, B, C)

Energy at A

At A, the body has no velocity, therefore kinetic energy at A=0 and potential energy at A= mgh.

∴ Total energy at A = mgh ……...............(i)

Energy at B

At B, the body has fallen through a distance(y).

Therefore velocity of the body at B

$= \sqrt{2gy}$

∴ Kinetic energy at B $= \frac{mv^2}{2} = \frac{m(\sqrt{2gy})^2}{2} = mgy$

Potential energy at B = mg(h-y)

Total energy at B = mgy + mg(h-y) =mgh ……(ii)

Energy at C

We know that at C , the body has fallen through a distance (h). Therefore ,

Velocity of the body at C $= \sqrt{2gh}$

Kinetic energy at C $= \frac{mv^2}{2} = \frac{m(\sqrt{2gh})^2}{2}$ = mgh

And potential energy at C=0

∴Total energy at C = mgh ………….(iii)

It shows that in all positions, the sum of kinetic and potential energies of a body remains constant under the action of gravity.

COLLISION OF ELASTIC BODIES

Collision means the interaction or the contact between two bodies for a short period of time. The bodies produce impulsive forces one another during collision.

The act of collision between two bodies that takes place in a short period of time and during which the bodies exert very large forces on each other, is known as impact.

The bodies come to rest for a moment immediately after collision. During the phenomenon of collision, the bodies tend to compress each other.

The bodies tend to regain their actual shape and size after impact, due to elasticity. The process of getting back the original shape is called restitution.

The time of compression is the time taken by the two bodies in compression, immediately after collision and the time of restitution is the time of regaining the original shape after collision. The period of collision is the sum of the time of compression and restitution.

NEWTON'S LAW OF COLLISION OF ELASTIC BODIES AND COEFFICIENT OF RESTITUTION

Newton's law of collision of elastic bodies states that " when two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach".

Let us consider two bodies A and B of masses m_1 and m_2 respectively move along the same line and produce direct impact.

Let u_1 = initial velocity of body A

u_2 = initial velocity of body B

v_1 = final velocity of body A after collision

v_2 = final velocity of body B after collision

The impact will take place when $u_1 > u_2$

Hence the velocity of approach = $u_1 - u_2$

After impact, the separation of the two bodies will take place if $v_2 > v_1$.

Hence the velocity of separation = $v_2 - v_1$

According to Newton's law Collision of Elastic bodies,

$$\Rightarrow \quad (v_2-v_1) = e\,(u_1-u_2)$$

where, e = a constant of proportionality known as coefficient of restitution.

The value of e" lies between 0 and1.

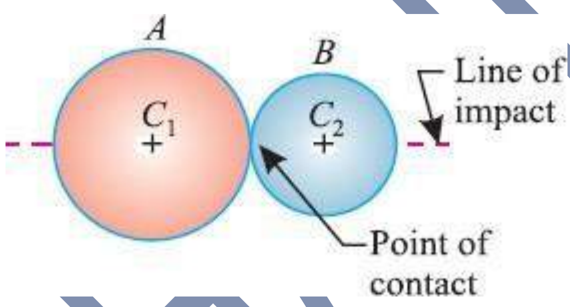
If e = 0, it indicates that the two bodies are inelastic.

If e =1, it indicates that the two bodies are perfectly elastic.

DIRECT COLLISION OF TWO BODIES:

If two bodies, before impact, are moving along the line of impact, the collision is called direct impact.

The line of impact, of the two colliding bodies, is the line joining the centres of these bodies and passes through the point of collision.



According to law of conservation of linear momentum, we have,

$$\mathbf{m_1u_1+m_2u_2=m_1v_1+m_2v_2}$$

DIRECT IMPACT OF A BODY WITH A FIXED PLANE

If one body is at rest initially, then such a collision is called direct impact.

Consider direct impact of a body with a fixed plane.

Let, u=initial velocity of the body

v =final velocity of the body

e=coefficient of restitution

Here, as the fixed plane will not move even after impact. Thus the velocity of approach is u and velocity of separation is v .According to Newton's law of elastic bodies, we have,

$$v = eu$$

1. In such cases we do not apply the principle of momentum (i.e. equating the initial momentum and the final momentum), since the fixed plane has infinite mass.

2. If a body is allowed to fall from some height on a floor, then the velocity, with which the body impinges on the floor, should be calculated by the relations of the plane motion as discussed below:

   Let H = Height from which the body is allowed to fall.

   Velocity with which the body impinges on the floor,

$$u = \sqrt{2gH}$$