QUESTION BANK

ENGINEERING MATHEMATICS –II

2ND SEMISTER (All Branches)

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MODULE-1

VECTOR ALGEBRA

SHORT ANSWER TYPE QUESTIONS (2MARKS)

1. Show that the points A (2,6,3), B (1,2,7) and C (3,10,-1) are collinear.
2. Find the value of P for which the vectors $\vec{a}= 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + P\hat{\jmath} + 3\hat{k}$ are perpendicular.
3. Find the area of parallelogram whose adjacent sides are determind by the vectors $\vec{a}= \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$.
4. Find the area of triangle whose adjacent sides are determind by the vectors $\vec{a}= 3\hat{\imath} + 4\hat{\jmath}$ and $\vec{b} = -5\hat{\imath} + 7\hat{\jmath}$.
5. If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$,find $\vec{a}.\vec{b}$.
6. If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$,find an angle between $\vec{a}$ *and* $\vec{b}$.
7. Find the magnitude and direction of $5\hat{\imath} + 3\hat{\jmath} - \hat{k}$.
8. If $\vec{a}.\vec{b} = 0$, then find the angle between $\vec{a}$ *and* $\vec{b}$.
9. What is the projection of vector of the vector $\vec{a}= 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$.
10. If vectors $\vec{a}= 2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ and $\vec{b} = \alpha\hat{\imath} - \hat{\jmath} + 2\hat{k}$ are parallel ,then find the value of α.

SHORT ANSWER TYPE QUESTIONS (5MARKS)

11. Show that the vectors $\hat{\imath}-2\hat{\jmath}+3\hat{k}$,$-2\hat{\imath}+3\hat{\jmath}-4\hat{k}$ and $\hat{\imath}-3\hat{\jmath}+5\hat{k}$ are coplanar.
12. Find the area of the triangle whose adjacent sides are determined by the vectors $\vec{a}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$ and $\vec{b}=3\hat{\imath}-2\hat{\jmath}+\hat{k}$.
13. Find the unit vector perpendicular to the vectors $\hat{\imath}-3\hat{\jmath}+\hat{k}$ and $-\hat{\imath}+2\hat{\jmath}-\hat{k}$.
14. If $\vec{a} + \vec{b} + \vec{c} = 0$, show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.
15. Find the sine angle between the vectors $\vec{a}$ *and* $\vec{b}$ where $\vec{a}= 2\hat{\imath}-\hat{\jmath}+3\hat{k}$ and $\vec{b} = \hat{\imath}+3\hat{\jmath}+2\hat{k}$.

LONG QUESTIONS (10 MARKS)

16. In a triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
17. If $\vec{a}, \vec{b}$, $\vec{c}$ are any three vectors , prove that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$.
18. Find the area of triangle formed by the point A(1,2,3) , B(-2,1,-4) and C(3,4,-2).

MODULE-2

LIMIT AND CONTINUITY

SHORT ANSWER TYPE QUESTIONS (2MARKS)

1. Evaluate $\lim_{x\to 1} \left(\frac{x^2-2x+1}{x^2-x}\right)$.
2. For what value of K $f(x) = \begin{cases} x^2 - a^2, & if\ x \neq a \\ K \quad , & if\ x = a \end{cases}$ is continuous at x=a.
3. Evaluate $\lim_{x\to 1} \left(\frac{\frac{1}{x^2} - \frac{1}{4}}{x-2}\right)$.
4. Evaluate $\lim_{x\to 0} \left(\frac{\tan 7x}{\tan 5x}\right)$.
5. Find $\lim_{x\to \frac{5}{2}} [x]$.
6. Evaluate $\lim_{x\to 0} \frac{1-\cos x}{x}$.
7. If f*(x)* = $\frac{x^2-16}{x-4}$, x$\neq 4$ is continuous at x=4, what is the value of *f(4).*
8. Find the value of "a" so that *f(x)*=$\frac{\sin^2 ax}{x^2}$,x$\neq a$, *f(0)*=1, is continuous at x=0.
9. Write the value of $\lim_{x\to 0} \frac{3^x-1}{4^x-1}$.
10. Find $\lim_{n\to \infty} \frac{n!}{(n+1)!-n!}$.
11. Find $\lim_{x\to 0} [x] + 10$.

