Lecture note

No

ENGINEERING MECHANICS (Th-4)

1st and 2nd Semester (Diploma Course)

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FUNDAMENTALS OF ENGINEERING MECHANICS

Definitions of Mechanics –

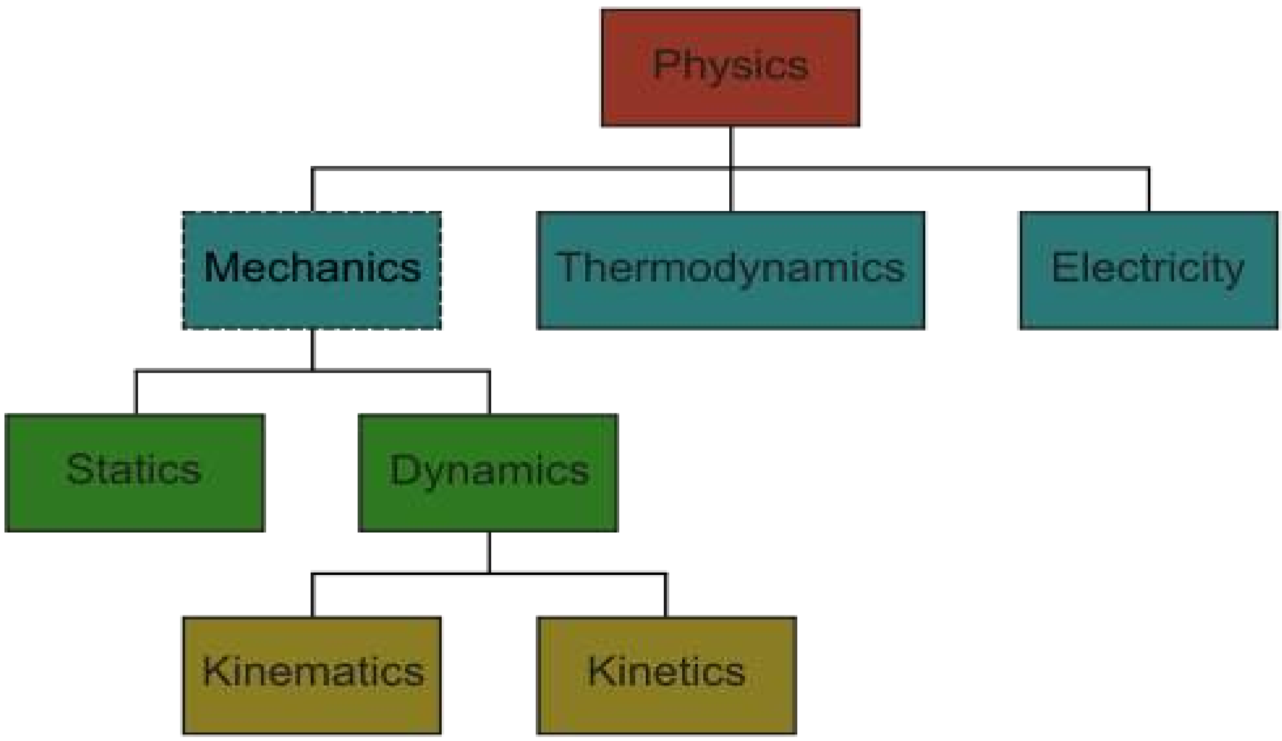
1. A branch of physical science that deals with energy and forces and their effect on bodies.

2. the practical application of **mechanics** to the design, construction, or operation of machines or tools

Definitions of enginnering Mechanics

The subject engineering mechanics is the branch of applied science which deals with the laws and principles of mechanics, along with their applications to engineering problems .

Sub division of Engg. Mechanics



1. Particle: A particle is defined as an object that has a mass but no size.
2. Body: A body is defined as the matter limited in all directions. It has a finite volume and finite mass.
3. Rigid Body: A body in which the particles do not change their relative positions under the action of any external force is called as Rigid Body. No body is perfectly rigid.
4. Deformable Body: A body in which the particles change their position under the action of any external force is called as Deformable body.
5. Mass: Mass of the body is the quantity of matter contained by the body.
6. Weight: The force with which the earth attracts any body to itself is called the weight of the body.

$$\boxed{W = m \cdot g}$$

7. Space: The unlimited universe in which all the materials are located is known as space. It is a three dimensional region.
8. Statics: It is the branch of engineering mechanics which deals with the study of bodies at rest under the action of forces.
9. Dynamics: It is the branch of engineering mechanics which deals with the study of bodies in motion.
10. Kinetics: This branch of dynamics is the study of the behaviour of bodies in motion without considering the forces which causing the motion.
11. Kinematics: The kinematics studies the behaviour of bodies in motion by considering the forces which causing the motion.
12. Force: It is the agent which changes or tends to change the state of rest or motion of a body.

Force

Defination –

Force is an external agent capable of changing the state of rest or motion of a particular body. It has a magnitude and a direction. The direction towards which the force is applied is known as the direction of the force and the application of force is the point where force is applied.

The Force can be measured using a spring balance. The SI unit of force is Newton(N).

Common symbols:	$\vec{F}$, F
SI unit:	Newton
In SI base units:	$kg \cdot m/s^2$

Other units:	dyne, poundal, pound-force, kip, kilo pond
Derivations from other quantities:	F = m a
Dimension:	LMT^{-2}

Classification of force system according to plane & line of action

System of Forces

When two, or more than two, forces act on a body, they are called to form a *system of forces*. Following systems of forces are important from the subject point of view :

1. *Coplaner forces*. The forces, whose lines of action lie on the same plane, are known as coplaner forces.
2. *Collinear forces*. The forces, whose lines of action lie on the same line, are known as collinear forces.
3. *Concurrent forces*. The forces, which meet at one point, are known as concurrent forces. The concurrent forces may or may not be collinear.
4. *Coplaner concurrent forces*. The forces, which meet at one point and their lines of action also lie on the same plane, are known as coplaner concurrent forces.
5. *Coplaner non-concurrent forces*. The forces which do not meet at one point, but their lines of action lie on the same plane, are known as coplaner non-concurrent forces.
6. *Non-coplaner concurrent forces*. The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplaner concurrent forces.
7. *Non-coplaner non-concurrent forces*. The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplaner non-concurrent forces.

Effects of a Force

A force may produce the following effects in a body, on which it acts :

1. It may change the motion of the body, *i.e.* if a body is at rest, the force may set the body in motion, and if the body is already in motion, the force may accelerate it.
2. It may retard the motion of a body.
3. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium. We shall study this effect in chapter 5 of this book.
4. It may give rise to the internal stresses in the body, on which it acts. We shall study this effect in chapters 12 and 13 of this book.

Characteristics of a Force

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force :

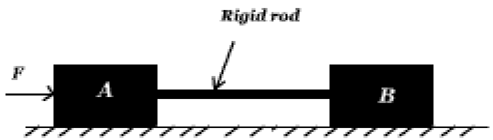
1. Magnitude of the force (*i.e.*, 10 kgf, 20 tf, 50 N, 15 kN, etc.)
2. The direction of the line, along which the force acts (*i.e.* along OX, OY or at 30° North or East etc.). It is also known as line of action of the force.
3. Nature of the force (*i.e.*, whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.
4. The point at which (or through which) the force acts on the body.

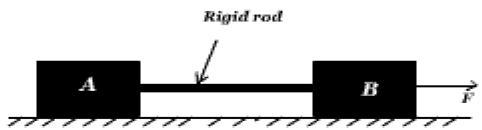
Principle of transmissibility

The state of rest or of motion of a rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the applied forces. In the following animation, two rigid blocks A and B are joined by a rigid rod. If the system is moving on a frictionless surface, the acceleration of the system in both the cases is given by,

Acceleration=Applied force/total mass

It is independent of the point of application

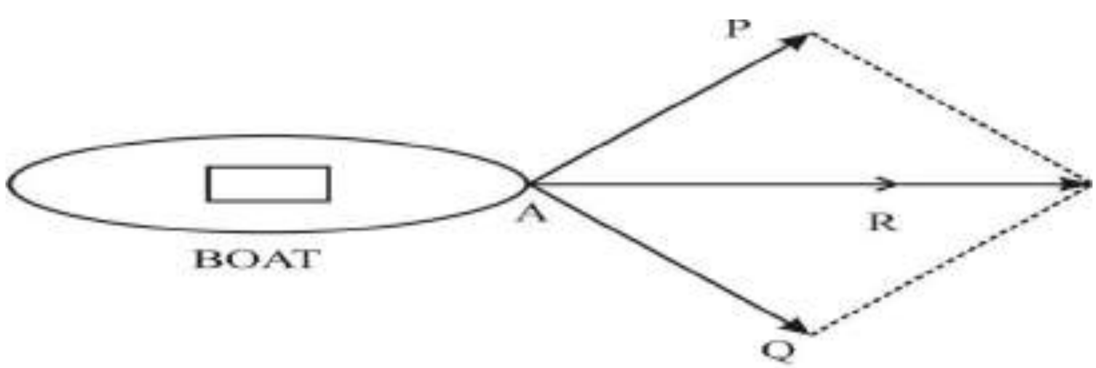




Principle of Superposition

This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces.

Consider two forces *P* and *Q* acting at *A* on a boat as shown in Fig.3.1. Let *R* be the resultant of these two forces *P* and *Q*. According to Newton's second law of motion, the boat will move in the direction of resultant force *R* with acceleration proportional to *R*. The same motion can be obtained when *P* and *Q* are applied simultaneously.



Principle of Superposition

<u>Action & Reaction Forces</u>

1. A force is a push or a pull that acts upon an object as a results of its interaction with another object.

2. Forces result from interactions but some forces result from *contact interactions* (normal, frictional, tensional, and applied forces are examples of contact forces) and other forces are the result of action-at-a-distance interactions (gravitational, electrical, and magnetic forces). According to Newton, whenever objects A and B interact with each other, they exert forces upon each other. When you sit in your chair, your body exerts a downward force on the chair and the chair exerts an upward force on your body. There are two forces resulting from this interaction - a force on the chair and a force on your body. These two forces are called *action* and *reaction* forces and are the subject of Newton's third law of motion. Formally stated, Newton's third law is:

For every action, there is an equal and opposite reaction.

The statement means that in every interaction, there is a pair of forces acting on the two interacting objects. The size of the forces on the first object <u>equals</u> the size of the force on the second object. The direction of the force on the first object is <u>opposite</u> to the direction of the force on the second object. Forces <u>always</u> come in pairs - equal and opposite action-reaction force pairs.

Concept of Free Body Diagram

Free-body Diagrams. To investigate the equilibrium of a constrained body, we shall always imagine that we remove the supports and replace them by the *reactions* which they exert on the body. Thus,

3.1. Free Body

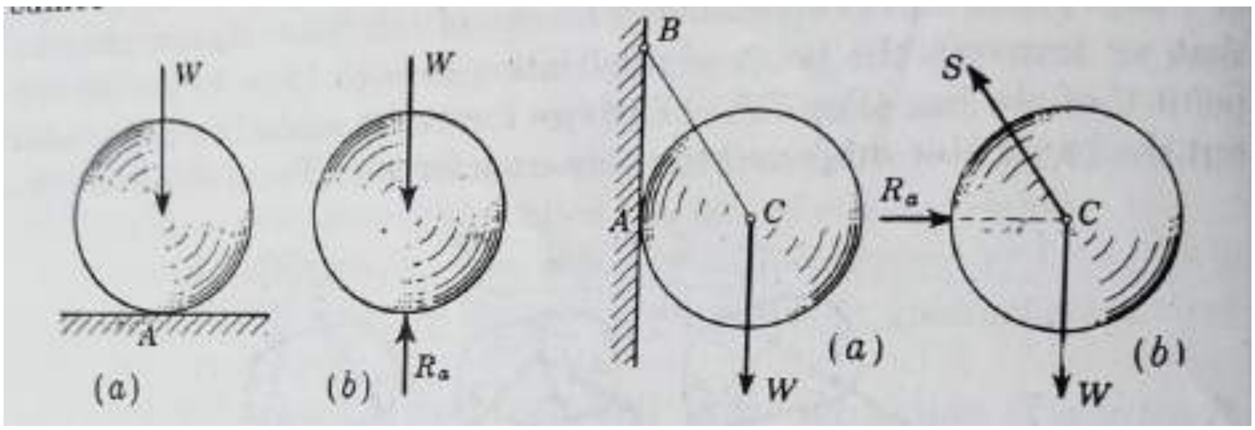
A body is said to be free body if it is isolated from all other connected members.

3.2. Free Body Diagram

Free body diagram of a body is the diagram drawn by showing all the external forces and reactions on the body and by removing the contact surfaces.

Steps to be followed in drawing a free body diagram

1. Isolate the body from all other bodies.
2. Indicate the external forces on the free body.
   (The weight of the body should also be included. It should be applied at the centre of gravity of the body.)
3. The magnitude and direction of the known external forces should be mentioned.
4. The reactions exterted by the supports on the body should be clearly indicated.
5. Clearly mark the dimensions in the free body diagram.



Resolution of a Force

The process of splitting up the given force into a number of components, without changing its effect on the body is called *resolution of a force*. A force is, generally, resolved along two mutually perpendicular directions.

In fact, the resolution of a force is the reverse action of the addition of the component vectors.

2·13. Principle of Resolution

It states, "*The algebraic sum of the resolved parts of a number of forces, in a given direction, is equal to the resolved part of their resultant in the same direction.*"

Proof

Now consider for simplicity, two forces P and Q; which are represented in magnitude and direction by the two adjacent sides OA and OB of a parallelogram $OACB$ as shown in Fig. 2·2.

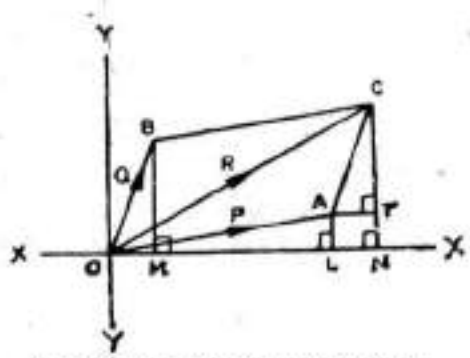


Fig. 2·2 Principle of resolution.

We know that the resultant (R) of these two forces P and Q will be represented, in magnitude and direction, by the diagonal OC of the parallelogram.

Let OX be the given direction, in which the forces are to be resolved. Now draw AL, BM, and CN perpendiculars from the points A, B and C on OX. Similarly, draw AT perpendicular from the point A on CN.

In the two triangles OBM and ACT, the two sides OB and AC are parallel and equal in magnitude. Moreover, the two sides OM and AT are also parallel.

$$\therefore \qquad OM = AT = LN$$

Now from the geometry of the figure, we find that

$$ON = OL + LN = OL + OM \quad \ldots (\because \; LN = OM)$$

But ON is the resolved part of the resultant R, OL is the resolved part of the force P, and OM is the resolved part of the force Q.

Hence resolved part of R along OX

= Resolved part of P along OX

+ Resolved part of Q along OX

Note: We have considered, for the sake of simplicity only, the two forces P and Q. But this principle may be extended for any number of forces.

2·14. Method of Resolution for the Resultant Force

The resultant force, of a given system of forces, may be found out by the method of resolution as discussed below:

1. Resolve all the forces vertically and find the algebraic sum of all the vertical components (*i.e.*, ΣV).

2. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (*i.e.*, ΣH).
3. The resultant R of the given forces will be given by the equation :

$$R = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}$$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan \theta = \frac{\Sigma V}{\Sigma H}$$

Note : The value of the angle θ will vary depending upon the values of ΣV and ΣH as discussed below :

1. When ΣV is +ve, the resultant makes an angle between 0° and 180°. But when ΣV is −ve, the resultant makes an angle between 180° and 360°.
2. When ΣH is +ve, the resultant makes an angle between 0° and 90° and 270° to 360°. But when ΣH is −ve, the resultant makes an angle between 90° to 270°.

Example 2·3. *A triangle ABC has its sides AB = 40 mm along positive x-axis and sides BC = 30 along positive y-axis. Three forces of 40 kgf, 50 kgf and 30 kgf act along the sides AB, BC and CA respectively. Determine the resultant of such a system of forces.*

(*Osmania University, 1985*)

Solution.

The system of the given forces is shown in Fig. 2·3. From the geometry of the figure, we find that the triangle ABC is a right angled triangle in which the *side AC = 50 mm. Moreover,

$$\sin \theta = \frac{30}{50} = 0{\cdot}6$$

and

$$\cos \theta = \frac{40}{50} = 0{\cdot}8$$

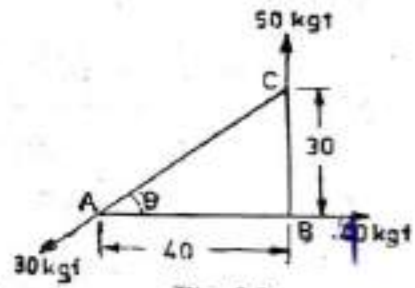


Fig. 2·3

Resolving all the forces horizontally (*i.e.* along AB)

$$\Sigma H = 40 - 30 \cos \theta = 40 - 30 \times 0{\cdot}8 = 16 \text{ kgf} \quad ...(i)$$

and now resolving all the forces vertically (*i.e.* along BC),

$$\Sigma V = 50 - 30 \sin \theta = 50 - 30 \times 0{\cdot}6 = 32 \text{ kgf} \quad ...(ii)$$

We know that the magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2} \text{ kgf}$$

$$= \mathbf{35{\cdot}8 \ kgf} \quad \textbf{Ans.}$$

Example 2·4. *The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting on one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.* (*Cambridge University*)

Solution.

The system of the given forces is shown in Fig. 2·4.

Magnitude of the resulant force

Resolving all the forces horizontally (*i.e.*, along AB),

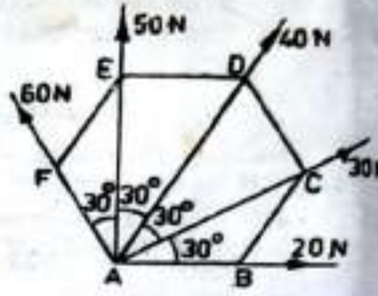


Fig. 2·4

$$\Sigma H = 20 \cos 0° + 30 \cos 30° + 40 \cos 60° + 50 \cos 90° + 60 \cos 120° \text{ N}$$

$$= (20 \times 1) + (30 \times 0{\cdot}866) + (40 \times 0{\cdot}5) + (50 \times 0) + 60(-0{\cdot}5) \text{ N}$$

$$= 36{\cdot}0 \text{ N} \qquad \ldots(i)$$

and now resolving the all forces vertically (*i.e.* at right angles to AB)

$$\Sigma V = 20 \sin 0° + 30 \sin 30° + 40 \sin 60° + 50 \sin 90° + 60 \sin 120° \text{ N}$$

$$= (20 \times 0) + (30 \times 0{\cdot}5) + (40 \times 0{\cdot}866) + (50 \times 1) + (60 \times 0{\cdot}866) \text{ N}$$

$$= 151{\cdot}6 \text{ N} \qquad \ldots(ii)$$

We know that magnitude of the resulant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(36{\cdot}0)^2 + (151{\cdot}6)^2} \text{ N}$$

$$= 155{\cdot}8 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant makes with the horizontal (*i.e.*, AB).

$$\therefore \quad \tan \theta = \frac{\Sigma V}{\Sigma H} = \frac{151{\cdot}6}{36{\cdot}0} = 4{\cdot}211$$

or $\theta = 76° \, 39'$ **Ans.**

Resultant Force

If a number of forces, P, Q, R......etc. are acting simultaneously on a particle, it is possible to find out a single force which could replace them *i.e.* which would produce the same effect as produced by all the given forces. This single force is called *resultant force*, and the given forces P, Q, R......etc. are called component forces.

Composition of Forces

The process of finding out the resultant force of a number of given forces is called *composition of forces* or compounding of forces.

Methods for the Resultant Force

Though there are many methods for finding out the resultant force of a number of given forces, yet the following are important from the subject point of view :

1. Analytical method, 2. Graphical method.

Analytical Method for Resultant Force

The resultant force, of a given system of forces, may be found out analytically by the following methods :

1. Parallelogram law of forces, 2. Method of resolution.

Parallelogram Law of Forces

It states "*If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram ; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection.*" Mathematically, resultant force,

$$R = \sqrt{P^2+Q^2+2PQ \cos\theta}$$

and $$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$$

where P and Q = Forces whose resultant is required to be found out,

θ = Angle between the forces P and Q, and

α = Angle which the resultant force makes with one of the forces (say P).

Note. If the angle (α) which the resultant force makes with the other force Q, then

$$\tan\alpha = \frac{P\sin\theta}{Q+P\cos\theta}$$

Cor.

1. If $\theta = 0$ *i.e.*, when the forces act along the same line, then
$R = P+Q$ ...(*since* $\cos 0° = 1$)

2. If $\theta = 90°$ *i.e.*, when the forces act at right angle, then
$R = \sqrt{P^2+Q^2}$ ...(*since* $\cos 90° = 0$)

3. If $\theta = 180°$ *i.e.*, when the forces act along the same straight line *but in opposite direction* then
$R = P-Q$ ...(*since* $\cos 180° = -1$)

In this case, the resultant force will act in the direction of the greater force.

4. If the two forces are equal *i.e.* when $P = Q$

then $R = \sqrt{P^2+P^2+2P^2\cos\theta} = \sqrt{2P^2(1+\cos\theta)}$

$$= \sqrt{2P^2\times 2\cos^2\frac{\theta}{2}} \qquad \cdots\left(\because\ 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right)$$

$$= \sqrt{4P^2\cos^2\frac{\theta}{2}} = 2P\cos\frac{\theta}{2}$$

Example 2·1. *Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.*

Solution

Given : $P = 40$ N ;

$\angle AOC = 120$;

$\angle BOC = 90°$

$\therefore\ \angle AOB$,

$\alpha = 120° - 90°$

$= 30°$

Let Q = Smaller force.

