QUESTION BANK

ENGINEERING MATHEMATICS –I

1ST SEMISTER MECHANICAL

MODULE-1

MATRICES AND DETERMINANT

SHORT ANSWER TYPE QUESTION (2MARKS AND 5MARKS)

1. Solve $\begin{vmatrix} 4 & x+1 \\ 3 & x \end{vmatrix} = 5$.
2. Find the value of $\begin{vmatrix} -6 & 0 & 0 \\ 3 & -5 & 7 \\ 2 & 8 & 11 \end{vmatrix}$.
3. Find the minimam value of $\begin{vmatrix} \sin x & \cos x \\ -\cos x & 1+\sin x \end{vmatrix}$.
4. If $\begin{vmatrix} a & b & c \\ b & a & b \\ x & b & c \end{vmatrix} = 0$, find x.
5. Solve by cramers rule 2x-y=3 , x+2y=4.

   Find the value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$.
6. Construct a 2×3 matrix having elements given by a_{ij}=i+j .
7. Find x and y when $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.
8. Find the adjoint of matrix $\begin{bmatrix} 1 & -w \\ w^2 & 1 \end{bmatrix}$.
9. Given $[x \quad y \quad z]$-$[-4 \quad 3 \quad 1]$=$[-5 \quad 1 \quad 0]$.
10. Write down the matrix $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ if a_{ij}=2i+3j.
11. Find the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$.
12. Solve the determinant $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}$=0.

13. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$=abc$\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

14. Solve by cramers rule x+y+z=3

2x+3y+4z=9

X+2y-4z=-1.

15. Prove that $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$= abc (a-b)(b-c)(c-a).

16. Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

17. Find the adjoint of the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 1 & 3 & -2 \end{bmatrix}$

18. Find the inverse of the matrix$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

19. Solve by matrix method x-y+z=4, 2x+y-3z=0,x+y+z=2.

<u>LONG QUESTIONS (10 MARKS)</u>

1. (a) find the adjoint of the matrix $\begin{bmatrix} -2 & 2 & 3 \\ 1 & 4 & 2 \\ -2 & -3 & 1 \end{bmatrix}$.

(b) Solve by matrix method x+y-z=6,2x-3y+z=1,2x-4y+2z=1.

2. Prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$=(a-b)(b-c)(c-a)(ab+bc+ca).

3. Solve by cramers rule x-y+z=1,2x+3y-5z=7,3x-4y-2z=-1.

4. Prove that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$=$x^2$(x+a+b+c).

5. (a) Verify $(AB)^T$=B^TA^T where A=$\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$,B=$\begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$.

(b) Find the inverse of $\begin{bmatrix}1 & 2 & 3\\2 & 1 & 4\\1 & 0 & 2\end{bmatrix}$.

MODULE-2

TRIGONOMETRY

SHORT ANSWER TYPE QUESTION (2MARKS AND 5MARKS)

1. Find the value of $\tan(-840)^\circ$.
2. Find the value of $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3$.
3. Prove that $2\sin 105^\circ.\sin 15^\circ$.
4. Find the value of $\frac{\tan 15^\circ}{1-\tan^2 15}$.
5. If $\tan\alpha=\frac{1}{2}$,$\tan\beta=\frac{1}{3}$ then find the value of $(\alpha+\beta)$.
6. If $\frac{1+\sin A}{\cos A}=\sqrt{2}+1$, then find the value of $\frac{1-\sin A}{\cos A}$.
7. Find the minimum value of $\sin\theta.\cos\theta$.
8. Find the value of $\tan\frac{\pi}{20}\tan\frac{3\pi}{20}\tan\frac{5\pi}{20}\tan\frac{7\pi}{20}\tan\frac{9\pi}{20}$.
9. Find the value of $\sin^2 24^\circ-\sin^2 6^\circ$.
10. Find the value of $\cos\left[\sin^{-1}\left(\frac{-1}{2}\right)\right]$.
11. Prove that $\sin 20^\circ.\sin 40^\circ.\sin 60^\circ.\sin 80^\circ=\frac{3}{16}$.
12. Find the value of $\sin 18^\circ\ and\ \cos 36^\circ$.
13. If A+B+C=π,then prove that $\sin 2A+\sin 2B+\sin 2C=4\sin A.\sin B.\sin C$
14. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}\ show\ that\ xy+yz+zx=1.$
15. Prove that $\frac{\cos 9^\circ+\sin 9^\circ}{\cos 9^\circ-\sin 9^\circ}=\tan 54^\circ$.
16. Find the maximum and minimum values of $5\sin x+12\cos x$.
17. Find the value of $\tan 75^\circ$ and hence prove that $\tan 75^\circ+\cot 75^\circ=4$
18. Prove that $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$.