SHORT ANSWER TYPE QUESTIONS (5MARKS)

12. Evaluate $\lim_{x\to 0} \left(\frac{x \tan x}{1-\cos x}\right)$.
13. Examine the continuity of the function defined by $f(x) = \begin{cases} \frac{|x|}{x}, if\ x \neq 0 \\ 0,\ if\ x = 0 \end{cases}$ at x=0 .
14. Evaluate $\lim_{x\to 0} \frac{x - x\cos 2x}{\sin^3 2x}$.

15. If $f(x) = \begin{cases} \frac{x-|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ at x=0 .

Show that $\lim_{x \to 0} f(x)$ does not exist .

16. Evaluate $\lim_{n \to \infty} \frac{1^2+2^2+\cdots\cdots+n^2}{n^3}$.

17. Show that the function $f(x) = \begin{cases} \frac{x}{|x|}, & when\ x \neq 0 \\ 1, & when\ x = 0 \end{cases}$ is discontinuous at x=0.

18. Evaluate $\lim_{x \to 0} \frac{(x+9)^{\frac{3}{2}}-27}{x}$.

19. Evaluate $\lim_{x \to 0} \left(\frac{\csc x - \cot x}{x}\right)$.

20. Examine the continuity of the function *f(x)* at x=0 by $f(x) = \begin{cases} \frac{\sin 3x}{x}, & if\ x \neq 0 \\ 3, & if\ x = 0 \end{cases}$ at x=0.

LONG QUESTIONS (10 MARKS)

21. Find the value of "a" if $\lim_{x \to 2} \frac{\log_e(2x-3)}{a(x-2)} = 1$.

22. If *f(x)* = $\begin{cases} ax^2 + b, & if\ x < 1 \\ 1, & if\ x = 1 \\ 2ax - b, & if\ x > 1 \end{cases}$ is continuous at x=1, then find a and b.

23. Examine the continuity of the functions at $f(x) = \begin{cases} 2x+1, & if\ x \leq 0 \\ x, & if\ 0 < x \leq 1 \\ 2x-1, & if\ x \geq 1 \end{cases}$ at x=0.

24. Evaluate :

a) $\lim_{x \to \infty} x^2\left\{\sqrt{x^4+a^2} - \sqrt{x^4-a^2}\right\}$.

b) $\lim_{x \to 1} \frac{\log_e(2x-1)}{(x-1)}$.

25. Discuss the continuity of the function at x=$\frac{1}{2}$, *f(x)* = $\begin{cases} x, & 0 \leq x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ 1-x, & \frac{1}{2} < x \leq 1 \end{cases}$.

MODULE-3

DERIVATIVES

SHORT ANSWER TYPE QUESTIONS (2MARKS)

1. Differentiate $\log(\sin x)$ w.r.t $\tan x$.

2. Find order and degree of the differential equation $\frac{d^2y}{dx^2} = \left\{2 + \left(\frac{dy}{dx}\right)^3\right\}^{\frac{1}{2}}$.
3. If z= $\tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$.
4. Find y_1 and y_2 if y=$\log(\cos x)$.
5. If u=t^2 and v=$\sin t^2$, then find $\frac{dv}{du}$.
6. If *f(x, y)* =e^{xy}, then find y.$\frac{\partial f}{\partial y}$.
7. Find derivatives of $\sqrt{x}$ w.r.t x^2.
8. If y=$c_1e^x + c_2e^{-x}$, then find $\frac{d^2y}{dx^2}$.
9. Find the derivative of $\sin^{-1}(3x)$.
10. Determine the slope of the curve y=$\tan x$ at x=$\frac{\pi}{4}$.

SHORT ANSWER TYPE QUESTIONS (5MARKS)

11. If y=$\sin^{-1} x$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$.
12. If y=$\tan^{-1} x$, prove that $(1 + x^2)y_2 + 2xy_1$.
13. If *f(x, y)* =$\frac{2x-3y}{x^2+y^2}$, find $f_x(1,2)$ *and* $f_y(1,2)$.
14. Find $\frac{dy}{dx}$ if $x^y = y^x$.
15. Obtain $\frac{dy}{dx}$ when x=$a(\cos u + u \sin u)$ *and* $y = a(\sin u + u \cos u)$.
16. If u=$x^2y + y^2z + z^2x$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x + y + z)^2$.
17. Differentiate, $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$.
18. Find $\frac{dy}{dx}$, where $(\cos x)^y$=$(\sin y)^x$.