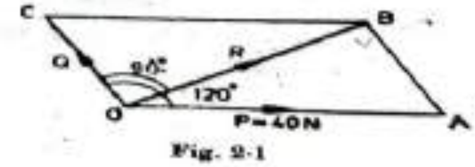


Fig. 2·1

We know that

$$\tan \alpha = \frac{Q \sin \theta}{P+Q \cos \theta}$$

$$\tan 30^\circ = \frac{Q \sin 120^\circ}{40+Q \cos 120^\circ} = \frac{Q \sin 60^\circ}{40+Q\,(-\cos 60^\circ)}$$

$$0{\cdot}577 = \frac{Q \times 0{\cdot}866}{40-Q \times 0{\cdot}5} = \frac{0{\cdot}866\,Q}{40-0{\cdot}5\,Q}$$

$$40-0{\cdot}5\,Q = \frac{0{\cdot}866\,Q}{0{\cdot}577} = 1{\cdot}5\,Q$$

$$\therefore \qquad 2Q = 40 \quad \text{or} \quad Q = 20 \text{ N} \quad \textbf{Ans.}$$

Example 2·2. *Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60°, their resultant is $\sqrt{13}$ N.* *(Bihar University, 1986)*

Solution

Let $\qquad P$ and Q = Two given forces.

First of all, consider the two forces acting at right angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{P^2+Q^2}$$

or $\qquad 10 = P^2+Q^2 \qquad$ ...(Squaring both sides)

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{P^2+Q^2+2PQ \cos 60^\circ}$$

$\therefore \qquad 13 = P^2+Q^2+2PQ \times 0{\cdot}5 \qquad$ ...(Squaring both sides)

$\qquad = 10+PQ \qquad$ ...(Substituting $P^2+Q^2 = 10$)

or $\qquad PQ = 13-10 = 3$

We know that $(P+Q)^2 = P^2+Q^2+2PQ = 10+6 = 16$

$\therefore \qquad P+Q = \sqrt{16} = 4 \qquad$ ...(*i*)

Similarly $\qquad (P-Q)^2 = P^2+Q^2-2PQ = 10-6 = 4$

$\therefore \qquad P-Q = \sqrt{4} = 2 \qquad$ ...(*ii*)

Solving equations (*i*) and (*ii*).

$$P = 3 \text{ N and } Q = 1 \text{ N} \quad \textbf{Ans.}$$

General Laws for the Resultant Force

The resultant force, of a given system of forces, may also be found out by the following general laws :

1. Triangle law of forces. 2. Polygon law of forces.

Triangle Law of Forces

It states, "*If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order ; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.*"

Polygon Law of Forces

It is an extension of Triangle Law of Forces for more than two forces, which states, *"If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order ; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."*

Graphical (Vector) Method for the Resultant Force

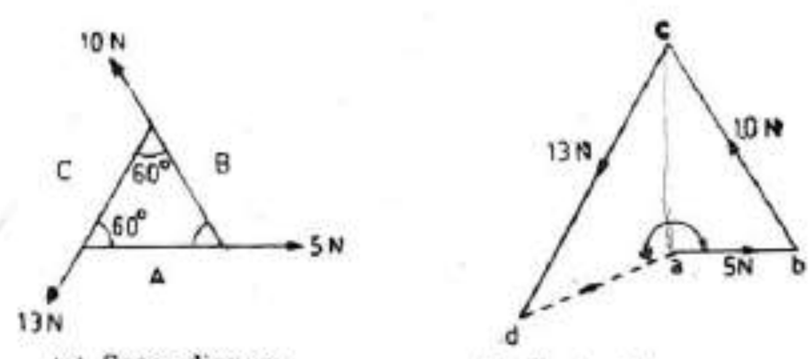
This is another name given to the method of finding out, graphically, magnitude and direction of the resultant force by the polygon law of forces. It is done as discussed below :

1. *Construction of space diagram (position diagram).* It means the construction of a diagram showing the various forces (or loads) alongwith their magnitude and lines of action.
2. *Use of Bow's notations.* All the forces in the space diagram are named by using the Bow's notations. It is a convenient method in which every force (or load) is named by two capital letters, placed on its either side in the space diagram.
3. *Construction of vector diagram (force diagram).* It means the construction of a diagram starting from a convenient point and then go on adding all the forces vectorially one by one (keeping in view the directions of all the forces) to some suitable scale.

Now the closing side of the polygon, taken in opposite order, will give the magnitude of the resultant force (to the scale) and its direction.

Example 2·7. *A particle is acted upon by three forces equal to 5 N, 10 N and 13 N, along the three sides of an equilateral triangle, taken in order. Find graphically the magnitude and direction of the resultant forces.* *(Madurai University, 1985)*

Solution.



(a) Space diagram (c) Vector diagram

Fig. 2·7

First of all, draw the space diagram for the given system of forces (acting along the sides of an equilateral triangle) and name the forces according to Bow's notations as shown in Fig. 2·7 (a). The 5 N force is named as *AB*, 10 N force as *BC* and 13 N force as *CD*.

Now draw the vector diagram for the given system of forces as shown in Fig. 2·7 (*b*) and as discussed below :

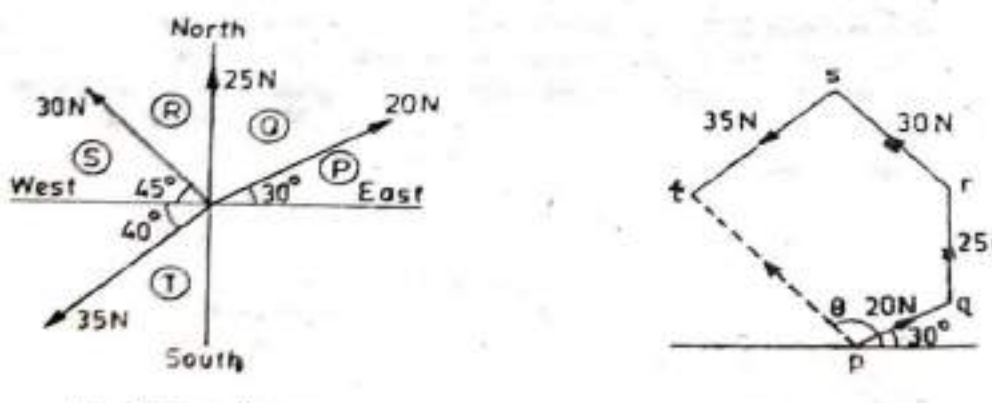
1. Select some suitable point *a* and draw *ab* equal to 5 N to some suitable scale and parallel to the force *AB* of the space diagram.
2. Through *b*, draw *bc* equal to 10 N to the scale and parallel to the force *BC* of the space diagram.
3. Similarly, through *c*, draw *cd* equal to 13 N to the scale and parallel to the force *CD* of the space diagram.
4. Join *ad*, which gives the magnitude as well as direction of the resultant force.
5. By measurement, we find the magnitude of the resultant force is equal to 7 N and acting at an angle of 200° with *ab*. **Ans**.

Example 2·8. *The following forces act at a point :*

(*i*) *20 N inclined at 30° towards North of East.*
(*ii*) *25 N towards North.*
(*iii*) *30 N towards North West, and*
(*iv*) *35 N inclined at 40° towards South of West.*

Find the magnitude and direction of the resultant force.
(*Jiwaji University, 1986*)

***Solution**



(*a*) Space diagram (*c*) Vector diagram

Fig. 2·8

First of all, draw the space diagram for the given system of forces (acting at point *O*) and name the forces according to Bow's notations as shown in Fig. 2·8 (*a*). The 20 N force is named as *PQ*, the 25 N force as *QR*, 30 N force as *RS* and 35 N force as *ST*.

Now draw the vector diagram for the given system of forces as shown in Fig. 2·8 (*b*) and as discussed below :

1. Select some suitable point *p* and draw *pq* equal to 20 N to some suitable scale and parallel to the force *PQ*.
2. Through *q*, draw *qr* equal to 25 N to the scale and parallel to the force *QR* of the space diagram.
3. Now through *r*, draw *rs* equal to 30 N to the scale and parallel to the force *RS* of the space diagram.
4. Similarly, through *s*, draw *st* equal to 35 N to the scale and parallel to the force *ST* of the space diagram.
5. Join *pt*, which gives the magnitude as well as direction of the resultant force.
6. By measurement, we find that the magnitude of the resultant force is equal to 45·6 N and acting at an angle of 132° with the horizontal *i.e.* East-West line. **Ans.**

2·19. Relation Between Mass and Weight

The term 'mass' is defined as the matter contained in a body, whereas the term 'weight' is defined as the force with which a body is attracted towards the centre of the earth. From the above mentioned two definitions, it is clear that the units of mass are kg, tonnes etc ; whereas the units of weight are N, kN and kgf etc.

It will be interesting to know that there is an important relation between the mass and weight of a body, which will be discussed in detail in chapter 23 of this book. But for the time being, it may be taken as

$$W\ P = mg = 9 \cdot 8\ m \qquad \ldots(g = 9 \cdot 8\ )$$

where P = Weight of the body in newtons,

m = Mass of the body in kg, and

g = Gravitational acceleration whose value is taken as $9 \cdot 8\ \text{m/sec}^2$.

Example 2·9. *A machine shaft BC 1·5 m long and of mass 100 kg is supported by two ropes AB and CD as shown in Fig. 2·9 given below :*

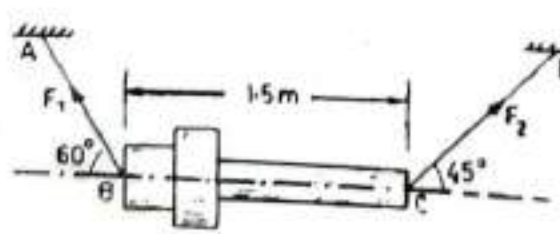


Fig. 2·9

Calculate the tensions F_1 and F_2 in the rope AB and CD.

(*London University*)

Solution. Given : Mass of shaft $= 100$ kg

We know that weight of the mass

$$= m.g = 100 \times 9{\cdot}8 = 980 \text{ N}$$

Resolving the forces horizontally (*i.e.* along *BC*) and equating the same,

$$F_1 \cos 60^\circ = F_2 \cos 45^\circ$$

$$\therefore \quad F_1 = \frac{\cos 45^\circ}{\cos 60^\circ} \times F_2 = \frac{0{\cdot}707}{0{\cdot}5} \times F_2 = 1{\cdot}414\, F_2 \quad \ldots(i)$$

and now resolving the forces vertically,

$$F_1 \sin 60^\circ + F_2 \sin 45^\circ = 980$$

$$(1{\cdot}414\, F_1)\, 0{\cdot}866 + F_2 \times 0{\cdot}707 = 980$$

$$1{\cdot}93\, F_2 = 980$$

$$\therefore \quad F_2 = 980/1{\cdot}93 = 507{\cdot}8 \text{ N} \quad \textbf{Ans.}$$

and $\quad F_1 = 1{\cdot}414 \times 507{\cdot}8 = 718 \text{ N} \quad \textbf{Ans.}$

Moment of a Force

It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required, and the line of action of the force. Mathematically, moment,

$$M = P \times l$$

where $\quad P =$ Force acting on the body, and

$\quad l =$ Perpendicular distance between the point about which the moment is required and the line of action of the force.

Graphical Representation of Moment

Consider a force P represented, in magnitude and direction, by the line AB. Let O be a point, about which the moment of this force is required to be found out, as shown in Fig. 3·1.

From O, draw OC perpendicular to AB. Join OA and OB.

Now moment of the force P about O

$$= P \times OC = AB \times OC$$

But $AB \times OC$ is equal to twice the area of the triangle ABO.

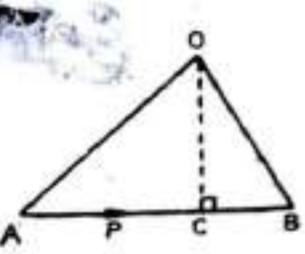


Fig. 3.1 Representation of moment

Thus the moment of a force, about any point, is geometrically equal to twice the area of the triangle, whose base is the line representing the force and whose vertex is the point, about which the moment is taken.

Units of Moment

Since the moment, of a force, is the product of force and distance, therefore the units of the moment will depend upon the units of force and distance. Thus, if the force is in Newton and the distance is in metres, therefore the units of moment will be Newton-metre (briefly written as N-m). Similarly, the units of moment may be kN-m (*i.e.* kN × m), N-mm (*i.e.* N × mm) kgf-m (kgf × m) etc

Types of Moments

Broadly speaking, the moments are of the following two types :

1. Clockwise moments. 2. Anticlockwise moments.

Clockwise Moment

(*a*) Clockwise moments (*b*) Anticlockwise moments

Fig. 3·2

It is the moment of a force, whose effect is to turn or rotate the body, in the *same* direction in which the hands of a clock move, as shown in Fig. 3·2 (*a*).

Anticlockwise Moment

It is the moment of a force, whose effect is to turn or rotate the bady, in the *opposite* direction in which the hands of a clock move, as shown in Fig. 3·2 (*b*).

Note. The general convention is to take clockwise moment as positive and anticlockwise moment as negative.

Varignon's Principle of Moments (or Law of Moments)

It states, "*If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.*"

Example 3·1. *A force of 15 N is applied perpendicular to the edge of a door 0·8 m wide as shown in Fig. 3·4 (a). Find the moment of the force about the hinge.*

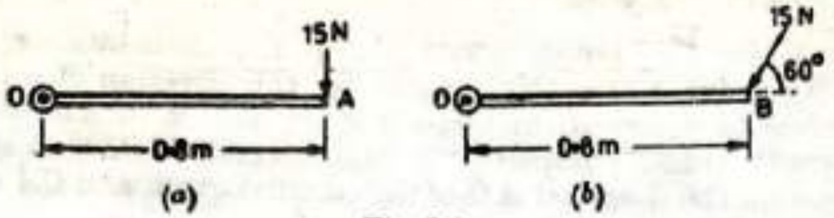


Fig. 3·4

If this force is applied at an angle of 60° to the edge of the same door, as shown in Fig. 3·4 (b), find the moment of this force.

(Gujarat University, 1984)

Solution. Given : $P = 15$ N ; $l = 0\cdot8$ m

Moment when the force acts perpendicular to the door

We know that the moment of the force about the hinge,

$$= P \times l = 15 \times 0\cdot8 = 12\cdot0 \text{ N-m} \qquad \textbf{Ans.}$$

Moment when the force acts at an angle of 60° to the door

This part of the example may be solved either by finding out the perpendicular distance between the hinge and the line of action of the force as shown in Fig. 3·5 (a) or by finding out the vertical component of the force as shown in Fig. 3·4 (b).

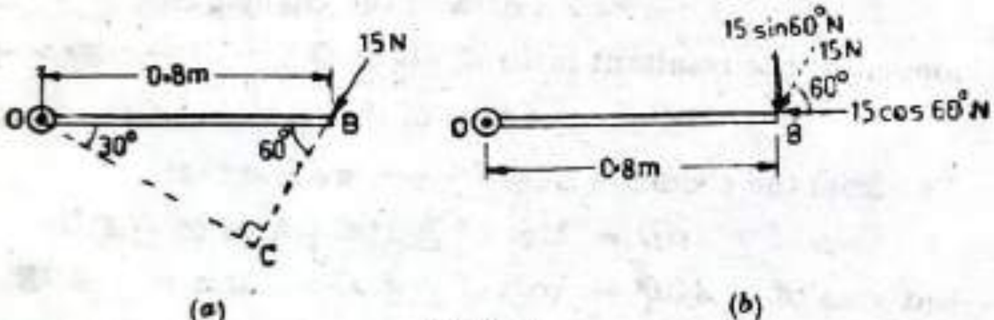


Fig. 3·5

From the geometry of Fig. 3·5 (a), we find that the perpendicular distance between the line of action of the force and hinge,

$$OC = OB \sin 60° = 0\cdot8 \times 0\cdot866 = 0\cdot693 \text{ m}$$

$$\therefore \quad \text{Moment} = 15 \times 0\cdot693 = 10\cdot4 \text{ N} \qquad \textbf{Ans.}$$

In the second case, we know that the vertical component of the force

$$= 15 \sin 60° = 15 \times 0\cdot866 = 13\cdot0 \text{ N}$$

$$\therefore \quad \text{Moment} = 13 \times 0\cdot8 = 10\cdot4 \text{ N} \qquad \textbf{Ans.}$$

Example 3·2. *A uniform plank ABC of weight 30 N and 2 m long is supported at one end A and at a point B 1·4 m from A as shown in Fig. 3·6.*

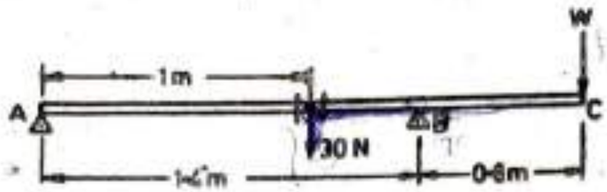


Fig. 3·6

Find the maximum weight W, that can be placed at C, so that the plank does not topple. (*Patna University, 1986*)

Solution. Given : $W = 30$ N ; Length $ABC = 2$ m

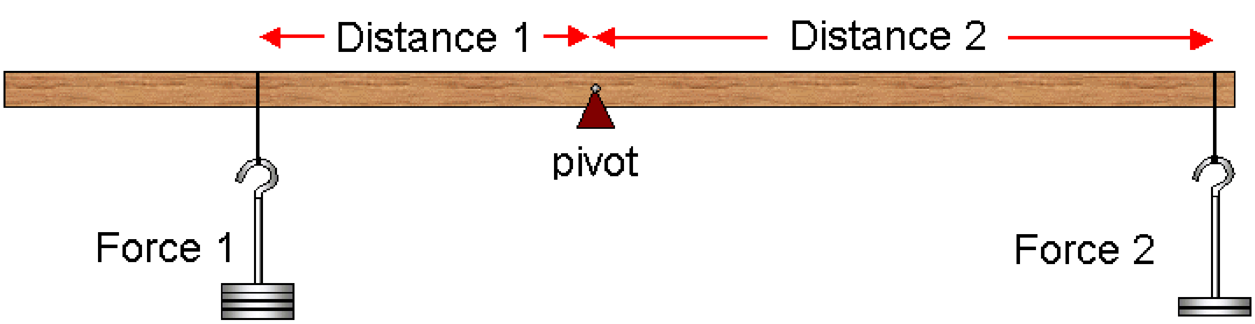
We know that weight of the plank (30 N) will act at its mid-point, as it is of uniform section. This point is at a distance of 1 m from A or 0·4 m from B.

We also know that if the plank is not to topple, then the reaction at A should be zero for the maximum weight at C. Now taking moments about B and equating the same,

$$30 \times 0{\cdot}4 = W \times 0{\cdot}6$$

$$\therefore \quad W = \frac{30 \times 0{\cdot}4}{0{\cdot}6} = 20 \text{ N}$$

Law of moments



When an object is balanced (in equilibrium) the sum of the clockwise moments is equal to the sum of the anticlockwise moments.

Force 1 x its distance from pivot = Force 2 x distance from the pivot

$$F_1\, d_1 = F_2\, d_2$$

COUPLE

Definition – Couple, in mechanics, pair of equal parallel forces that are opposite in direction. The only effect of a couple is to produce or prevent the turning of a body.

- The turning effect, or moment, of a couple is measured by the product of the magnitude of either force and the perpendicular distance between the action lines of the forces.

Arm of a Couple

The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known as *arm of the couple* as ~~shown in~~ Fig. 4·12.

Moment of a Couple

The moment of a couple is the product of the force (*i.e.* one of the forces of the two equal and opposite parallel forces) and the arm of the couple. Mathematically :

Fig 4·12. Couple

Moment of a couple $= P \times a$

where P = Force, and

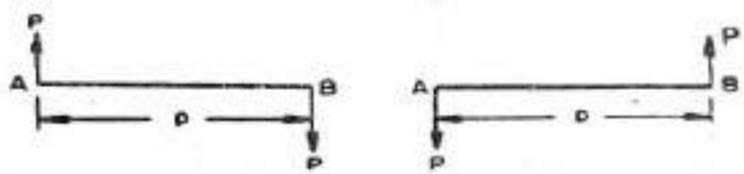
a = Arm of the couple.

Classification of Couples

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which they act :

1. Clockwise couple, and 2. Anticlockwise couple.

Clockwise Couple



(*a*) Clockwise couple (*b*) Anticlockwise couple

Fig. 4·13

A couple, whose tendency is to rotate the body, on which it acts, in a *clockwise direction,* is known as a clockwise couple as shown in Fig. 4·13 (*a*). Such a couple is also called *positive* couple.

Anticlockwise Couple

A couple, whose tendency is to rotate the body, on which it acts, in an *anticlockwise direction,* is known as an anticlockwise couple as shown in Fig. 4·13 (*b*). Such a couple is also called a *negative* couple.

Characteristics of a Couple

A couple (whether clockwise or anticlockwise) has the following characteristics :

1. The algebraic sum of the forces, constituting the couple, is zero.

2. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.