LONG QUESTIONS (10 MARKS)

1. If A+B=45° Prove that

(i) (1+tan A)(1+tan B)=2. Deduce the value of tan $22\frac{1}{2}^{\circ}$.

(ii) (cot $A-1$)(cot $B-1$)=2.

2. (a) Prove that cot $7\frac{1}{2}^{\circ} = \sqrt{6}+\sqrt{3}+\sqrt{2}+2$.

(b) Prove that $\sin A+\sin B+\sin C = 4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$.

3. If $\sin A = K\sin B$, prove that $\tan\frac{1}{2}(A-B) = \frac{K-1}{K+1}\tan\frac{1}{2}(A+B)$.

4. Prove that $2\cos\frac{\pi}{16} = \sqrt{2+\sqrt{2+\sqrt{2}}}$.

5. (i) If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$, *show that* $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}$.

(ii) If $\sec(\emptyset+\alpha)+\sec(\emptyset-\alpha) = 2\sec\emptyset$,show that $\cos\emptyset = \sqrt{2}\cos\frac{\alpha}{2}$.

6. (a) Prove that $\cos^2 A+\cos^2 B+2\cos A.\cos B.\cos C = \sin^2 C$.

(b) Prove that $2\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{4} = \tan^{-1}\frac{8}{53}$.

MODULE-3

TWO DIMENSIONAL GEOMETRY

SHORT ANSWER TYPE QUESTION (2MARKS AND 5MARKS)

1. Find the distance between the points P(-3,2) and Q(2,-1) .
2. If the area of the triangle with the vertices (0,0), (1,0), (0,a) is 10 units,find the value of a?
3. Find the equation of a line which cuts off an intercept -2 on the axis of "y" and makes an angle 45° with positive direction of x-axis.
4. Find the co-ordinate of the point dividing the joining of (3,7) and (-1,-5) internally in the ratio 2:3.
5. Find the equation of the line passing through (-1,2) and making intercepts on the y-axis.
6. Reduce 3x+5y+4=0 to the intercept form and y-intercept.
7. Find the centre and radius of the circle $2x^2+2y^2-5x+3y-11=0$.
8. Determine the distance between the parallel lines x+5=0 and x-5=0.
9. Find the equation of a circle with centre (–3, 2) and radius 7.
10. Determine the equation of the straight line parallel to x-axis and passing through (3,4).

11. Find the equation of a straight line that passes through the point (3,4) and perpendicular to the line 3x+2y+5=0.

12. Find the equation of line passing through the point of intersection of lines x+3y+2=0 and x-2y-4=0 and perpendicular of the line 2y+5x-9=0.

13. Find the equation of straight line passing through (-2,3) and sum whose intercept is 2.

14. Find the equation of bisecting the line segment joining (3,-4) and (1,2) at right angle.

15. Show that the points A(-1,4),B(0,2),C(2,-2) are collinear.

16. Find the equation of a circle whose end points of diameter are (-5, 3) and (7,5).

17. Find the equation of the line through the point of intersection of 3x+4y-7=0 and x-y+2=0 and which is parallel to the line 5x-y+11=0.

18. Show that the points (1,1),(4,4),(4,8) and (1,5)are the vertices of the parallelogram.

19. Find the co-ordinate of the point which divide internally and externally the line joining (1,-3) and (-3,9) in the ratio 1:3.