LONG QUESTIONS (10 MARKS)

19. Differentiate $\sin^2\left\{\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right\}$.
20. If *V*=$(x^2 + y^2 + z^2)^{\frac{1}{2}}$, show that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$.
21. Differentiate $\tan^{-1}(\sec x + \tan x)$.
22. If y=$(\sin^{-1} x^2)^2$, show that $(1 - x^2)y_2 - xy_1 - 2 = 0$.
23. If y=$e^{m \sin^{-1} x}$, then prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$.

24. If z=$\sin^{-1}\left(\frac{xy}{x+y}\right)$ show that x$\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\tan z$.

MODULE-4

INTEGRATIONS

SHORT ANSWER TYPE QUESTIONS (2MARKS)

1. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$.
2. Evaluate $\int \frac{e^{2x}+1}{e^x}dx$.
3. Evaluate $\int e^{(5x+3)}dx$.
4. Evaluate $\int \sqrt{1+sin2x}\,dx$.
5. Write the value of$\int |x|dx$, when x<0.
6. Evaluate $\int x\cos x\,dx$.
7. Evaluate $\int \frac{dx}{1+e^{-x}}$.—
8. Evaluate $\int_{-1}^{1}|x|dx$.
9. Evaluate $\int_1^3 [x]\,dx$.
10. Evaluate $\int e^x.a^x\,dx$.
11. Write the value of $\int \sqrt{x^2-a^2}dx$?

SHORT ANSWER TYPE QUESTIONS (5MARKS)

12. Evaluate $\int \tan^{-1}(\sec x+\tan x)\,dx$.
13. Evaluate $\int \sin^4 x.\cos^3 x\,dx$.
14. Prove that $\int \frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}+c$.
15. Evaluate $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$.
16. Evaluate $\int \log(1+x^2)dx$.
17. Evaluate $\int_0^{\frac{\pi}{2}} \log\tan x dx$.

18. Evaluate $\int_{-2}^{2}([x] + |x|)\, dx$.

19. Find the area bounded by the curve $xy = C^2$, the x-axis and x=2 ,x=3.

20. Find the area under the curve $y = \sqrt{a^2 - x^2}$ between x=0 and x=a.

LONG QUESTIONS (10 MARKS)

21. Find the whole area of circle $x^2 + y^2 = a^2$.

22. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$.

23. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}} dx$.

24. Evaluate $\int_0^{1} x \log(1 + x)\, dx$.

25. Prove that $\int \sqrt{a^2 + x^2 dx} = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|\left(\frac{x+\sqrt{x^2+a^2}}{a}\right)\right| + c$.

MODULE-5

DIFFERENTIAL EQUATION

SHORT ANSWER TYPE QUESTIONS (2MARKS)

1. Solve the differential equation $\frac{dx}{dy} + \sqrt{\frac{1-x^2}{1-y^2}} = 0$.
2. What is the order and degree of the equation $\left(\frac{dy}{dx}\right)^4 + \frac{d^3y}{dx^3} = 3yx\frac{dy}{dx}$.
3. Solve $\frac{dy}{dx}$=$\log x$.
4. Solve $\frac{dy}{dx}$=$\sec^2 x$.
5. Define Homogeneous differential equation and give an example.
6. Find I.F of 4$\frac{dy}{dx}$+8y=5e^{-3x}.

SHORT ANSWER TYPE QUESTIONS (5MARKS)

7. Solve the equation : x(1+y^2)dx +y(1+x^2)dy=0.

8. Solve (1+x^2)dy+(1+y^2)dx=0.
9. Solve $\frac{dy}{dx} + y \tan x = \sec x$.
10. Solve:4$\frac{dy}{dx} + 8y = 5e^{-3x}$.
11. Solve (1+x^2)$\frac{dy}{dx} + 2xy - x^3 = 0$.

LONG QUESTIONS (10 MARKS)

12. Solve $\frac{dy}{dx} + (\sec x)y = \tan x$.
13. If $\frac{dy}{dx} + 2y \tan x = \sin x$, and y=0 for x=$\frac{\pi}{3}$, show that maximum value of y is $\frac{1}{8}$.
14. Solve $\frac{dy}{dx} = \frac{y-x}{x+y}$.
15. Solve $e^x \tan y dx + (1 + e^x) \sec^2 y dy = 0$.
16. Find the differential equation of the family of curves $y = e^x(A \cos x + B \sin x)$.