3. A couple cannot be balanced by a single force, but can be balanced only by a couple ; but of opposite sense.

4. Any number of coplaner couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Example 4·6. *A square ABCD has forces acting along its sides as shown in Fig. 4·14. Find the values of P and Q, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 1 m.* *(Allahabad University, 1985)*

Solution. Given : Length of square = 1 m

Values of P and Q

We know that if the system reduces to a couple, the resultant force in horizontal and vertical directions is zero. Therefore resolving the forces horizontally,

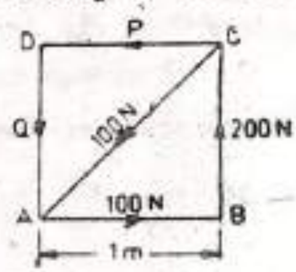


Fig. 4·14

$$100 - 100 \cos 45° - P = 0$$

$$\therefore \quad P = 100 - 100 \cos 45° \text{ N}$$
$$= 100 - 100 \times 0{\cdot}70 \text{ N}$$
$$= 29{\cdot}3 \text{ N} \quad \textbf{Ans.}$$

Now resolving the forces vertically,

$$200 - 100 \sin 45° - Q = 0$$

$$\therefore \quad Q = 200 - 100 \times 0{\cdot}707 = 129{\cdot}3 \text{ N} \quad \textbf{Ans.}$$

Magnitude of the Couple

We know that moment of the couple is equal to the algebraic sum of the moments about any corner. Therefore moment of the couple (taking moments about A)

$$= (-200 \times 1) + (-P \times 1) = -200 - 29{\cdot}3 \times 1 \text{ N-m}$$

$$= -229{\cdot}3 \text{ N-m} \quad \textbf{Ans.}$$ ...(Minus sign due to anticlockwise)

CHAPTER - 02 EQUILIBRIUM OF FORCES

2.1 If a system of forces acting simulteneously on a body produces no change in the state of rest or the state of motion of the body, the system of forces is said to be in equilm.

A system of forces can be in equilm under two situations.

↳* If the resultant of a number of forces acting at a point is zero.

↳ When the resultant of a system of forces applied on a particle has a non-zero value, then the particle will remain at rest by applying a force equal in magnitude but opposite in dirn of the resultant.

Principles of Equilibrium

Two - force principle

When a body is acted upon by two, equal opposite collinear forces, the resultant force is zero. The system of forces is said to be in equilibrium.

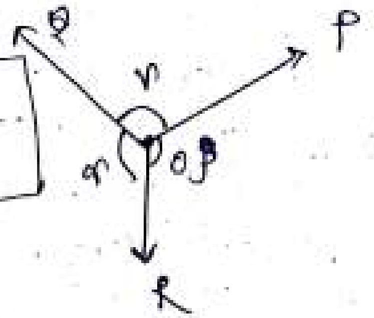
Three Force principle

Three non- parallel forces will be in equilm when they lie in one plane, intersect at one point and their free vectors form a closed triangle.

2.2 Lami's Theorem

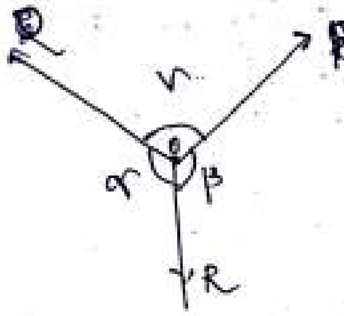
If three coplanner concurrent forces are acting on a body kept in equilibrium, then each force is proportional to the sine angle between other two forces and the const. of proportionality is the same.

$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} = k = \text{const}}$$



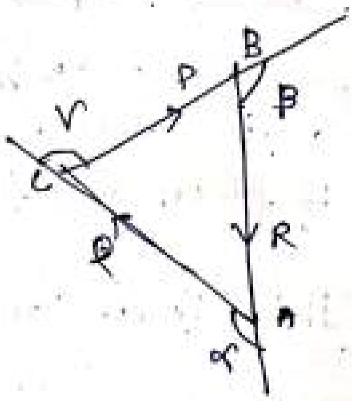
proof

Let force P, Q, R acting at point O.



Since P, Q, R are in equilibrium the triangle of forces should be a closed one. (Vector diagram)

Draw a line AB ∥ to force R. From end A draw a line ∥ to Q. name it AC. From 'C' draw a line ∥ to P. It will intersect the line AB at B.



$\angle A = \pi - \alpha$

$\angle B = \pi - \beta$

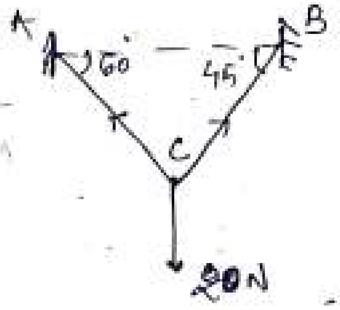
$\angle C = \pi - \gamma$

Applying sine rule to the $\triangle ABC$.

$$\frac{P}{\sin(\pi-\alpha)} = \frac{Q}{\sin(\pi-\beta)} = \frac{R}{\sin(\pi-\gamma)}$$

$$\Rightarrow \boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

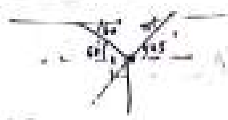
Q) An electric lamp weighing 20N is suspended from a point C supported by 2 wire AC & BC. The point A, B are at same level. AC makes an angle 60° and BC makes 45° to horizontal as shown in fig. Determine the tension in the string AC & BC.



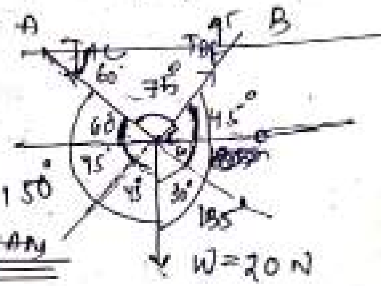
Sol^n W at C = 20

T_{AC} - tension in AC

T_{BC} = " " BC.



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



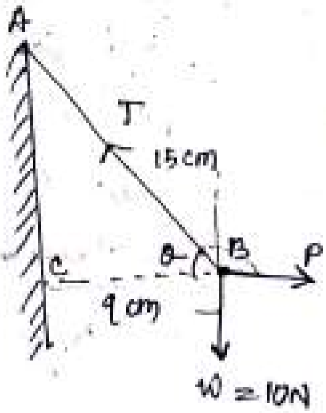
$$\Rightarrow \frac{20}{\sin 75^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{T_{AC}}{\sin 135^\circ}$$

$$T_{AC} = \frac{20 \times \sin 135}{\sin 75} = \frac{14.14}{\sin 75} = 14.93 \text{ Ans}$$

$$T_{BC} = \frac{20 \times \sin 150}{\sin 75^\circ} = \frac{10}{\sin 75^\circ} = \frac{10}{.965} = 10.35$$

Q) Body weighing 10N is suspended from a fixed point by a string 15cm long & is kept at rest by a horizental force P at a distance of 9 cm from the vertical line drawn through the point of suspension. What are the tension of the string & the value of P?

Sol^n Let tension T developed in the string AB. The point B is in equil^m, under the three forces 10, T_{AB} & P.



Let ∠ABC = θ

Applying Lami's theorem

$$\frac{P}{\sin(90+\theta)} = \frac{T_{AB}}{\sin 90} = \frac{10}{\sin(180-\theta)}$$

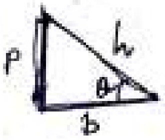
$$\frac{P}{\cos\theta} = \frac{T}{1} = \frac{10}{\sin\theta}$$

From ΔABC

$$AB^2 = AC^2 + BC^2$$
$$\Rightarrow AC^2 = AB^2 - BC^2$$
$$= 15^2 - 9^2$$
$$= 225 - 81$$
$$AC = \sqrt{144}$$
$$= 12\text{cm Ans}$$

$$\sin\theta = \frac{AC}{AB} = \frac{12}{15} = 0.8$$

$$\cos\theta = \frac{BC}{AB} = \frac{9}{15} = 0.6$$



$\sin\theta = p/h$

$\cos\theta = b/h$

$\tan\theta = \frac{p}{b}$

$$\frac{T}{1} = \frac{P}{0.6} = \frac{10}{0.8}$$

$$\Rightarrow P = \frac{10 \times 0.6}{0.8} = \frac{6.0}{.8} = 7.5\text{N Ans}$$

$$\Rightarrow T = \frac{10}{0.8} = 12.5\text{ N Ans}$$

2) A fine light string ABCDE with one end A fixed, has weights W_1 & W_2 attached to it at B and C. The string passes round a smooth pulley D carrying wt 60N at free end E as shown in fig. If the position of equ^n, BC is horizontal with AB & CD makes an angle 150° & 120° with BC. Find

i) Tension in portion AB, BC, DE.

ii) magnitude of W_1 & W_2

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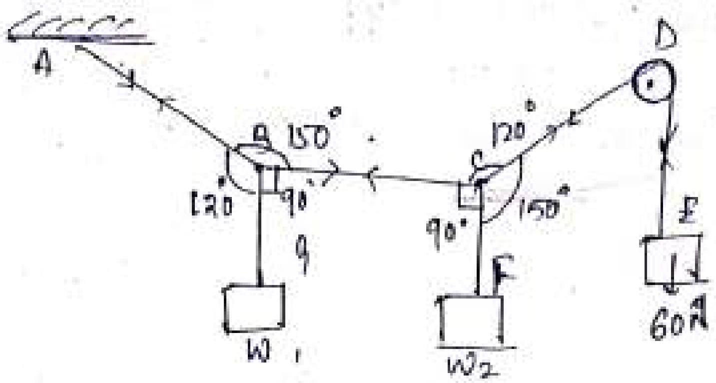
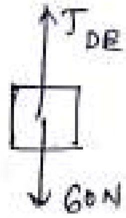


T_{AB} = tension in AB

T_{BC} = 〃 〃 BC

T_{CD} = 〃 〃 CD



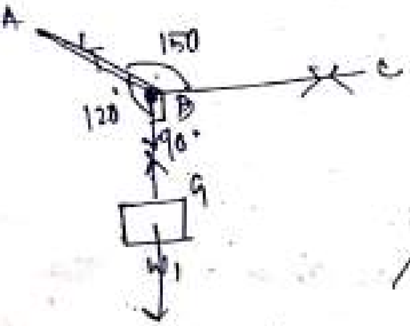
pulley is smooth no friction $T_{CD} = T_{DE}$

$T_{DE} = 60N = T_{CD}$

Apply Lami's theorem at C & B.

known from (i)

ii)

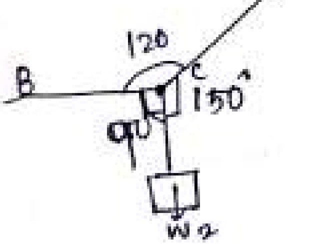


$$\frac{T_{AB}}{\sin 90} = \frac{T_{BC}}{\sin 120} = \frac{W_1}{\sin 150}$$

$$\Rightarrow T_{AB} = \frac{T_{BC}\times \sin 90}{\sin 120} = \frac{\overset{30}{\cancel{51.96}}\times 1}{\sin 120} = 34.63N$$

$$\Rightarrow W_1 = \frac{T_{BC}\times \sin 150}{\sin 120} = \frac{30\times .5}{.866} = 17.32N$$

i)



$$\frac{T_{CD}}{\sin 90^\circ} = \frac{T_{BC}}{\sin 150} = \frac{W_2}{\sin 120^\circ}$$

$$\Rightarrow T_{BC} = \frac{T_{CD}\times \sin 150}{\sin 90} = \frac{60\times .5}{1} = \underline{30N}$$

$$\Rightarrow W_2 = \frac{T_{CD}\times \sin 120}{\sin 90^\circ} = \underline{51.96\ N}$$

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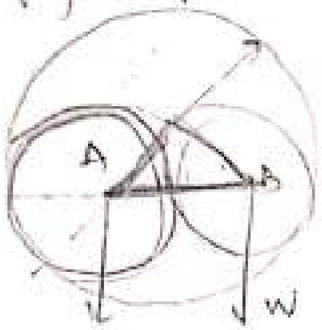
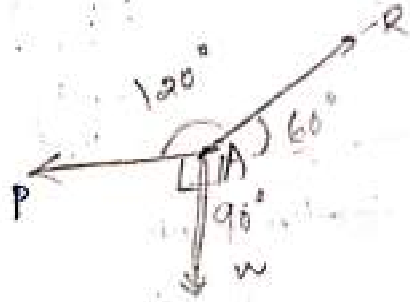
Q) Two equal and heavy spheres of 40 mm radius are in equilib with in a cup of radius 120 mm. Show that the reaⁿ betⁿ the cup & one sphere is double of that betⁿ the two spheres. As shown in the fig





$$\frac{R}{\sin 90^\circ} = \frac{w}{\sin 120} = \frac{P}{\sin 150^\circ}$$

$\Rightarrow R = \frac{w}{\sqrt{3}/2} = \frac{P}{1/2}$

$\Rightarrow R = \frac{P}{1/2}$

$\Rightarrow R = 2P$ ✓ Ans

Q) A uniform wheel 600 mm dia weighing 5 kN rests against a rigid rectangular block of 150mm height as shown in the fig. Find the minimum force reqd. to turn the wheel over the corner A & find the reaction on the block.

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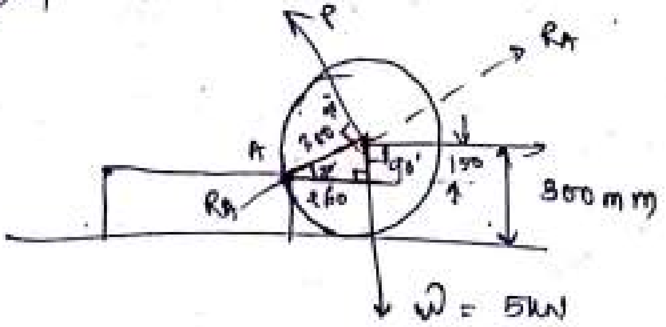
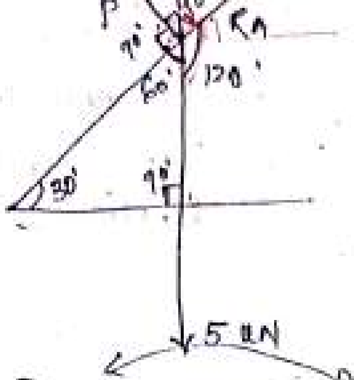
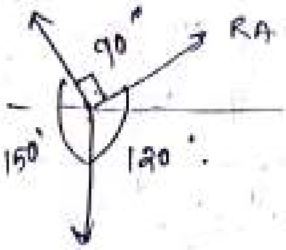






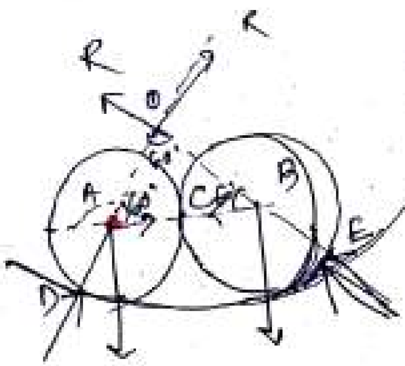
$$\frac{P}{\sin 120} = \frac{R_A}{\sin 150} = \frac{5000}{\sin 90^\circ}$$

$\Rightarrow$ $P = 4330\,N = 4.33\,kN$

$R_A = 2500\,N$

$= 2.5\,kN$

→



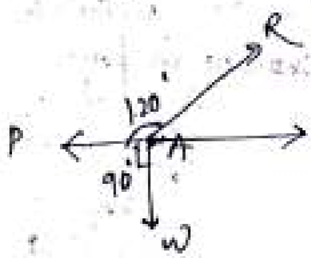
Two spheres with center A & B, lying in equilm, in cup with center 'O', let the 2 sphere contact at pt C. and sphere A with cup D & sphere B with cup E.

$R \rightarrow$ reacn at D & E

$P \rightarrow$ reacn at C.

From geometry. OD = 120 mm AD = 40 mm so AO = 120 − 40 = 80.

similarly OB = 80. AB = AC + CB = 40 + 40 = 80



OAB becomes equilateral Δ.

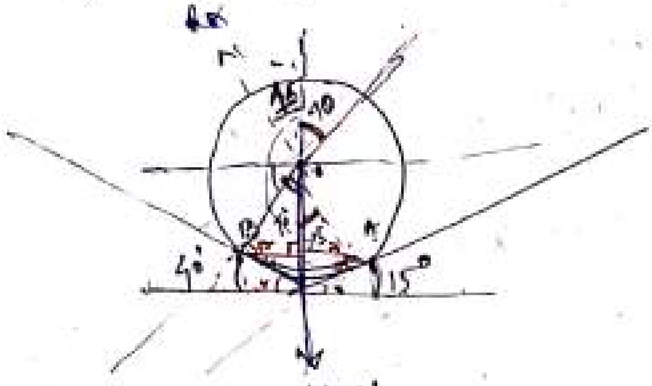
$$\frac{R}{\sin 90} = \frac{W}{\sin 120} = \frac{P}{\sin 150^\circ}$$

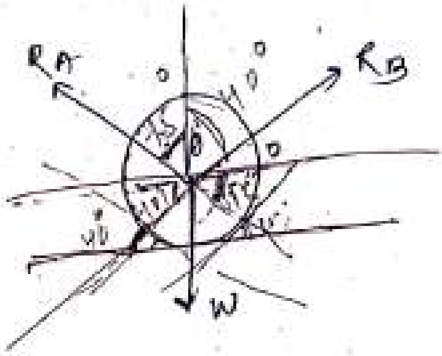
$\Rightarrow R = \frac{W}{\sqrt{3}/2} = \frac{P}{1/2}$

$\Rightarrow R = P/(1/2)$

$\Rightarrow R = 2P$

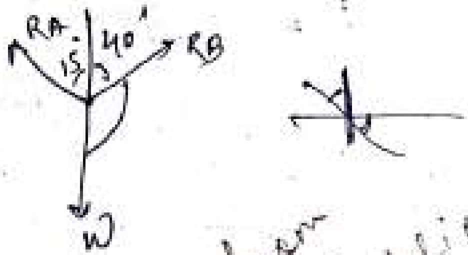
Q) A smooth circular cylinder of radius 1.5 meter is laying in triangular groove. One side of which makes 15° angle & other 40° angle with horizontal. Find the reactions at the surface of contact. If there is no friction & the cylinder weighs 100N.





$R_A \rightarrow$ Reac^n of A

$R_B \rightarrow$ Reac^n of B



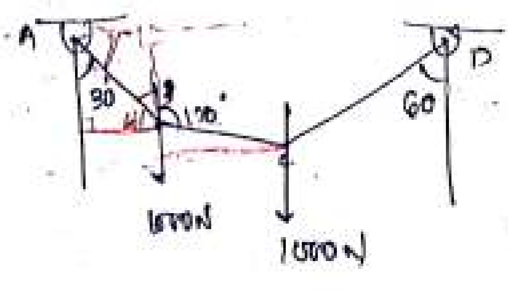
from friction force

$$\frac{R_A}{\sin(180-40)} = \frac{R_B}{\sin(180-15)} = \frac{100}{\sin(15+45)}$$

$R_A = 78.5N$

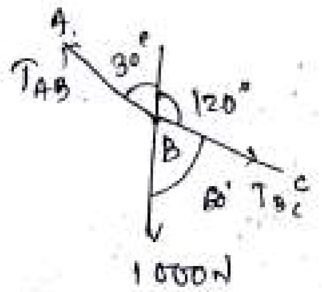
$R_B = 81.6N$

Q) A string ABCD attached to fixed points A & D has two equal weights of 1000N attached to B & C. The weight rest with the portions AB & CD inclined angle as shown in fig.



Find the tension in AB, BC & CD

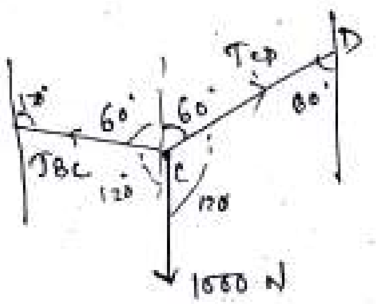
Soln. Free body diagram.



$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin(180-30)} = \frac{1000}{\sin 150°}$$

$\Rightarrow T_{AB} = 1732\ N$

$\Rightarrow T_{BC} = 1000\ N$

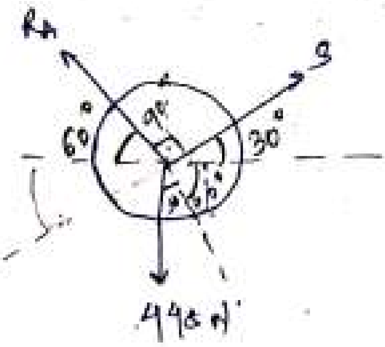


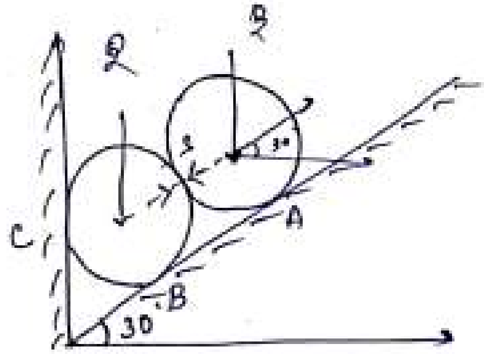
$$\frac{T_{BC}}{\sin 120°} = \frac{T_{CD}}{\sin 120°} = \frac{1000}{\sin 120°}$$

$T_{CD} = 1000\ N$ Ans

Q) Two identical rollers each of weight Q = 445 N are supported by an inclined plane and a vertical wall as shown in the fig. Assuming smooth surface, Find the reactions induced at pt A, B, C

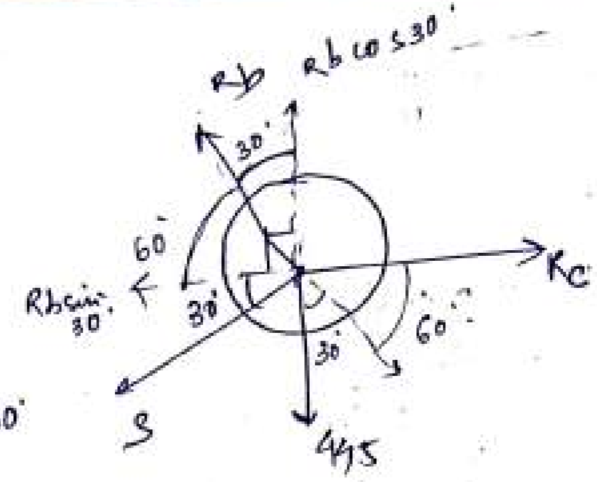
Soln





$$\frac{R_a}{\sin 120} = \frac{S}{\sin 150} = \frac{445}{\sin 90°}$$

$\Rightarrow R_a = 385.38\ N$ $S = 225.5\ N$



Resolving vertically

$\Sigma y = 0$

$R_b \cos 30 = 445 + S \sin 30°$

$\Rightarrow \quad R_b = (\quad) \ N$

Resolving horizontally

$\Sigma x = 0$

$R_b \sin 30° + S \cos 30° = R_c$

$\Rightarrow \quad R_c = (\quad) \ N$

CHAPTER → 03 < FRICTION >

3.1 When a body slides or tends to slide over another surface, an opposing force, called as force of friction. It acts tangent to the surface and opposite to the direction the body is moving or tends to move.