20. Find the equation of the circle passing through the points (3, 4) (4, –3) and (–3, 4).

LONG QUESTIONS (10 MARKS)

1.(a) Find the equation of the line passing through the intersection of 2x – y – 1 = 0 and 3x-4y+ 6 = 0 and parallel to the line x + y – 2 = 0

(b) Find the equation of the circle passing through the points (1,-2) and its centre at the point of intersection of lines 2x-y+3=0 and x+2y-1=0 .

2. (a) Find the co-ordinates of the foot of the perpendicular from the point (2, 3) on the line 3x-4y +7=0

(b) Find the equation of the line passing through (-4, 2) and parallel to the line 4x-3y=0.

3. Find the equations of straight lines passing through the point (3,-2) and making an angle 45° with the line 6x+5y=1.

4. Find the distance of the point (3,2) from the line x+y-1=0,measured parallel to the line 3x-4y+1=0 .

5. Find the equation of the circle whose Centre is on the line 8x+5y=0 and the circle passing through the points (2,1) and (3,5) .

MODULE-4

THREE DIMENSIONAL GEOMETRY

1. Determine the Centre and radius of the sphere $x^2 + y^2 + z^2 - 4x + 6y - 8z + 1 = 0$.
2. Determine the value of k such that the planes r + 3y + kz = 5 and kr +y+ 22= 0 are perpendicular to each other.
3. Find the image of the point (-6, 2, -3) w.r.t yz-plane.
4. Find the value of k such that the points (1,-2,3)(3,-1,2)and(7,1,k) are collinear .
5. Find out the equation of the plane passing through (1,1,2)and parallel to x+y+z-1=0.
6. Find the distance between the parallel planes x-y+z+1=0andy-z-x+1=0 .
7. Find the equation of the sphere with Centre (3,-2,5) and radius 4.
8. Find the direction cosines of the line passing through the two points (– 2, 4, – 5) and (1,2,3).
9. Find the distance of the point P(x,y,z) from z-axiz.
10. Find the projection of the line segment joining (1,3,-1)and(3,2,4) on z-axis .
11. Find the equation of the plane which passes through the point (1,-1, 4) and is parallel to the Plane 2x+3y+7z=11.
12. Find the angle between two planes 2x+2y-3z=5 and 3x-3y+5z=3.
13. Find the foot of the perpendicular drawn from the point (0,0,0) on the plane 2x+y+z-3=0.
14. Find the equation of the sphere on the join of (2,3,5) and (4,9,-3) as diameter ?
15. Find the equation of the sphere with its centre at (1,-2,3) and touching the plane 2x-3y+z+6=0.
16. Show that points (0,1,2),(2,5,8),(5,6,6) and (3,2,0) are the vertices of the parallelogram.
17. Find the ratio in which the line joining the points (2,-3,1),(3,-4,-5) by the laces 2x+y+z=7.
18. Show that A(0,0,0),B(3,4,5),C(-3,-4,-5) are collinear.

LONG QUESTIONS (10 MARKS)

1. Find the equation of the sphere passing through the point (1, 2, -3) and (3, -1, 2) and centre lying on y-axis.
2. Show that the points A(1,2,3),B(-1,-2,-1),C(2,3,2) and D(4,7,6) are the vertices of a parallelogram *ABCD*,but it is not a rectangle.
3. Find the equation of the plane which is perpendicular to the plane 5x+3y+6z+8=0 and contains the line of intersection of the plane x+2y+3z-4=0 and 2x+y-z+5=0.
4. Find the equation of the sphere which passes through the points (0,0,0),(0,1,0),(1,0,0) and (0,0,1).

5. (a) Find the equation of sphere with its centre at (1,-2,3) and touching the plane 2x-3y+z+6=0.

(b) Find the equation of the plane through the points (2,1,0) and passing through intersection of the planes 3x-2y+z-1=0 and x-2y+3z-1=0

6. Find the equation of the plane containing the line of intersection of the plane x+y+z+1=0 , 2x-3y+5z-2=0 and passing through the point (-1,2,1).