- Friction
  - Static
  - Dynamic
    - Sliding
    - Rolling

↳ Static Friction

It is experienced by a body when it is at rest or when the body is tends to move.

↳ Sliding Friction

It is experienced when a body slide over another body.

↳ Rolling Friction

It is experienced when a body rolls over another body.

Limiting Friction

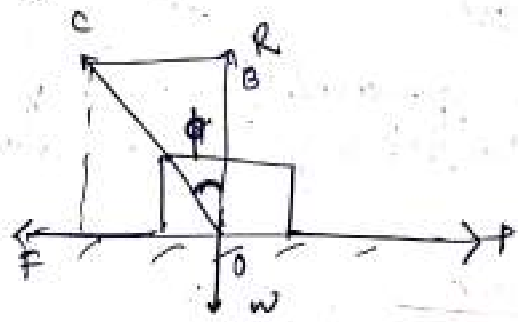
This is the maximum value of frictional force which comes into play, when a body just begins to slide over another body, known as limiting friction.

If the applied force is less than the limiting friction, the body remains at rest & the friction is called static friction, which may have any value betn. zero to limiting friction.

Angle of friction

Angle of friction is the angle which the resultant of force of limiting friction & normal reaction makes with the normal reacn.

- Let mass m kept on horizontal, pulled by a force P. When the body is just about to slide a limiting (F) friction will act on the opposite side. R be the normal reacn of wt. W.



Let OC is the resultant betn R & F, makes an angle ϕ with R.

$\Delta OBC \quad \tan\phi = \frac{BC}{BO} = \frac{F}{R}$

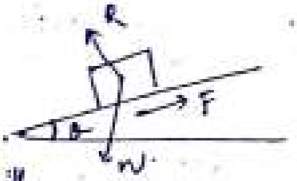
Coefficient of friction

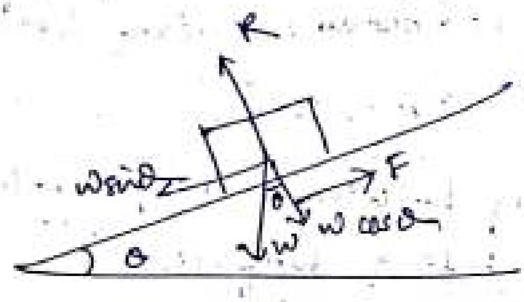
It is the ratio of friction to the normal reaction betn 2 bodies denoted by μ

$$\mu = \frac{F}{R} = \tan\phi \Rightarrow \boxed{F = \mu R}$$

Angle of repose

consider the block of weight W resting on an inclined plane which makes an angle θ with horizontal. When θ is very small the block will rest on the plane. If θ increases gradually, a stage is reached at which the block will starts to slide. That angle is called as angle of repose.





$\Sigma V = 0$

$R = W\cos\theta$ —— ①

$\Sigma H = 0 \quad F = W\sin\theta$ —— ②

$$\frac{\cancel{W}\sin\theta}{\cancel{W}\cos\theta} = \frac{F}{R}$$

$$\Rightarrow \boxed{\tan\theta = \frac{F}{R}}$$

$\therefore \tan\phi = \tan\theta$

$\Rightarrow \phi = \theta$

Angle of friction = Angle of repose.

Laws of friction

↳ Laws of static friction

→ The force of friction always act opposite in the direcⁿ of applied force.

→ The magnitude of force of friction is exactly equal to the applied force, which tend to move the body.

→ The magnitude of the limiting friction bears a const ratio to normal reaction betⁿ the two surface.

$$F/R = \text{const.}$$

→ The force of friction is independent of the area of contact betⁿ 2 surface.

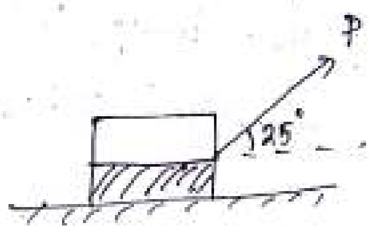
→ The force of friction depends upon the surface roughness.

↳ Laws of Dynamic friction

→ The force of friction always act in a direction opposite in which the body is moving.

→ For moderate speed the force of friction remains const, but it decreases with increase of the speed.

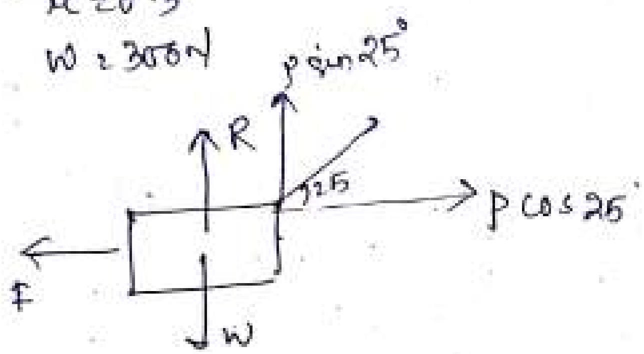
Q) A body of weight 300N is lying on a rough horizontal plane having a co-efficient of friction 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.



Soln

$\mu = 0.3$

$W = 300N$



$\Sigma H = 0 \Rightarrow p\cos 25° = F \Rightarrow F = 0.9063p$

$\Sigma V = 0 \Rightarrow R = W - p\sin 25°$

We know that $F = \mu R$

$\Rightarrow 0.9063p = \mu [W - p \times .4226]$

$\Rightarrow 0.9063p = 0.3[300 - .4226p]$

$\Rightarrow 0.9063p = 90 - .1268p$

$\Rightarrow p = 87.1\ N.$ Ans

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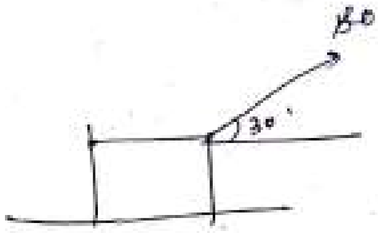
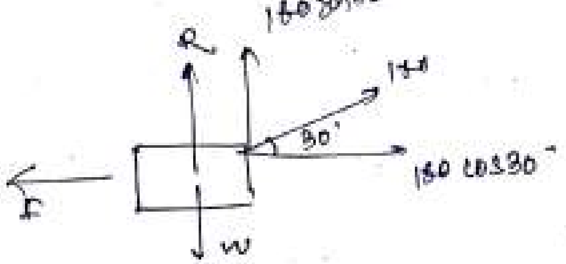
Q) A body resting on a rough horizontal plane required a pull of 180N inclined at 30°, to the plane to move it. It was found that that a push of 220N inclined at 30° to the plane just m the body. determine the weight of the body and the co-efficient of friction.

Soln



①

FBD of fig 1



$\Sigma H = 0$

$F_1 = 180 \cos 30°$ N

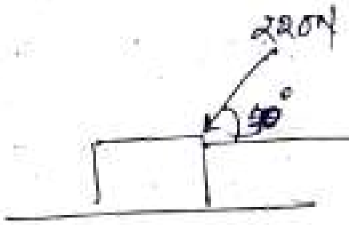
$\Sigma V = 0$

$R = W - 180 \sin 30°$

$\Rightarrow R = W - 90$

$F_1 = \mu R$

$\Rightarrow 155.88 = \mu (W - 90)$ —— ①



$\Sigma V = 0$

$R = W + 220 \sin 30°$

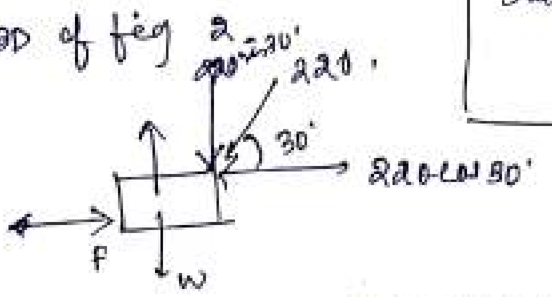
$\Rightarrow R = W + 110$

$F = \mu R$

$\Rightarrow 190.52 = \mu (W + 110)$ —— ②

Adding eqn ① & ② substracting

FBD of fig 2



$\Sigma H = 0$

$F_2 = 220 \cos 30°$

$\Rightarrow F_2 = 190.52$ N

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$155.88 = \mu W - 90\mu$

$- \quad 190.52 = \mu W + 110\mu$

$(-) \quad (-) \quad (-)$

$+34.64 = +200\mu$

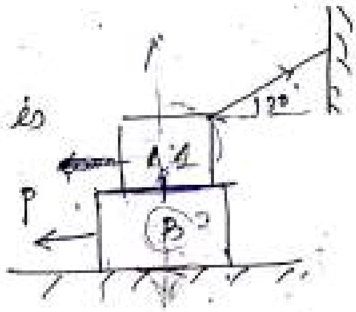
$\Rightarrow \mu = 0.1732$ Ans

puting value of μ in eqn ①

we get $155.88 = 0.1732(W - 90)$

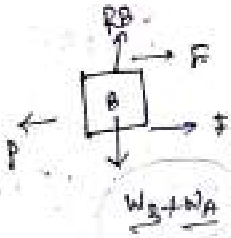
$W = 991.68N$

2) if co-efficient betn the 2 blocks is 0.3. Find force p req to move the block.

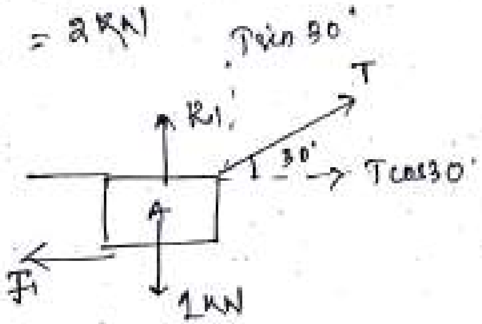


$W_A = 1kN$

$W_B = 2kN$



soln



$R_1 + T\sin 30° = 1kN$ (vertically)

$\Rightarrow \cancel{R_1} \Rightarrow T\sin 30° = 1 - R_1$ —— ①

Horizontally

$T\cos 30° = F_1$

$\Rightarrow T\cos 30° = \mu R_1$ —— ①

$\Rightarrow T\cos 30° = 0.3R_1$

Dividing eqn ① & ②

$\dfrac{T\sin 30°}{T\cos 30°} = \dfrac{1 - R_1}{0.3R_1} \Rightarrow \tan 30° = \dfrac{1 - R_1}{0.3R_1}$

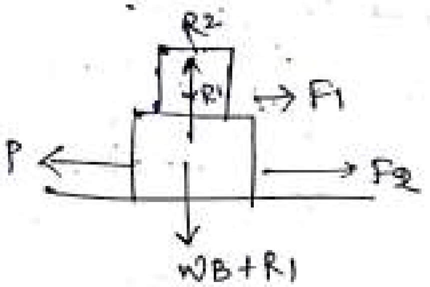
$$\Rightarrow \quad 0.5774 = \frac{1-R_1}{0.3R_1}$$

$$\Rightarrow \quad 0.5774 \times 0.3R_1 = 1-R_1$$

$$\Rightarrow \quad 0.173R_1 = 1-R_1$$

$$\Rightarrow \quad R_1 = 0.85 \text{ kN}$$

$$F = \mu R_1 = 0.3 \times 0.85$$
$$= .255 \text{ kN}$$



$$R_2 = 2 + R_1$$
$$= 0.85 + 2 = 2.85 \text{ kN}$$

$$F_2 = \mu R_2$$
$$= 0.03 \times 2.85 = .855 \text{ kN}$$

$$P = F_1 + F_2$$
$$= .255 + .855$$
$$= 1.11 \text{ kN}$$

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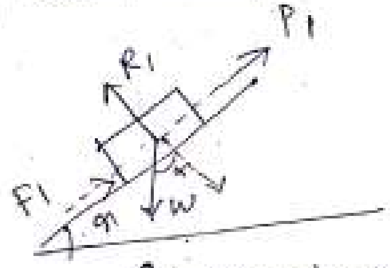
9.2 Equilᵐ of a body on Rough Inclined plane

Consider a body laying on a rough inclined plane subjected to force P. as shown in fig

1. Minimum force (P_1) which will keep the body in equilᵐ when it is sliding down ward.



(moving downward)

$F_1 = \mu R_1$

Net horizontal force.

$P_1 = W\sin\alpha - F_1$

$\Rightarrow P_1 = W\sin\alpha - \mu R_1$ —— ①

Net vertical force.

$W\cos\alpha = R_1$ —— ②

putting value of R_1 in equ ① we get

$P_1 = W\sin\alpha - \mu(W\cos\alpha)$

$= W(\sin\alpha - \mu \times \cos\alpha)$

$= W(\sin\alpha - \tan\phi \times \cos\alpha)$ $(\because \mu = \tan\phi)$

$= W\left(\sin\alpha - \frac{\sin\phi}{\cos\phi} \times \cos\alpha\right)$ $\because \left(\tan\phi = \frac{\sin\phi}{\cos\phi}\right)$

$\Rightarrow P_1 \cos\phi = W(\sin\alpha \times \cos\phi - \sin\phi \times \cos\alpha)$

$\Rightarrow P_1 \cos\phi = W\sin(\alpha - \phi)$

$$\Rightarrow \boxed{P_1 = \frac{W\sin(\alpha-\phi)}{\cos\phi}}$$

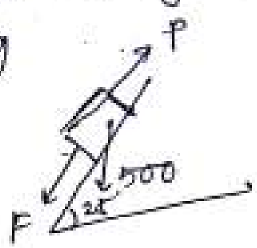
2. Minimum force (P_1) which will keep the body in equᵐ when moving upward.

$P_1 = W\sin\alpha + F_1$ —— ①

$R_1 = W\cos\alpha$

$$\boxed{P_1 = \frac{W\sin(\alpha+\phi)}{\cos\phi}}$$

Q) A body of wt 500 N is lying on a rough plane inclined at an angle of 25°. supported by horizontal force P as shown in fig Determine P for both upward & downward motion.

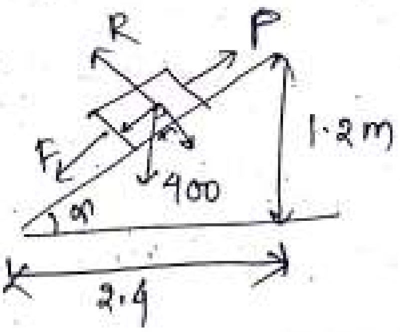


Sol^n

$$P_1 = \frac{W \sin(\alpha - \varphi)}{\cos \varphi} = 46.4 \text{ N}$$

$$P_2 = \frac{W \sin(\alpha + \varphi)}{\cos \varphi} = 376.2 \text{ N}$$

2) An inclined plane as shown in fig is used to unload a body of wt 400N. from a height 1.2 m. $\mu = 0.3$. (State weather it is necessary to push the body down the plane or hold it back from sliding down. what minim force is req. parallel for this purpose) Find P



Sol^n $\tan \alpha = \frac{1.2}{2.4} = 0.5$

$\alpha = 26.5°$

& normal reac^n

$R = W \cos \alpha$

$= 400 \times \cos 26.5°$

$= 357.9$ N

$F = \mu R$

$4 \sin \alpha + \mu R = P$

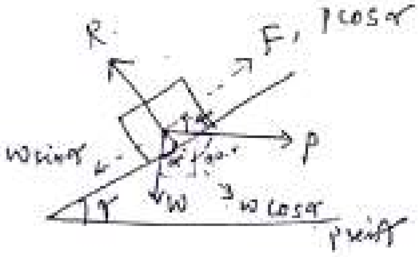
$\Rightarrow P = 400 \times \sin 26.5 + 0.3 \times 357.9$

$=$

Equillibrium of a body on a rough inclined plane subjected to a force acting horizontally

Consider a body lying on a rough inclined plane subjected to a force acting horizontally.

1. Minimum force (P_1) which will keep the body in equilm, when it is at the point of sliding downward



$F = \mu R$

$\Sigma H = 0$

$P \cos\alpha + F = W \sin\alpha$

$\Rightarrow P\cos\alpha = W\sin\alpha - F$

$\Rightarrow P\cos\alpha = W\sin\alpha - \mu R$ —① ($\because F = \mu R$)

$\Sigma V = 0$

$R = W\cos\alpha + P\sin\alpha$ —— ②

putting the value of R in eqn ①

$P_1 \cos\alpha = W\sin\alpha - \mu(W\cos\alpha + P_1 \sin\alpha)$

$\Rightarrow P_1\cos\alpha + \mu P_1 \sin\alpha = W\sin\alpha - \mu W\cos\alpha$

$\Rightarrow P_1(\cos\alpha + \mu\sin\alpha) = W(\sin\alpha - \mu\cos\alpha)$

put $\mu = \tan\phi$

$$\Rightarrow P_1 = W \frac{(\sin\alpha - \mu\cos\alpha)}{\cos\alpha + \mu\sin\alpha}$$

$$= W \frac{(\sin\alpha - \tan\phi \cdot \cos\alpha)}{(\cos\alpha + \tan\phi \cdot \sin\alpha)}$$

$$= W \frac{\left(\sin\alpha - \frac{\sin\phi}{\cos\phi} \cdot \cos\alpha\right)}{\left(\cos\alpha + \frac{\sin\phi}{\cos\phi} \cdot \sin\alpha\right)}$$

$$= W \left(\frac{\sin\alpha \cdot \cos\phi - \sin\phi \cdot \cos\alpha}{(\cos\alpha \cdot \cos\phi + \sin\phi \cdot \sin\alpha)}\right)$$

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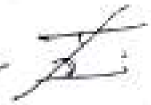
$$P_1 = W\frac{\sin(\alpha-\varphi)}{\cos(\alpha-\varphi)}$$

$$\Rightarrow \boxed{P_1 = W\tan(\alpha-\varphi)}$$

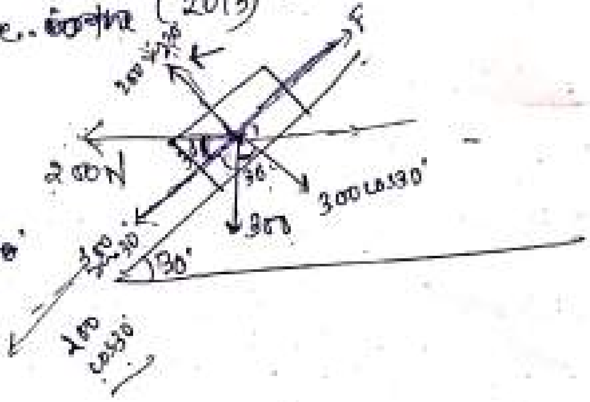
ii) Maximum For force (P_1), when the body is moving upward.

$$P_1 = W\frac{\sin(\alpha+\varphi)}{\cos(\alpha+\varphi)}$$

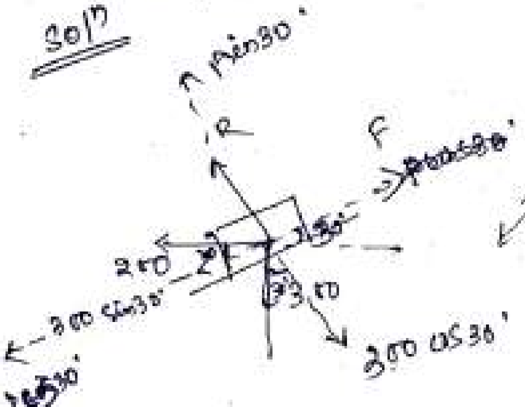
$$\Rightarrow \boxed{P_1 = W\tan(\alpha+\varphi)}$$



Q) Find the total force... (2013)
μ ?



Sol^n



$\Sigma V = 0$

$300\cos30° = R + 200\sin30°$

$\Rightarrow R = 300\cos30° - 200\sin30°$

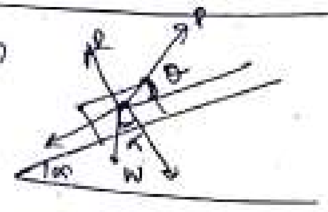
$= (\quad)$

$\Sigma H = 0$.

$200\cos30° + 300\sin30° = F$

$\Rightarrow \mu R = 200\cos30° + 300\sin30°$

Minimum force (P_1), keep the body in equ^m when sliding downward



$$P_1 = W\frac{\sin(\alpha - \varphi)}{\cos(\theta+\varphi)}$$

$$P_{max} = P_2 = \frac{W\sin(\alpha+\varphi)}{\cos(\theta-\varphi)}$$

$$P_1 = W\frac{\sin(\alpha-\varphi)}{\cos(\alpha-\varphi)}$$

$\Rightarrow$ ~~P_1 ...~~

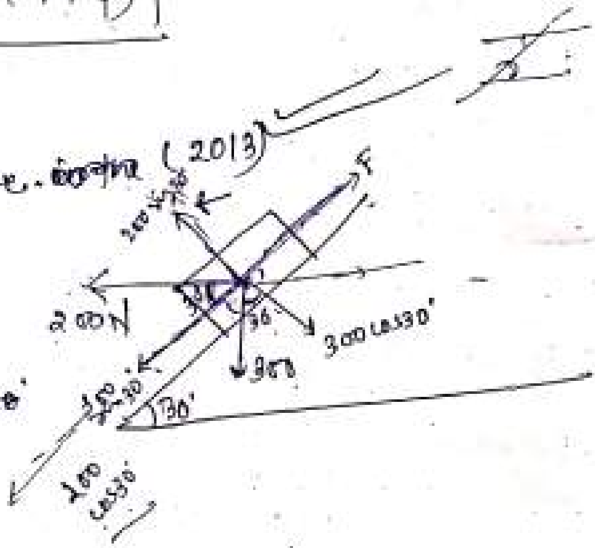
$$\Rightarrow \boxed{P_1 = W\tan(\alpha-\varphi)}$$

ii) Maximum force (P_1), when the body is moving upward.

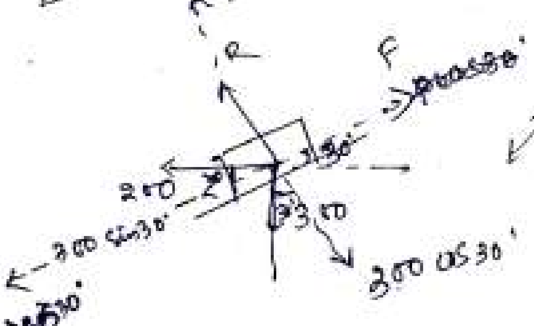
$$P_1 = W\frac{\sin(\alpha+\varphi)}{\cos(\alpha+\varphi)}$$

$$\Rightarrow \boxed{P_1 = W\tan(\alpha+\varphi)}$$

20) Find the total force. (2013) μ ?



Sol^n



$\Sigma V = 0$

$300\cos 30' = R + 200\sin 30'$

$\Rightarrow R = 300\cos30' - 200\sin30'$

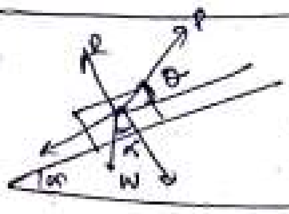
$= (\quad)$

$\Sigma H = 0$

$200\cos30' + 300\sin30' = F$

$\Rightarrow \mu R = 200\cos30' + 300\sin30'$

Minimum force (P_1), keep the body in equi^m when sliding downward



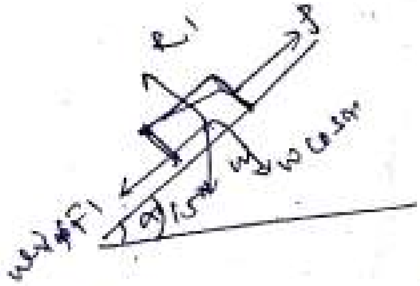
$$P_1 = W\frac{\sin(\alpha-\varphi)}{\cos(\theta+\varphi)}$$

$$P_{max} = P_2 = \frac{W\sin(\alpha+\varphi)}{\cos(\theta-\varphi)}$$

2) An effort of 200N is required just to move certain body up an inclined plane at an angle 15° the forces acting ∥ to plane. If angle of inclⁿ is 20°, then the effort req is found to be 230N. Find weight of the body & μ.

given $P_1 = 200N$ $P_2 = 230N$

$\alpha = 15°$ $\alpha = 20°$



$\Sigma F_x = 0$

$F_1 + W \sin \alpha_1 = 200$

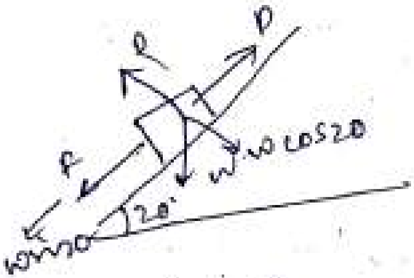
$\Rightarrow \mu R_1 + W \sin 15 = 200$

$\Rightarrow \mu W \cos \alpha + W \sin 15 = 200$

$\Rightarrow W(\mu \cos \alpha + \sin \alpha) = 200$ — (1)

$\Sigma F_y = 0$

$R_1 = W \cos \alpha$



$\Sigma F_x = 0$

$P = W \sin 20 + F$

$\Rightarrow \mu R + W \sin 20 = 230$

$\Rightarrow \mu W \cos 20 + W \sin 20 = 230$

$\Rightarrow W(\mu \cos 20 + \sin 20) = 230$ — (2)

$\Sigma F_y = 0$

$R_2 = W \cos 20°$

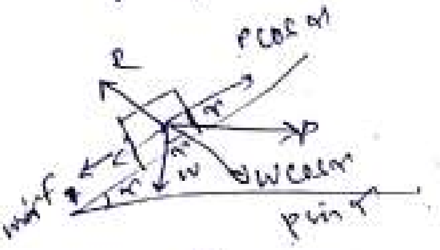
$$\frac{\text{eq}^n (2)}{\text{eq} (1)} = \frac{\mu \cos 20 + \sin 20}{\mu \cos 15 + \sin 15} = \frac{230}{200}$$

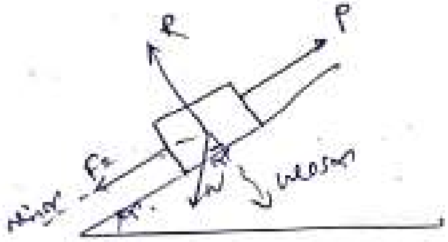
$\Rightarrow \mu = 0.259$

eqⁿ (1) → $W(0.259 \times \cos 15 + \sin 15) = 200$

$\Rightarrow W = 392\,N$ Ans

Q) A load of 1.5kN resting on an inclined rough plane, can be moved up the plane by a force of 2kN applied horizontally & by a force of 1.25 kN applied ∥ to the plane. Find angle of inclination & μ.





$p = w\tan(\alpha+\varphi)$

$\Rightarrow 2 = 1.5\tan(\alpha+\varphi)$

$\Rightarrow \alpha+\varphi = 53.1^\circ$

$p = w\dfrac{\sin(\alpha+\varphi)}{\cos\varphi}$

$\Rightarrow 1.25 = 1.5\dfrac{\sin(53.1)}{\cos\varphi}$

$\Rightarrow \cos\varphi = .96$

$\Rightarrow \varphi = 16.3^\circ$

$\alpha = 53.1 - 16.3^\circ$

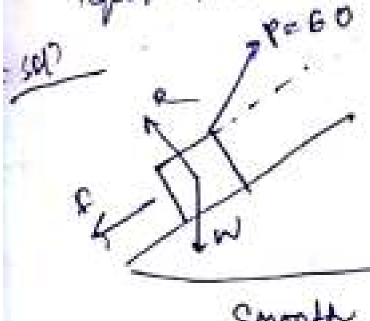
$= 36.8^\circ$

$\mu = \tan\varphi = \tan 16.3^\circ$

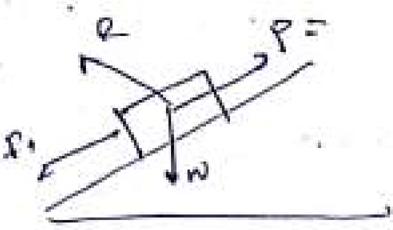
$= .292$

Q) Find the force req. to move a load 300N up a rough plane the force being ∥ to the plane. The inclination of the plane is such that when the same load is kept on a perfectly smooth plane inclined at same angle, a force 60N applied at an inclination of 30° to the plane, keep the same load in equilibrium. $\mu = 0.3$.

Sol



Smooth



Rough.

For smooth $\mu = 0 \therefore \varphi = 0$

$P = W\dfrac{\sin(\alpha+\varphi)}{\cos(\theta-\varphi)} \Rightarrow 60 = \dfrac{300\sin\alpha}{\cos 30^\circ} \Rightarrow \alpha = 10^\circ$

For Rough

$P = W\dfrac{\sin(\alpha+\varphi)}{\cos\varphi} \Rightarrow P = 140.7N$

$\mu = 0.3$

$\tan\varphi = 0.3$

$\varphi = \tan^{-1}0.3$

$= 16.7$

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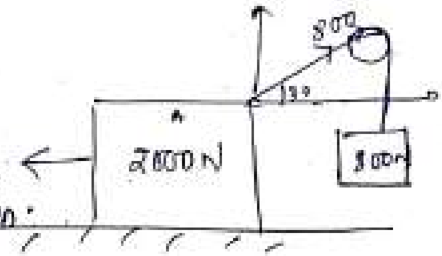
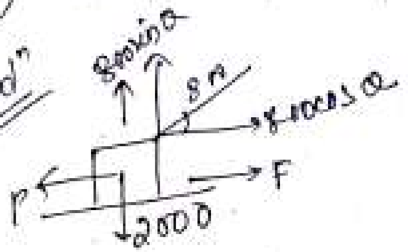
Q) $\mu = 0.35$

2014 Determine value of P.
Consider the pulley is frictionless.



Sol^n



$P = F + 800\cos 30^\circ \Rightarrow P = \mu R_n + 800\cos 30^\circ$

$2000 = R_n + 800\sin 30^\circ$

$\Rightarrow R_n = 2000 - 800\sin 30^\circ$

$\Rightarrow$ putting value of R_n.

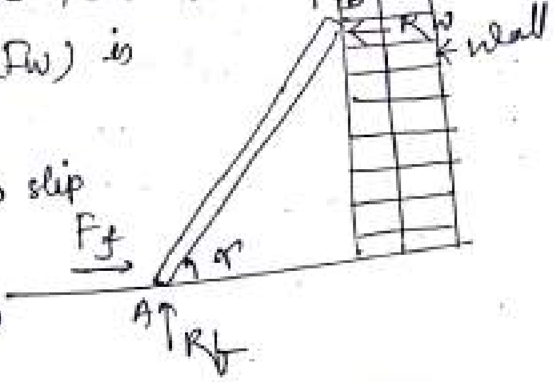
$P = \mu \times (2000 - 800\sin 30^\circ) + 800\cos 30^\circ$

$= (1252.82)$ ✓

Application of friction.

3.3 LADER FRICTION

A ladder is a device for climbing on walls.



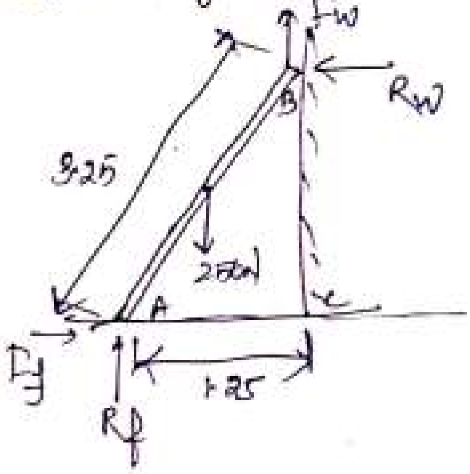
- As upper end of the ladder tends to slip downward, friction (F_w) is upward.
- As the lower end tries to slip away from wall, so friction (F_f) is towards the wall.
- Since the system is in equil^m, therefore the algebraic sum of horizontal & vertical components of the forces must also be equal to zero.

2014 1) A uniform ladder of length 3.25m and weighing 250 N placed against a smooth vertical wall. It's lower end 1.15 m from the wall. The co-efficient of friction bet^n ladder & floor is 0.3. Determine ~~...~~ frictional force acting on ladder at point of contact bet^n ladder & floor.

Soln

$\Sigma V = 0$

$R_f = 250 N$



from geometry

$BC^2 = \sqrt{AB^2 - AC^2}$

$= 30m$

Taking moment about 0.

$$R_f \times 1.25 - 250 \times \left(\frac{1.25}{2}\right) = F_f \times 3$$

$\Rightarrow R_f = 521. N$

2) 2015

A ladder 5 meter long rest on a horizontal ground and leans against a smooth vertical wall at an angle 70° with horizontal. The weight of ladder is 900N and acts at it's middle. The ladder is at the point of sliding, when a man weighing 750N stands on the ladder 1.5m from bottom. calculate μ..

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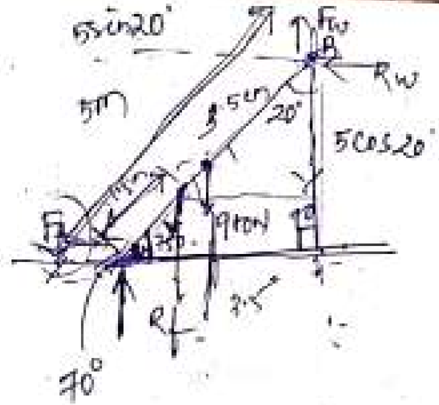
Sol^n $l = 5m$

$\alpha = 70^\circ$

$W_1 = 900\,N$

$W_2 = 750\,N$



$R_f = 900 + 750 = 1650\,N$

$F_f = \mu_f \times R_f = \mu_f \times 1650\,N$

Taking moment about B

$R_f \times 5\cos 70 - 900 \times 2.5\cos 70 - 750 \times 3.5\cos 70 = F_f \times 5\sin 70.$

$R_f \times 5\sin 20^\circ - 900 \times 2.5\sin 20 - 750 \times 3.5\sin 20^\circ = F_f \times 5\cos 20^\circ$

put the value of F_f

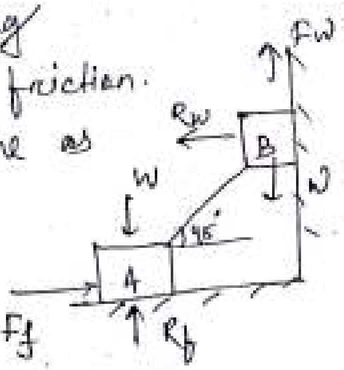
$R_f \times 5\sin 20 - 900 \times 2.5\sin 20 - 750 \times 3.5\sin 20 = \mu_f \times 1650 \times 5\cos 20^\circ$

$\Rightarrow 1650 \times 5\sin 20^\circ = (\mu_f \times 1650 \times 5\cos 20^\circ) + 975$

$= 4533\mu_f + 975$

$\Rightarrow \mu_f = 0.15$ Ans

2) Two identical blocks of weight W are supported by a rod inclined at 45° with horizontal, as shown in fig. If Both the blocks are limiting equilibrium, Find the coefficient of friction (μ). assuming it to be same as floor as well as at wall.



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Sol^n Resolving forces vertically.

$$F_w + R_f = 2W$$

$$\Rightarrow \mu R_w + R_f = 2W \quad \text{——— (1)}$$

Now resolving the forces horizontally.

$$R_w = F_f$$

$$\Rightarrow R_w = \mu R_f \quad \text{——— (2)}$$

Substituting R_w in equ^n (1).

$$\mu(\mu R_f) + R_f = 2W$$

$$\Rightarrow \mu^2 R_f + R_f = 2W$$

$$\Rightarrow R_f = \frac{2W}{(1+\mu^2)} \quad \text{——— (3)}$$

putting value of R_f in equ^n (2)

$$R_w = \mu \times \frac{2W}{\mu^2+1}$$

Taking moment of the forces about block A

$$R_w \times l \cos 45^\circ + F_w \times l \cos 45^\circ = W \times l \cos 45^\circ.$$

$$R_w + F_w = W$$

$$\Rightarrow R_w + \mu R_w = W$$

$$\Rightarrow R_w(1+\mu) = W$$

putting value of R_w $\quad \dfrac{\mu \times 2W}{\mu^2+1}(1+\mu) = W$

$$\Rightarrow 2\mu(1+\mu) = \mu^2+1$$

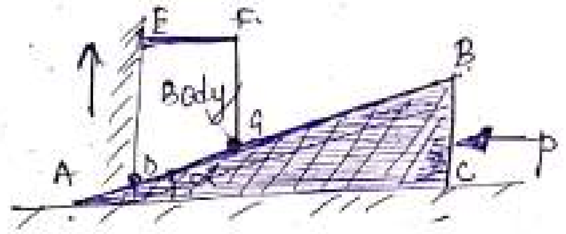
$$\Rightarrow 2\mu + 2\mu^2 = \mu^2+1$$

$$\Rightarrow \mu^2 + 2\mu - 1 = 0$$

$$\mu = \frac{-2 \pm \sqrt{(2)^2+4}}{2} = 0.414 \text{ Ans}$$

WEDGE FRICTION

A wedge is usually, of a triangular in cross-section & is, generally, used for slight adjustments in the position of a body i.e for tightening fits or keys for shafts. Sometimes, a wedge is also used for lifting heavy weight. It is made of up wood or metal.



wedge ABC, used to lift the body DEFG.

↑ effect
← P (effort)

W = weight of the body DEFG

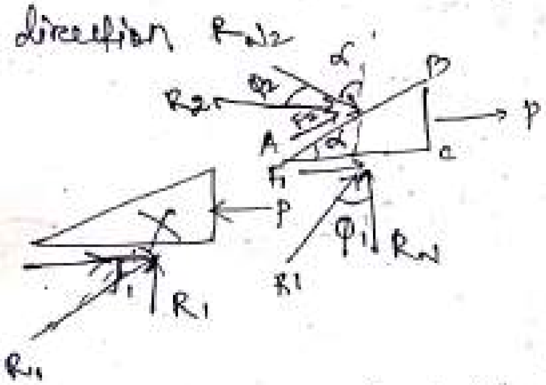
P = Force req. to lift the body

due to horizontal movement we get vertical.

μ = co-efficient of friction = tanφ

W_{wedge} → Not considered.

When force P is applied in, the body will lift in upward direction.



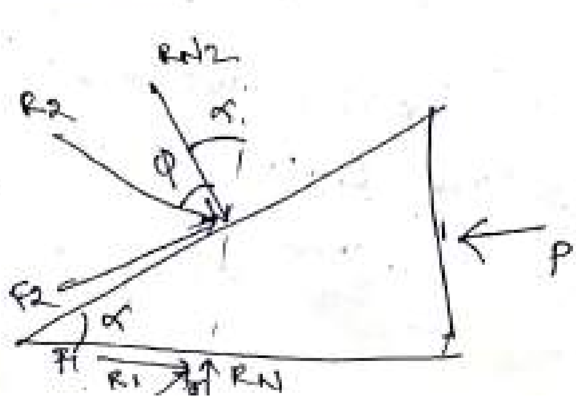
R_1 → resultant of frictional force & normal rxn between floor & wedge. F_1 & R_N

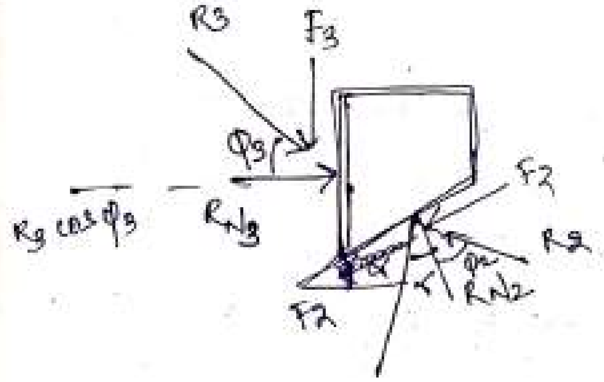
φ_1 & φ_2 → angle of friction.

R_{N2} → normal rxn at AC,

& frictional force F_2.

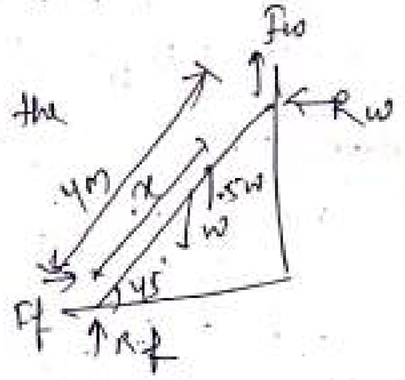
The resultant of both is R_2, making an angle φ_2.





2) A uniform ladder of 4m length rests against a vertical wall with which it makes an angle of 45°. The co-effi of friction betn ladder & wall 0.4 & that betn ladder & ~~wall~~ floor 0.5. If a man whose weight is one-half of that ladder accesends it. how high it will be when the ladder slips?

Soln $x \rightarrow$ distance betn A & the man.



weight of man $= \frac{W}{2}$
$= .5W$

$F_f = \mu_f R_f = 0.5 R_f$

$F_w = \mu_w R_w = 0.4 R_w$

$R_w = F_f = 0.5 R_f$

$R_f = 2R_w$

Resolving vertically $R_f + F_w = W + 0.5W$

$\Rightarrow 2R_w + 0.4 \cdot R_w = 1.5W$

$\Rightarrow R_w = \frac{1.5W}{2.4} = 0.625W$

$F_W = .4 \times .625W$

$= 0.25W$

Taking moment about A.

$(W \times 2\cos 45^\circ + .5W \times x\cos 45^\circ)$

$= R_W \times 4\sin 45^\circ + F_W \times 4\cos 45^\circ$

put value of R_W & F_W

$x = 3.0\,m$.

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CHAPTER → 04 Centre of Gravity

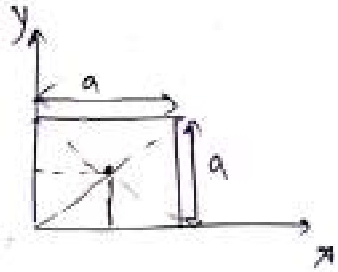
Centre of gravity can be defined as a point through which the whole weight of the body acts, irrespect of it's position. It may be noted that every body has one and only one centre of gravity.

4.1 Centroid

The plane figures like triangle, rectangle, circle etc have only area, but no mass. The centre of area of such fig is known as centroid.

Centroid of basic geometrical figures

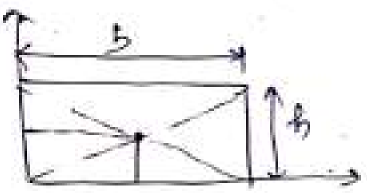
1)



$\bar{x} = a/2$

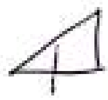
$\bar{y} = a/2$

2)



$\bar{x} = b/2$

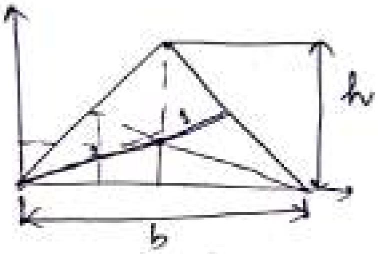
$\bar{y} = h/2$



$x = \frac{2b}{3}$

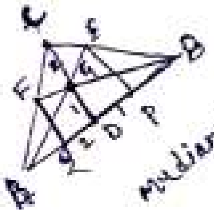
$y = h/3$

3)



$\bar{x} = b/2$

$\bar{y} = h/3$



median divided into 2:1 ratio.

Draw a line FQ

BFQ & DAGD.

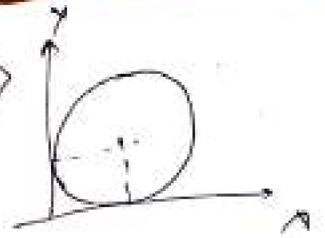
$\frac{AQ}{\ldots} = \frac{AF}{AC}$

$\left.\right\}$ DQ BF & D DBF

$FG/GB = \frac{QD}{DB} = \frac{1}{2}$

QD:DB = 1:2

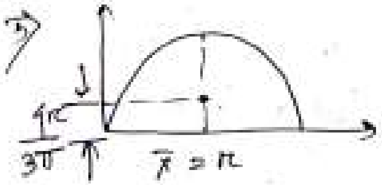
4)



$\bar{x} = r$

$\bar{y} = r$

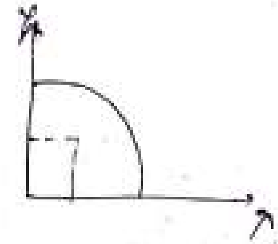
5)



$\bar{x} = r$

$\bar{y} = 4r/3\pi$

6)

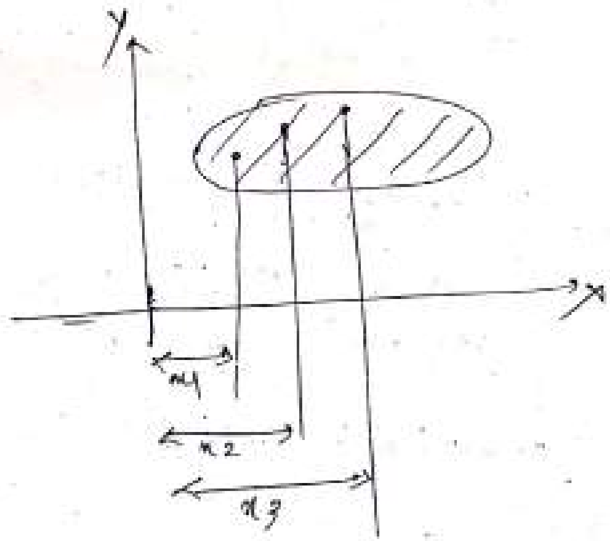


$\bar{x} = 4r/3\pi$

$\bar{y} = 4r/3\pi$

Where $\bar{x}$ & $\bar{y}$ is the co-ordinates of centroid given

<u>Centre of gravity by Moments</u>



Consider a body of mass M whose centre of gravity is required to be found out. Let it is divided into small masses $m_1, m_2, m_3 \ldots$ & the co-ordinates are (x_1, y_1) (x_2, y_2) & $(x_3\ y_3)$

$$M\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 \ldots$$

$$\bar{x} = \frac{\Sigma mx}{M}$$

$$\bar{y} = \frac{\Sigma my}{M}$$

$M = m_1 + m_2 + m_3 + \cdots$

Axis of Reference

The centre of gravity of a body is always calculated with reference to some assumed axis known as axis of reference, called as axis of reference, from where $\bar{x}$ & $\bar{y}$ is calculated.

Centre of gravity of plane figure

The plane geometrical sections such as T, I, L sections only have area but no mass. For these the centroid & centre of gravity is same.

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$$

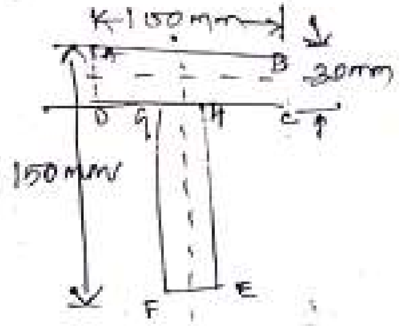
$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$$

Center of gravity of Symmetrical Sections

- If the given section is symmetrical about X-X axis then we have to find $\bar{x}$.

- If it is symmetrical to Y-Y axis then we have to find $\bar{x}$ & $\bar{y}$.

2) Find the centre of gravity of 100 mm x 150 mm x 30 mm of T-section.



Sol:

This section of is symmetrical about Y-Y axis.

Split the section in 2 section.

ABCD ; EFGH

For rectangle ABCD.

$a_1 = 100 \times 30 = 3000 \text{ mm}^2$

$y_1 = (150 - \frac{30}{2}) = 135 \text{ mm}$

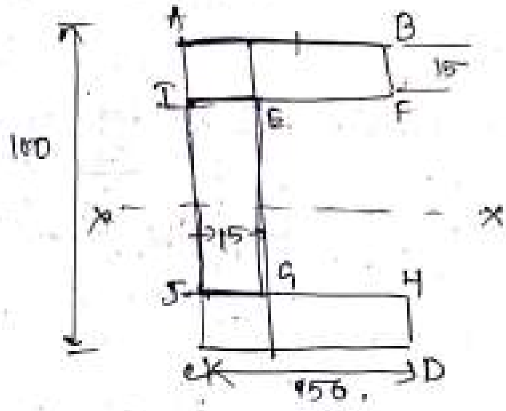
rectangle EFGH $a_2 = (150-30) \times 30 = 120 \times 30$
$= 3600 \text{ mm}^2$

$y_2 = 120/2 = 60 \text{ mm}.$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{3000 \times 135 + 3600 \times 60}{3000 + 3600}$$

$= 94.1 \text{ mm}.$

2) Symmetrical about X-X axis.



1) Rectangle ABIF.

$a_1 = 15 \times 50 = 750 \text{ mm}^2$

$x_1 = 50/2 = 25 \text{ mm}$

2) Rectangle CDHJ

$a_2 = 50 \times 15 = 750 \text{ mm}^2$

$x_2 = 50/2 = 25 \text{ mm}.$

3) Rectangle IEJG

$a_3 = 15 \times (100 - 30)$

$= 1050 \text{ mm}^2$

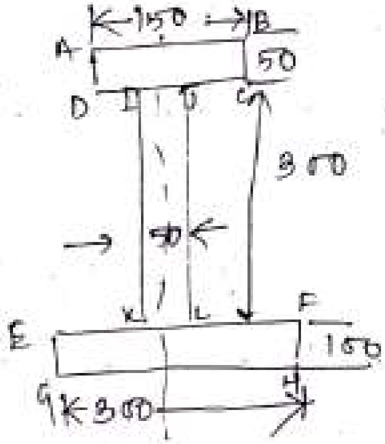
$x_3 = 15/2 = 7.5 \text{ mm}$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{750\times25 + 750\times25 + (1050\times7.5)}{750 + 1050 + 750}$$

$$= 17.8\,mm$$

Q2)



$a_1 = 150\times50$

$y_1 = 100 + 300 + \frac{50}{2}$

$= 400 + 25 = 425\,mm$

$a_2 = 300\times100$

$y_2 = 100/2 = 50\,mm$

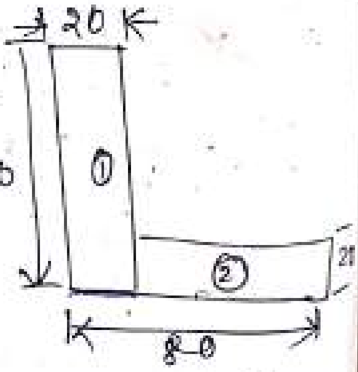
$a_3 = 300\times50$

$y_3 = 100 + \frac{300}{2} = 250\,mm$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

Center of gravity of unsymmetrical section

Q2) Find C.G of the given L section



Rectangle (1)

1 $a_1 = 20\times100 = 2000\,mm^2$

$y_1 = 100/2 = 50\,mm$

$x_1 = 20/2 = 10\,mm$

Rectangle (2) $a_2 = (80-20)\times20 = 1200\,mm^2$

$y_2 = 20/2 = 10\,mm$ $x_2 = 20 + \frac{(80-20)}{2}$

$= 50\,mm$

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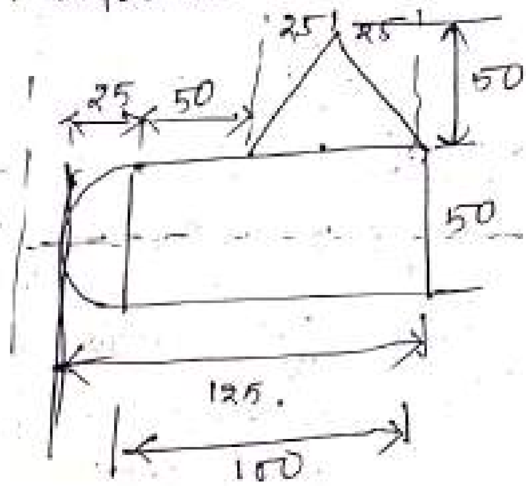
$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = 25mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = 35mm$$

2) A uniform lamina is shown in fig. Determine the C.G of the lamina.



a) for the rectangle:

$a_1 = 100 \times 50 = 5000\ mm^2$

$x_1 = 25 + 100/2 = 75mm$

$y_1 = 50/2 = 25mm$

for semicircle: $a_2 = \frac{\pi r^2}{2} = \pi/2\,(25)^2 = 982mm^2$

$x_2 = 25 - 4r/3\pi = 14.4\ mm$

$y_2 = 50/2 = 25mm$

for Δ. $a_3 = \frac{1}{2} \times b \times h = \frac{1}{2}\, 50 \times 50 = 1250\ mm^2$

$x_3 = 25 + 50 + 25 = 100\ mm$

$y_3 = 50 + 50/3 = 66.7mm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = 71.1\ mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = 32.2\ mm$$

4.2 MOMENT OF INERTIA

Moment of force = F × ⊥ distance. (1st moment of force.)

F × ⊥r distance × ⊥r distance (2nd moment of force) or moment, moment of force)

M.M.O.F / Second moment of force.

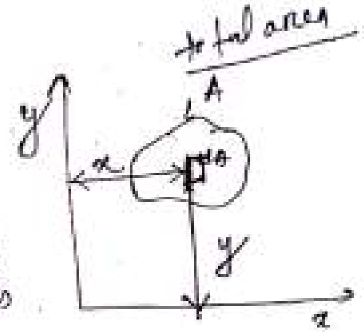
Sometime Area & mass can be found out by above methode.

⇒ also known as Moment of inertia.

{ M.M.O.A
{ M.M.O.M

$I_{yy} = \Sigma dA \cdot x$ (M.I about yy)

$= \Sigma dA \cdot x \cdot x$



$\boxed{I_{yy} = \Sigma dA \cdot x^2}$ — MI about yy axis

$\boxed{I_{yy} = \int dA \cdot x^2}$

$\boxed{I_{xx} = \int dA \cdot y^2}$ — M.I about xx axis

$\boxed{\text{Moment of inertia} = \text{Force} \times (\text{perpendicular distance})^2}$

$\boxed{\text{unit} = N\,m^2}$

Moment of inertia of a rectangular section.

Considere a rectangular section ABCD.

b → width of the section

d → depth of the section

Consider a small strip PQ of thickness dy ∥ to X-X axis at a distance y from the Centre axis.

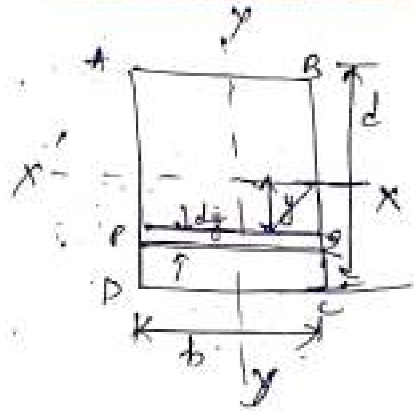
Area of small strip = $dA = b \times dy$

M.O.I of strip about $x-x$ axis

$= \text{Area} \times y^2$

$= dA \cdot y^2$

$= b \times dy \cdot y^2$



$$I_{x-x} = \int_{-d/2}^{d/2} dA \cdot y^2$$

$$= \int_{-d/2}^{d/2} b \cdot dy \cdot y^2$$

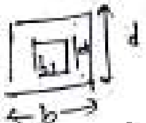
$$= b \int_{-d/2}^{d/2} y^2 \cdot dy = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right]$$

$$= b\left[\frac{d^3/8 - (-d^3/8)}{3}\right]$$

$$= b\left[\frac{2d^3}{8 \times 3}\right]$$

$$I_{xx} = bd^3/12.$$

for hollow 

$I_{xx} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$

$= db^3/12 - d_1 b_1^3/12$

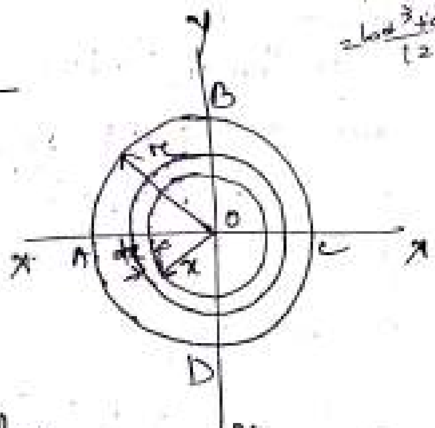
I_{yy} similarly $I_{yy} = \frac{db^3}{12}$.

$I_z = \frac{bd^3}{12} + \frac{db^3}{12}$

$= \frac{bd^3 + db^3}{12}$

M.I of a circular section

- Consider a circle ABCD with centre O.
- Consider a ring of radius x and thickness dx.



area of the ring $da = 2\pi x \cdot dx$

M.O.I about xx axis = area $\times$ distance^2

or yy axis $= 2\pi x \cdot dx \times x^2$

$= 2\pi x^3\, dx$.

Now M.I about the central axis let it be I_{xx}.

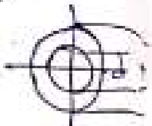
$$I_{zz} = \int_0^r 2\pi r^3 \cdot dx = 2\pi \int_0^r x^3\, dx$$

$$= 2\pi \left[\frac{x^4}{4}\right]_0^r$$

$$= \frac{\pi}{2} \times r^4 = \frac{\pi}{32} d^4 \quad (r = d/2)$$

$$\therefore I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{\pi}{64} d^4$$

for hollow



$$I_{xx} = \frac{\pi}{64}(D^4 - d^4) = I_{yy}$$

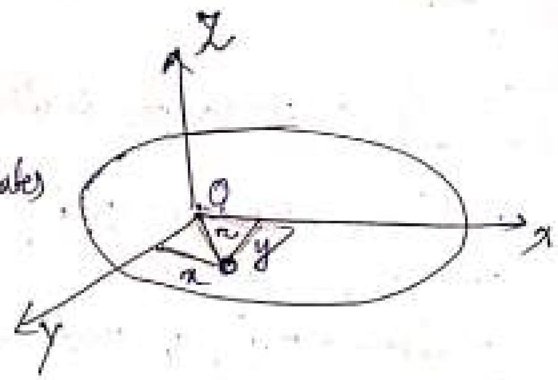
Theorem of perpendicular Axis

It states that if I_{xx} & I_{yy} be the moment of inertia of a plane section about 2 perpendicular axis metting at O, the moment of inertia about I_{zz} about the zz axis perpendicular to the plane and passing through intersection of x-x & y-y is given by

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Proof

consider a lamina 'p' of area da having coordinates x & y along ox & oy axis as shown in fig.



consider a plane oz $\perp$ to ox & oy. Let r be the distance of lamina p from zz axis. op = r

from geometry $r^2 = x^2 + y^2$

M.I about XX $I_{xx} = da \cdot y^2$

YY $I_{yy} = da \cdot x^2$

$I_{zz} = da \cdot r^2$

$= da(x^2+y^2)$

$= da\,x^2 + da \cdot y^2$

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Theorem of parallel axis

It states that if the M.I of a plane area about an axis through it's centre of gravity is denoted by I_G, then moment of inertia of the area about any other axis AB, parallel to the 1st, and at a distance h from the C.G is given by

$$\boxed{I_{AB} = I_G + ah^2}$$

$I_{AB} \rightarrow$ M.I of the area about axis AB.

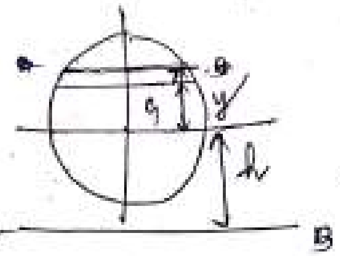
$I_G \rightarrow$ M.I . . . about C.G

$a \rightarrow$ area of section

$h \rightarrow$ distance betn C.G & secn AB.

proof

consider a strip of a circle, where M.I required to be found out



let δa = area of strip

y = distance of strip from C.G.

h = distance of C.G from axis AB

M.I of whole section about an axis passing through C.G $= \delta a \cdot y^2$

$I_G = \Sigma \delta a \cdot y^2$ M.I of whole secn passing through C.G.

M.I of section about AB

$$I_{AB} = \Sigma\, \delta a (h+y)^2$$

$$= \Sigma\, \delta a (h^2+y^2+2hy)$$

$$= (\Sigma h^2 \delta a) + (\Sigma y^2 . \delta a) + (\Sigma \cancel{2hy}\, \delta a)$$

$$\boxed{I_{AB} = ah^2 + I_G}$$

$\Sigma h^2 \delta a = ah^2$ sum of moments

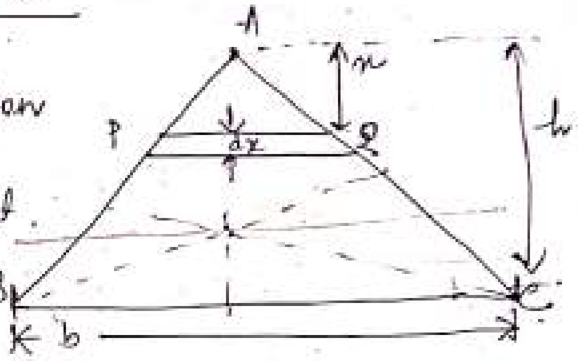
$\Sigma y^2 \delta a = I_G$

M.I of a triangular section

Consider a triangular section ABC whose ~~and~~ M.I is required to be found out.

b → base

h → height



(BC = base = b)

Consider a small secⁿ PQ of thickness dx at a distance from vertex A.

for △APQ , △ABC

$$\frac{PQ}{BC} = \frac{x}{h}$$

$$\Rightarrow PQ = \frac{BC . x}{h} = \frac{b.x}{h}$$

Small area of ~~dx~~ PQ $= \frac{b.x}{h} \times dx$

M.I of strip about BC $=$ Area $\times$ (distance)^2

$$= \frac{bx}{h} . dx \times (h-x)^2$$

$$= \frac{bx}{h} (h-x)^2 . dx$$

M.I of whole ~~section~~ △ can be found out by integrating the above from 0 to h

$$I_{BC} = \int_0^h \frac{bx}{h}(h-x)^2\,dx$$

$$= \frac{b}{h}\int_0^h x\cdot(h^2+x^2-2hx)\,dx$$

$$= \frac{b}{h}\int_0^h (xh^2+x^3-2hx^2)\,dx$$

$$= \frac{b}{h}\left[\frac{x^2h^2}{2}+\frac{x^4}{4}-\frac{2hx^3}{3}\right]_0^h$$

$$= \frac{b}{h}\left[\frac{h^4}{2}+\frac{h^4}{4}-\frac{2h^4}{3}\right] = \frac{b}{h}\left[\frac{2h^4+h^4}{4}-\frac{2h^4}{3}\right]$$

$$= \cancel{\frac{b}{h}\left[h^4-\frac{2h^4}{3}\right]}$$

$$= \cancel{\frac{b}{h}\left[\frac{3h^4-2h^4}{3}\right]} = \cancel{\frac{b}{h}\left[\frac{h^4}{3}\right]} = \cancel{\frac{bh^3}{3}}$$

$$= \frac{b}{h}\left[\frac{3h^4}{4}-\frac{2h^4}{3}\right] = \frac{b}{h}\left[\frac{9h^4-8h^4}{12}\right] = \frac{bh^3}{12}$$

M.I. of triangular section through axis of the centre of gravity, parallel to x-axis

$$I_G = I_{BC} - ad^2 \qquad d = h/3$$

$$= \frac{bh^3}{12} - \frac{bh}{2}\times\left(\frac{h}{3}\right)^2 \qquad I_{BC} = I_G + ah^2$$

$$\boxed{I_G = \frac{bh^3}{36}}$$

Moment of Inertia of a Composite Section.

Steps

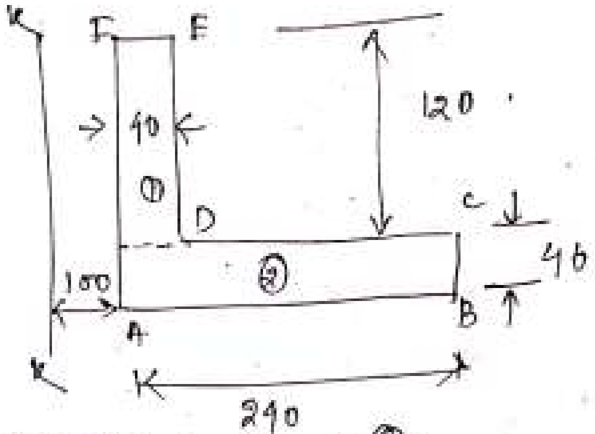
↳ 1st split up the given section into plane areas.

↳ Find M.I of these areas about their respective C.G.

↳ Apply parallel axis theorem.

↳ Obtain the M.I.

2) Find M.I about axis KK



Split up the secⁿ into ① & ②.

for secⁿ ①. I_{G1} = M.I about c.G about the axis K-K.

$$I_{G1} = \frac{db^3}{12} = \frac{120 \times 40^3}{12} = 640 \times 10^3 \text{ mm}^4$$

$h_1 = 100 + \frac{40}{2} = 120$ mm. (distance betⁿ c.G of secⁿ ① & axis K-K)

M.I of secⁿ ① axis K-K.

$$I_{KK} = I_{G1} + a_1 h_1^2$$
$$= [(640 \times 10^3) + (120 \times 40) \times (120)^2]$$
$$= 69.76 \times 10^6 \text{ mm}^4$$

Similarly M.I of section ② above it's c.G & parallel to axis K-K.

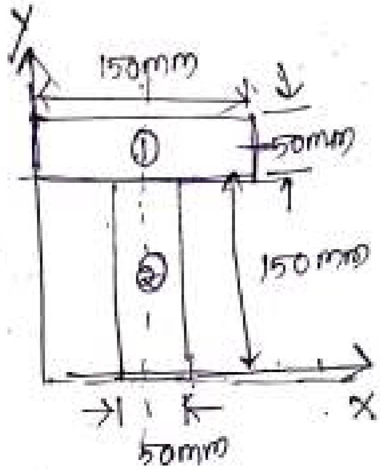
$$I_{G2} = \frac{db^3}{12} = 46.08 \times 10^6 \text{ mm}^4$$

$$h_2 = 100 + \frac{240}{2} = 220 \text{ mm}$$

$$I_{G2} + a_2 h_2^2$$
$$= [(46.08 \times 10^6) + (240 \times 40) \times (220)^2]$$
$$= 510.72 \times 10^6 \text{ mm}^4$$

$$I_{KK} = 69.76 \times 10^6 + 510.72 \times 10^6$$
$$= 580.48 \times 10^6 \text{ mm}^4$$

2) Find the M.I of a T-section with as 150 mm × 50 mm and web 150 mm × 50 mm about x-x & y-y axis through the centre of gravity of the section.



Soln Rectangle ①

$a_1 = 150 \times 50 = 7500 \text{ mm}^2$

$y_1 = 150 + \frac{50}{2} = 175 \text{mm}$

Rectangle ②

$a_2 = 150 \times 50 = 7500 \text{ mm}^2$

$y_2 = \frac{150}{2} = 75 \text{ mm}$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500} = 125 \text{mm}$$

M.I of ① about X-X axis

$I_{G1} = \frac{db^3}{12} = \frac{150 \times 50^3}{12} = 1.5625 \times 10^6 \text{mm}^4$

y → distance from c.G.

$h_1 = \overbrace{150 + \frac{50}{2}}^{y_1} \overset{-\bar{y}}{-} 125 = 50 \text{mm}$

M.I about x-x axis $I_{G1} + a_1 h_1^2$

$= 1.5625 \times 10^6 + 7500 \times (50)^2$.

$= 20.3125 \times 10^6 \text{mm}^4$

Similarly M.I of ② about X-X axis

$I_{G2} = \frac{bd^3}{12} = \frac{50 \times (150)^3}{12} = 14.06 \times 10^6 \text{ mm}^4$

$h_2 = 125 - \frac{150}{2} = 50 \text{ mm}$

M.I about XX axis $I_{G2} + a_2 h_2^2$

$= 14.06 \times 10^6 + 7500 \times 50^2$.

$= 32.8125 \times 10^6 \text{ mm}^4$

$I_{xx} = 20.3125 \times 10^6 + 32.8125 \times 10^6$

$= 53.125 \times 10^6 \text{ mm}^4$ Ans

Moments about yy axis of

$$I_{G_1} = \frac{db^3}{12} = \frac{50\times150^3}{12} = 14.0625\times10^6\ mm^4$$

$$I_{G_2} = \frac{db^3}{12} = \frac{150\times50^3}{12} = 1.5625\times10^6\ mm^4$$

From y axis the distance is zero.

M.I about y-y axis ①

$$I_{G_1} + \cancel{a_1 h_1^2}^{\,0} = 14.0625\times10^6\ mm^4$$

M.I about y-y axis ②

$$I_{G_2} + \cancel{a_1 h_1^2}^{\,0} = 1.5625\times10^6\ mm^4$$

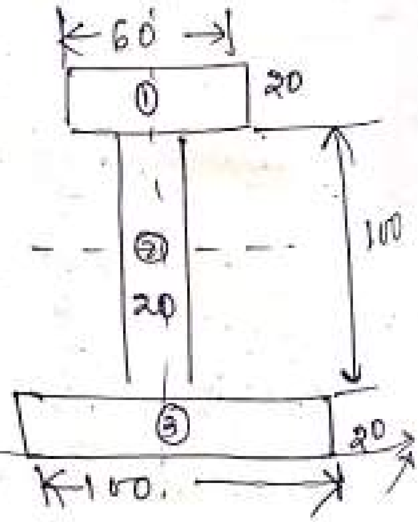
$$I_{yy} = 14.0625\times10^6 + 1.5625\times10^6$$

$$= 15.625\times10^6\ mm^4 \ \underline{\underline{Ans}}$$

~~2019~~ Q)

Find the M.I of the given section about horizontal axis passing through C.G. Find M.I about X-X axis

~~2017~~ This secn is symmetric about y axis. CG pass



Rect ① $a_1 = 60\times20 = 1200\ mm^2$

$x_1 = 60/2 = 30$

$y_1 = 120 + \frac{20}{2} = 130\ mm$

② $a_2 = 100\times20 = 2000$

$y_2 = 20 + \frac{100}{2} = 70\ mm$

③ $a_3 = 100\times20 = 2000$

$y_3 = 20/2 = 10\ mm$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1+a_2+a_3} = 60.8\ mm$$

$I_{G_1} = \frac{bd^3}{12} = \frac{60 \times 20^3}{12} = 40 \times 10^3 \, mm^4$

$h_1 = y_1 - \bar{y} = 130 - 60.8 = 69.2 mm$

M.I of rectangle 1 about X-X

$I_{G_1} + a_1 h_1^2 = 40 \times 10^3 + [1200 \times (69.2)^2]$

$= 5786 \times 10^3 mm^4$

for 2

$I_{G_2} = \frac{bd^3}{12} = \frac{20 \times 100^3}{12} = 1666.7 \times 10^3 mm^4$

$h_2 = y_2 - \bar{y} = 70 - 60.8 = 9.2 mm$

$I_{xx(2)} = I_{G_2} + a_2 h_2^2 = 1836 \times 10^3 mm^4$

for 3 $I_{G_3} = \frac{100 \times 20^3}{12} = 66.7 \times 10^3 mm^4$

$h_3 = \bar{y} - y_3 = 60.8 - 10 = 50.8 mm.$

$I_{xx(3)} = I_{G_3} + a_3 h_3^2 = 5228 \times 10^3 mm^4$

$I_{xx} = (5786 \times 10^3) + (1836 \times 10^3) + (5228 \times 10^3)$

$= 12850 \times 10^3 mm^4$

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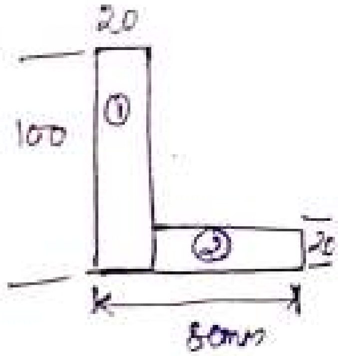
2018 (S) Find the M.I about the centroidal X-X & Y-Y axis of the angle section.

Soln: This section is not symmetrical about x axis axis.



Rectangle (1)

$$a_1 = 100 \times 20 = 2000 \ mm^2$$

$$y_1 = 100/2 = 50 \ mm$$

(2) $$a_2 = 80 \times 20 = 1600 \ mm^2$$

$$y_2 = \frac{20}{2} = 10 \ mm$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{2000 \times 50 + 1600 \times 10}{2000 + 1600} = 35 mm$$

M.I of ① about X-X axis.

$$I_{G1} = \frac{bd^3}{12} = \frac{20 \times (100)^3}{12} = 1.667 \times 10^6 \ mm^4$$

$$h_1 = y_1 - \bar{y} = 50 - 35 = 15 mm$$

$$I_{xx(1)} = I_{G1} + a_1 h_1^2 = 1.667 \times 10^6 + 2000 \times (15)^2$$
$$= 2.117 \times 10^6 \ mm^4$$

M.I of ② about X-X-axis

$$I_{G2} = \frac{bd^3}{12} = \frac{\cancel{80}^{60} \times 20^3}{12} = 0.04 \times 10^6 \ mm^4$$

$$h_2 = \bar{y} - \bar{y}_2 = 35 - 10 = 25 mm$$

$$I_{xx(2)} = I_{G1} + a_2 h_2^2 = 0.79 \times 10^6 \ mm^4$$

A

$I_{x-x} = I_{xx(1)} + I_{xx(2)} = 2.407 \times 10^6 \ mm^4$

M.I about y axis

$x_1 = 20/2 = 10 \ mm$

$x_2 = 20 + 60/2 = 50 \ mm$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = 25 mm.$$

M.I of ① about y-y axis

$$I_{G1} = \frac{db^3}{12} = \frac{100 \times 20^3}{12} = 0.06 \times 10^6 \ mm^4$$

$h_1 = \bar{x} - x_1 = 25 - 10 = 15 mm$

$$I_{yy(1)} = I_{G1} + a_1 h_1^2 = 0.06 \times 10^6 + 2000 \times 15^2$$
$$= 0.517 \times 10^6 \ mm^4$$

M.I of ② y-y

$$I_{G2} = \frac{db^3}{12} = \frac{20 \times 80^3}{12} = 0.36 \times 10^6 \ mm^4$$

$h_2 = x_2 - \bar{x} = 50 - 25 = 25 mm$

$$I_{yy(2)} = I_{G2} + a_2 h_2^2 = 1.11 \times 10^6 \ mm^4$$

$$I_{yy} = I_{yy(1)} + I_{yy(2)}$$
$$= 1.627 \times 10^6 \ mm^4$$

CHAPTER – 05 Principle of Lifting Machines.

5.1 ↳ Machine :- It is an assembly of interconnected components arranged to transmit or modify force in order to perform useful work.

↳ Simple machine :- It is defined as a machine which helps to do some work at some point when effort of force is applied to it.

↳ Compound machine :- It can be defined as a device which consist of no. of simple machine which enable us to do some work at a faster speed with less effort as compare to simple machine.

↳ Lifting Machine :- The machine which are use to lift heavy load are called lifting machine. In a lifting machine a force or load (W) applied at one point by means of another force called effort (P) applied at another point.

1) Mechanical Advantage (M.A)

$$M.A = \frac{\text{Weight load lifted}}{\text{effort applied}} = \frac{W}{P}$$

$$M.A = \frac{W}{P}$$

2) Velocity Ratio (V.R)

$$V.R = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{y}{x}$$

3) **Input** :- It can be defined as work done on the machine. It is measured by the product of effort applied & the distance covered by the effort.

$i/p = P \times y$ or effort × effort distance.

4) **output** :- It is defined as the work done by the machine. It is the product of load lifted & distance covered by the load.

$\text{output} = W \times x$ Load × load distance.

Efficiency (η) / Relation bet^n η, M.A, V.R

Ratio of $\dfrac{\text{work done by the machine}}{\text{work done on the m/c}}$

$$= \frac{W \times x}{P \times y} = \frac{W}{P} \times \frac{x}{y}$$

$$= \frac{W}{P} \times \frac{1}{y/x} = \frac{M \cdot A}{1} \times \frac{1}{V \cdot R}$$

$$\boxed{\eta = \frac{M \cdot A}{V \cdot R}} < 1.$$

Ideal Machine

$$\eta = \frac{M \cdot A}{V \cdot R} = 100\%$$

i.e $\boxed{O/P = i/P}$.

Q) In a certain weight lifting m/c a weight of 1 kN is lifted by an effort of 25 N. While wt. moves by 100mm, the point of application of effort moves by 8 m. Find MA, VR & η.

Soln

$W = 1 \text{KN}$

$P = 25 \text{ N}$

$x = 100 \text{mm} = .1 \text{ m}$

$y = 8$

$MA = W/P = 40$

$VR = y/x = 80$

$\eta = MA/VR = 0.5 = 50\%$

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3) Effort = 50 N (P)

Load (W) = 500 N

Effort distance = (y) = 55 cm = .55 m

Load distance = (x) = 5 cm = 0.05 m

$VR = y/x = \frac{.55}{.05} = 11$

$M.A = \frac{500}{50} = 10$

$\eta = \frac{10}{11} = 0.9\% = 90\%$

5) $V.R = 50$ Determine W & P = 60

$\eta = 70\%$

$VR = y/x$

$\eta = \frac{MA}{VR}$

$\Rightarrow .70 = \frac{MA}{50}$

$\Rightarrow MA = 35$

$MA = \frac{W}{P}$

$\Rightarrow W = 2100\ N.$

Reversibility of a Machine

Sometimes, a machine is also capable of doing [same work in the reversed direction, after effort is removed. Such a m/c is called a reversible m/c & known as reversibility of a machine.

Condition for Reversibility of a m/c

W → load lifted by the m/c

P → effort req to lift the load

y → distance moved by effort

x → distance moved by load.

$i/p = P \times y$

$o/p = W \times x$

We know that m/c friction = $i/p - o/p$

$= P \times y - W \times x$

If the m/c is reversible then the o/p of the machine should be more than friction.

$W \times x > P \times y - W \times x$

$\Rightarrow 2W \times x > P \times y$

$\Rightarrow \frac{W \times x}{P \times y} > \frac{1}{2}$

$\Rightarrow \frac{W/P}{y/x} > 1/2$

$\frac{M.A}{VA} > 1/2$

$\frac{MA}{VR} > 50\%$

$\eta > 50\%$

So the condition is if the machine is reversible the efficiency is more than 50%.

Self locking m/c

Some time a machine is not capable of doing any work when the effort is removed. Such machine is called as self locking machine. Here the efficiency should not be more than 50%.

Law of Machine.

Law of machine may be defined as the relationship between effort applied & load lifted.

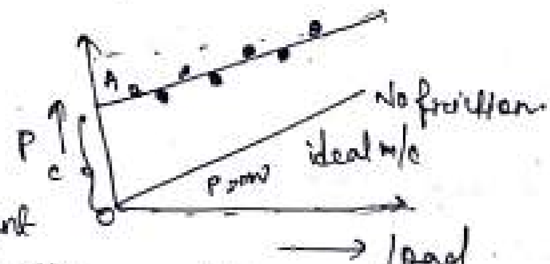
Mathmetically it is $P = mW + C$

$P \rightarrow$ effort

$W \rightarrow$ Load lifted

(slope) $m \rightarrow$ const

$C \rightarrow$ Another const. represent m/c friction.



amount of friction need to overcome by the machine.

2) What load can be lifted by an effort of 120N, if the velo. ratio is 18 & $\eta = 60\%$. Determine the law of the machine, if it is observed that an effort of 200N is req. to lift a load of 2600N & find the effort req. to run the m/c at a load of 3.5kN.

Soln $V.R = y/x = 18$ $P = 120$

$\eta = .6$

$$\frac{W/P}{V/R} = .6 \Rightarrow \frac{W}{P} = V.R \times .6$$
$$= 18 \times .6$$
$$= 10.8$$

$\Rightarrow W = 120 \times 10.8$
$= 1296N$

Laws of m/c $P = 200$
$W = 2600$

$P = mW + c$

$120 = m \times 1296 + c$ — (1)
$200 = m \times 2600 + c$ — (2)

$-80 = -m\,1304$

$\Rightarrow m = 0.061$

put the value of m in equn (2)

~~$120 = 0.061 \times 1296 + C$~~ $200 = 0.061 \times 2600 + c$

~~$\Rightarrow C = 1.15$~~ $\Rightarrow C = 44$

Now effort req. to lift a load of $3.5kN = 3.5 \times 10^3 N$

$P = .061 \times 3.5 \times 10^3 + 44$

$P = 257N$ Ans

6) In a lifting m/c an effort of 40N raised a load of 1kN. If efficiency of the m/c is 0.5, what is its velocity ratio? If on this m/c an effort of 74N raised a load of 2kN, what is new efficiency? What will be the effort req. to raise a load of 5kN.

Sol: $P = 40N$ $\quad \eta = 0.5$

$W = 1kN = 1000N$ $\quad P = 74N$ $\quad W = 2kN = 2000N$.

velocity ratio when effi is 0.5

$MA = \frac{W}{P} = \frac{1000}{40} = 25$

$\eta = \frac{M.A}{V.R} = \frac{25}{V.R} \Rightarrow V.R = \frac{25}{0.5} = 50$

Effi when P is 74 & W = 2000N

$M.A = \frac{W}{P} = \frac{2000}{74} = 27$

$\eta = \frac{M.A}{V.R} = \frac{27}{50} = 54\%$

effort req. to raise a load of 5kN or 5000N

$P = mW + C$

$40 = m \times 1000 + \cancel{C}$

$74 = m \times 2000 + \cancel{C}$

$\Rightarrow 34 = 1000m$

$\Rightarrow m = 0.034$

value of c.

$40 = m \times 1000 + C$

$\Rightarrow 40 = 0.034 \times 1000 + C$

$\Rightarrow C = 6$

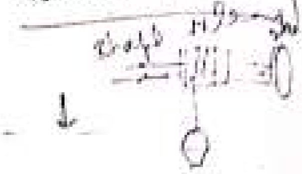
$P = 0.034W + 6$

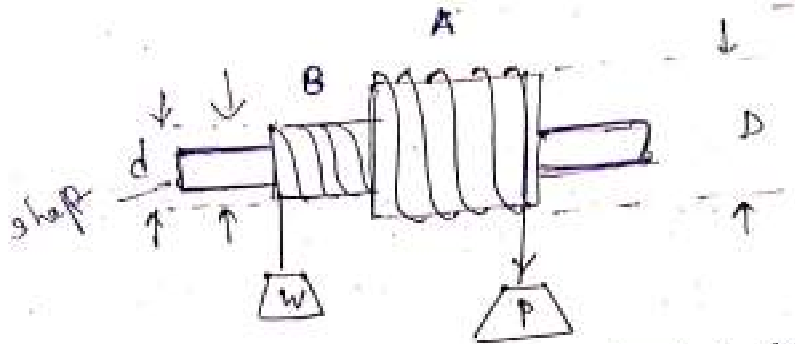
$\Rightarrow P = 0.034 \times 5000 + 6 = \underline{176N}$

5.2 Simple Lifting Machine

Simple Wheel & Axle

Well & wheel





The above is the fig of simple wheel & Axle.

↳ The wheel A & axle B are keyed to the same shaft. The shaft is mounted on ball bearing, to reduce the frictional resistance minimum.

↳ A string is wound round the axle B, which carries the load to be lifted. A second string is wound round the wheel A in the opposite direcⁿ to that of the string on B.

D → Dia of effort wheel   W → load lifted
d → " " " load axle   P → effort applied

↳ one end of the string is fixed to the wheel, while the other is free & the effort is applied to this end.

↳ Since the two strings are wound in opposite directions, therefore a downward motion of the effort (P) will raise the load (W)

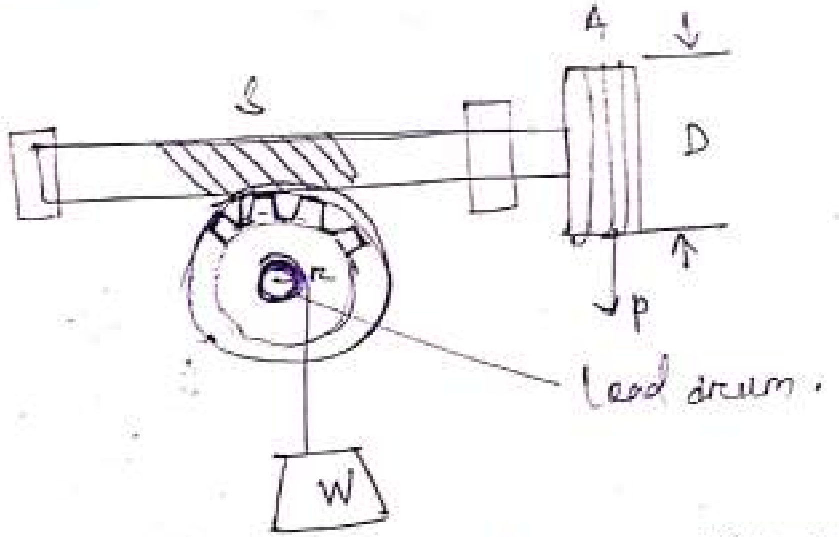
$$M.A = \frac{W}{P}$$

Distance / Displacement by the wheel $= \pi D$
" " " Axle $= \pi d$

$$V.R = \frac{\pi D}{\pi d} \Rightarrow V.R = \frac{D}{d}$$

$$\eta = \frac{M.A}{V.R}$$

Worm & Wheel



↳ It consist of a square threaded screw. S (known as worm) & a toothed wheel (known as worm wheel) geared to each other..

↳ A wheel A is attached to the worm, over which passes a rope as shown in fig.

- D → Dia of effort wheel
- r → radius of the load drum.
- W → load
- P → Effort applied
- T → No. of teeth on the worm wheel.

$2\pi r$ $2\pi \times \frac{d}{2}$

$$M.A = \frac{W}{P}$$

~~Displace~~ Distance moved by wheel $= \pi D$

" " " Load drum $= \frac{2\pi r}{T}$

$$V.R = \frac{\pi D}{2\pi r / T} = \frac{DT}{2r}$$

if the there is thread of n no.

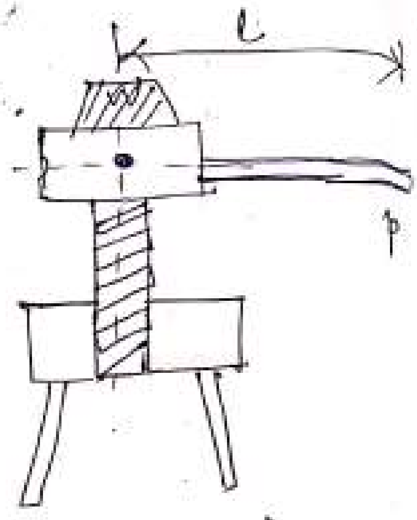
then $V.R = \frac{DT}{n \times 2 \times r}$

$$\eta = \frac{M.A}{V.R}$$

Simple Screw Jack

It consist of a screw, fitted in a nut, which forms the body of the Jack. The principle, on which a screw works, is similar to that of an inclied plane.

↳ The fig shows a simple screw Jack.

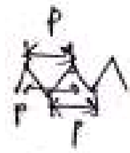


↳ L → length of effort arm

P → effort

W → load

p → pitch of the screw



The distance moved by the effort in one revolution = $2\pi l$

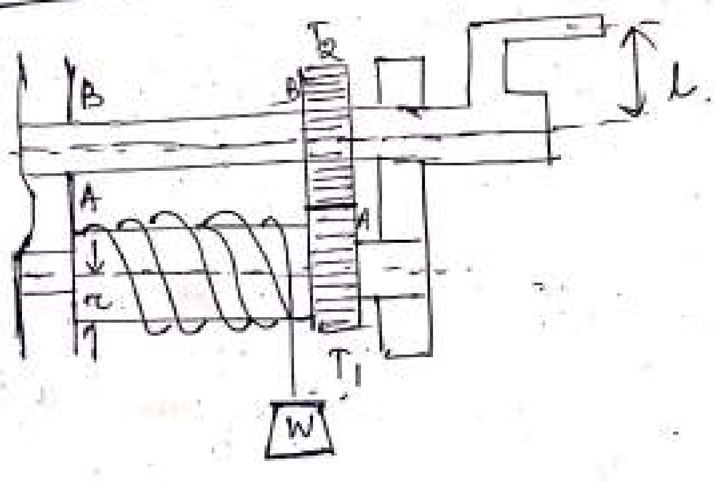
Distance moved by the load = p

$$V \cdot R = \frac{2\pi l}{p}$$

$$M \cdot A = \frac{W}{P}$$

$$\boxed{\eta = \frac{M \cdot A}{V \cdot R}}$$

Single purchase crab winch



In a single purchase crab winch, a rope is fixed to the drum & is wound a few turns around it.

The free end of the rope carries a load W.

↳ A toothed wheel A is rigidly mounted on the load drum

↳ Another toothed wheel B called pinion is geared with wheel A.

$T_1 \rightarrow$ No. of teeth in wheel/gear A.

$T_2 \rightarrow$ " " " " / " B.

$l \rightarrow$ length of handle

$r \rightarrow$ radius of load drum

$W \rightarrow$ load

$P \rightarrow$ effort.

Distance moved by the effort in one revolution of handle

$= 2\pi l$

No. of revolⁿ made by pinion B = 1

" " " " A = $\frac{T_2}{T_1}$

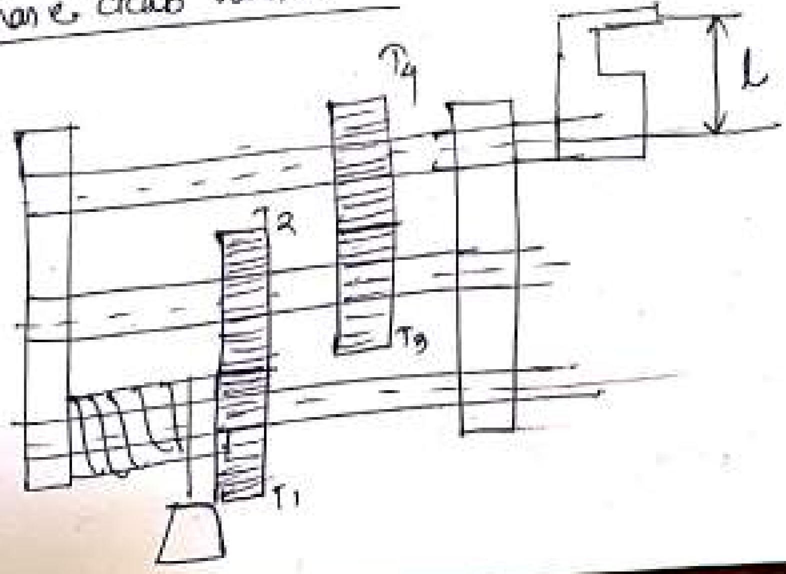
" " " " load drum = T_2/T_1

distance moved by load = $2\pi r \times T_2/T_1$

$$V.R = \frac{2\pi l}{2\pi r \times T_2/T_1} = \frac{T_1 \times l}{T_2 \times r}$$

$$M.A = \frac{W}{P} \qquad \boxed{\eta = \frac{M.A}{V.R}}$$

Double purchase crab winch



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It is the improved version of single purchase crab winch. Here there are 2 spur wheel & 2 pinion.

T_1 meshed with T_2 (pinion)

T_3 " " T_4 (pinion)

l = length of the handle.

T_1 & T_3 = No. of teeth in spur wheels

T_2 & T_4 = " " " pinion "

r = radius of drum

W = load

P = effort

Distance moved by effort in one revolution of handle

$= 2\pi l$

No. of revol^n made by pinion 4 = 1

" " " " spur 3 = T_4/T_3

" " " " pinion 2 = T_4/T_3

" " " spur 1 = $\frac{T_2}{T_1} \times \frac{T_4}{T_3}$

Distance moved by load $= 2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

$$V.R = \frac{2\pi l}{2\pi r (T_2/T_1)(T_3/T_4)} = \frac{l}{r}\left(T_1/T_2 \times T_4/T_3\right)$$

$M.A = W/P$

$$\boxed{\eta = \frac{M.A}{V.R}}$$

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CHAPTER-06 | DYNAMICS

6.1 Dynamics :- It is the study of motion of rigid body and their relation with the forces causing them.

The entire system of dynamics is based on 3 laws of motion. Also known as newton law's of motion.

Newton's 1st law

It states that " Every body continues in its state of rest or of uniform motion in a straight line, unless it is acted up on by some external force.
It is also called as law of inertia.

↳ A body at rest has a tendency to remain at rest called inertia of rest.

↳ A body in uniform motion in a straight line has a tendency to preserve its motion. known as inertia of motion.

Newton's 2nd Law

" The rate of change of momentum is directly proportional to the impressed force and takes place in the same dirn in which the force acts".

m = mass of a body
U = initial velo. of the body
v = Final velo of the body
a = const. accln
t = time in seconds req. to change the velo U to v.
F = Force req to change velo from u to v in t sec.

Initial momentum = mu
final " = mv

Rate of change of momentum = $\frac{mv - mu}{t} = \frac{m(v-u)}{t}$

$= ma$

Acc to 2nd law $F \propto ma$

$\Rightarrow F = Kma$ $\quad (\because \frac{v-u}{t} = a)$

$K \rightarrow$ const.

For convenience, the unit of force adopted in such that it produces unit accelⁿ in unit mass.

$F = ma$ = mass x accelⁿ

In S.I system unit of force is Newton → N.

A Newton may be defined as the force which acting upon a mass of 1 kg, produces an accelⁿ of $1 m/s^2$ in the direⁿ of which it acts.

— Also known as law of dynamics.

If accelⁿ is due to gravity $a = 9.8\ m/s^2$ = 1 kg-wt

$F = ma$

$\Rightarrow F = 9.8$ ~~kg-wt~~ N $\quad$ (1 kg-wt = 9.8 N)

~~= 9.8 wt~~ $\quad$ [1 kg-F = 9.8 N)

= 1 kg-wt

= 1 kg-F

Q) body has 50 kg mass on earth. Find a where $g = 9.8 N/m^2$

b) on moon $g = 1.7 m/s^2$

c) " sun $g = 270 m/s^2$

$F_1 = 50 \times 9.8$

$F_2 = 50 \times 1.7$

$F_3 = 50 \times 270$

Kinetic equations

1) $v = u + at$

2) $s = ut + \frac{1}{2}at^2$

3) $v^2 = u^2 + 2as$

2) A particle of mass 50gm starts from rest & moves under the influence of a const force. It acquires a speed of 6m/s after 12s.

i) Find force on the particle

ii) Find speed at $t = 16s$

iii) Find distance covered by the particle in 1st 10s.

iv) Find " " " " during time interval 5 to 15s.

Sol:

m = mass of the particle = 50g = 0.05 kg

$u = 0$

$v = 6$ m/s

$t = 12$.

$F = ma$

i) $v = u + at$

$\Rightarrow 6 = 0 + a \times 12$

$\Rightarrow a = 6/12 = 0.5$ m/s^2

i) $F = m \times a$

$= .05 \times 0.5 = 0.025$ N

ii) $v = u + at$

$= 0 + (0.5 \times 16) = 8.0$ m/s

iii) $s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 0.5 \times 10^2 = 25$ m

iv) $s_2 - s_1 = \left(ut_2 + \frac{1}{2}at_2^2\right) - \left(ut_1 + \frac{1}{2}at_1^2\right)$

$= (\quad)$

Newton 3rd law of Motion

To every action there is an equal & opposite reacn.

Momentum :- It is the product of mass with velocity.

$m \times v$

Force :- Any external agent which produce or tends to produce, destroys or tends to destroy the motion of any body known as Force. unit N.

$F = m \times a$

Inertia :- The property which offers resistance to change state of rest or motion is known as inertia.

Newton 3rd law for recoil of gun

When bullet is fired from a gun, the opposite reaction of the bullet is known as recoil of gun.

$M \rightarrow$ Mass of gun.

$m \rightarrow$ Mass of bullet.

$V \rightarrow$ velo. of gun

$v \rightarrow$ velo of bullet after being fired.

momentum ~~before~~ of the gun $= MV$

" " , bullet $= mv$

$$\boxed{MV = mv}$$

Law of conservation of Momentum.

D'alembert's Principle

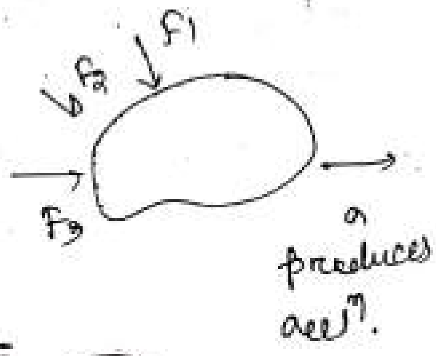
A system of forces acting on a body in motion is in dynamic equil^m with inertia force of the body.

inertia $\rightarrow$ Resist motion

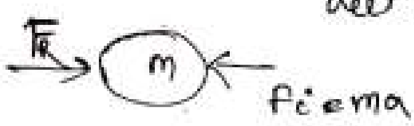
$\rightarrow$ resist to be at rest

The resultant of F_1, F_2, F_3 let R.

Let a mass m.



Ifulnant to bring the body at rest, we have to apply a force ~~xxx~~ in opposite direc whose value is equal to ma.



~~known as inertia force~~

~~whose value is~~

known as inertia force, to bring the body in static equlm.

$$\sum F = 0$$

$$F_i - ma = 0$$

$$\Rightarrow F_i = ma \Rightarrow \boxed{F_i = ma}$$

$-ma \rightarrow$ inertia force $= F_i$, Also known as reversed force.

6.2 Work, power, Energy

Work

When force acts on a body, the body undergoes a displacement, work is said to be done on the body by the force.

$$W = F \cdot S$$

unit

$$W = F \cdot S$$

$$= N\text{-}m = 1 \text{ Joule} \quad (SI)$$

$1 \text{ erg} = CGS = 1 \text{ dyne} = 10^{-7} \text{ Joule}$

power

It is the rate of doing work.

unit = Watt = J/s = N-m/s

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Energy

It is the capacity to do work.
It exists in many forms, mechanical, electrical, Chemical, heat, light etc.

unit

Same as work = Joule

Mechanical Energy

- Kinetic = $\frac{1}{2}mv^2$
- potential = mgh

Kinetic Energy

Energy possed by a body, by virtue of its mass & velocity.

P.E

Energy possed by a body, by virtue of its position.

Q) A truck of mass 15 tonnes travelling at 1.6 m/s. impact with a spring

Law of conservation of Energy

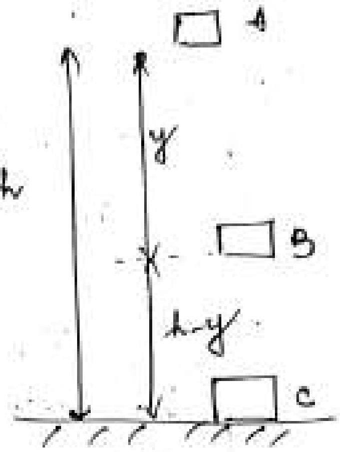
It states that "Energy can neither be created nor destroyed. though it can transformed from one form to another form.

Transformation of Energy

Consider a body of mass m which is released from rest from height h above the ground.

m = mass of the body

h = height



Energy at A

Since at A body has 0 velocity

KE = 0

PE = mgh

total energy = PE + KE = mgh

Energy at B

The body travelled y distance from A to B. So

$v = \sqrt{2gy}$

KE at B $= \frac{mv^2}{2} = \frac{m \times (\sqrt{2gy})^2}{2} = mgy$

PE $= mg(h-y) = mgh - mgy$

total energy = KE + PE $= \cancel{mgy} + mgh - \cancel{mgy}$

$= mgh$

Energy at C

At C body has fallen a height h.

$v = \sqrt{2gh}$

KE $= \frac{mv^2}{2} = \frac{m(\sqrt{2gh})^2}{2} = mgh$

PE = 0

total energy = KE + PE = mgh

Q) A 100gm ball is released from rest from the top of 20m high building. Find the change in P.E & K.E when it is at a height of 10 from the ground.

Soln

$m = 100gm = 0.1kg$

$h_1 = 20m$

$PE_1 = mgh_1 =$

$KE_1 = 0$

Impulse → When a const. force F acts on a body for a time interval. t. Known as Impulse.

$\boxed{I = F \times t}$ unit N-s

Linear momentum -

Law of conservation of linear momentum

Acc to newton's 2^{nd} law, the net external force acting on a body is equal to rate of change of linear momentum / momentum.

This leads to the law of conservation of linear momentum for a body.

Which states that the linear momentum of a body remains const. if the external force on a body is zero.

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6.3 Collision of Elastic Bodies

When two bodies strikes with each other with certain velocity it is known as collision.

↳ If one body is in rest and even if another body strikes to it (wall or floor) also known as collision.

↳ Let any ball strikes to the floor, it rises certain height or rebounded.

↳ This property of bodies by virtue of which, they rebounded after impact is called elasticity.

↳ But if a body does not rebound at all, after impact called as inelastic collision.

Phenomenon of collision

- The bodies, immediately after collision, come momentarily to rest.
- The two bodies tend to compress each other, so long as they are compressed to the maxi^m value called as time of compression. (t_c)
- The process of regaining of original shape from the deformed shape of the bodies called restitution. Time taken for that called as time of restitution (t_r)

Time of collision = Time of compression + Time of restitution

Law of conservation of Momentum

It states that the total momentum of two bodies remains const. after their collision.

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

m_1 = mass of 1st body

m_2 = " " 2nd body

u_1, u_2 = initial velocity of mass m_1 & m_2 respectively

v_1, v_2 = final " " " m_1 & m_2 "

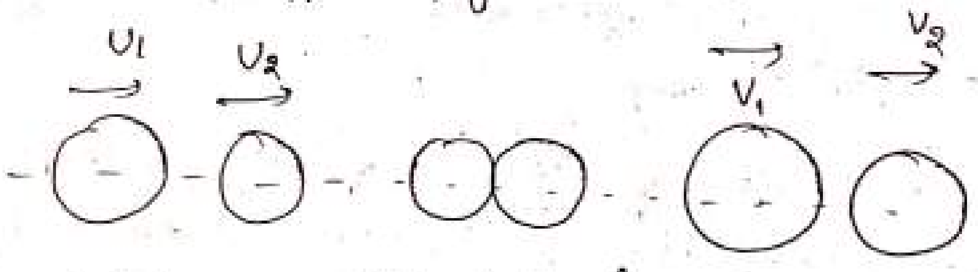
Newtons Law of collision of elastic bodies

It states when two moving bodies collide with each other, their velo. of separation bears a const ratio to their velo. of approach.

$$(v_2 - v_1) = e\,(u_1 - u_2)$$

$$e = \frac{u_1 - u_2}{v_2 - v_1}$$

e = co-efficient of restitution.



$u_1 > u_2 \rightarrow$ collision takes place.

$v_2 > v_1 \rightarrow$ separation takes place.

Types of collision

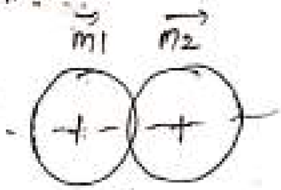
→ Direct collision

→ Indirect "

Direct collision

The line of impact of the two coliding bodies, is in the line joining the centers of the 2 bodies, known as point of contact or point of collision.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$



The value of e is in betn 0 to 1

if e = 0 collision is inelastic

e = 1 " " elastic.

2016

Q) A ball of mass 2 kg moving with a velocity 2m/sec hit another ball of mass 4 kg at rest, after impact the 1st ball comes to rest. Cal. velo. of the 2nd ball after impact & coeffi of restitution.

$m_1 = 2kg$ $u_1 = 2 m/s$

$m_2 = 4 kg$ $v_1 =$

$v_2 =$

$u_2 =$

$e =$

$v_1 = 0$ (comes to rest m_1)

$u_2 = 0$ at rest (m_2)

$$m_1 u_1 + m_2 \overset{0}{u_2} = m_1 \overset{0}{v_1} + m_2 v_2$$

$\Rightarrow 2 \times 2 = 4 \times v_2$

$\Rightarrow v_2 = 1 \ m/s$

$(v_2 - v_1) = e (u_1 - u_2)$

$\Rightarrow e = \dfrac{1-0}{2-0} = \dfrac{1}{2} = 0.5$ Ans

Q) 2015, 2018

Two balls of masses 2kg & 3kg are moving with velo 2m/s & 3m/s towards each other. If e = 0.5. find velocity of the two balls after collision.

Sol)

$m_1 = 2$ $u_1 = 2$

$m_2 = 3$ $u_2 = 3$

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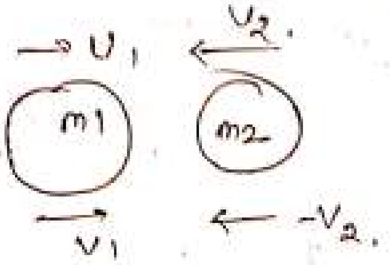
$$e = \frac{v_2 - v_1}{u_1 - u_2}$$



$$\Rightarrow \frac{1}{2} = \frac{v_2 - v_1}{\cancel{2(-3)}} = \frac{-v_2 - v_1}{2-(-3)}$$

$$\Rightarrow \cancel{v_2 - v_1 = \ldots \tfrac{1}{2}}$$

$$\Rightarrow \frac{1}{2} = \frac{-v_2 - v_1}{5}$$

$$\Rightarrow -v_2 - v_1 = 5/2 \text{ ——— } (2)$$

$$\Rightarrow v_2 + v_1 = -5/2$$

$$\Rightarrow v_2 = -5/2 - v_1$$

put the value at equ^n (2)

$$2v_1 - 3(-5/2 - v_1) = -5$$

$$\Rightarrow 2v_1 + 15/2 + 3v_1 = -5$$

$$\Rightarrow v_1 = -2.5 \text{ m/s}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow 2\times 2 + 3(-3) = 2v_1 + (-3v_2)$$

$$\Rightarrow 2v_1 - 3v_2 = -5 \text{ ——— } (1)$$

multiply 2 in eq (2) (2×2)

$$\begin{aligned} \cancel{2v_1} - 3v_2 &= -5 \\ -\cancel{2v_1} - 2v_2 &= +5 \\ \hline -5v_2 &= 0 \end{aligned}$$

$$v_2 = 0 \text{ m/s}$$

Now $v_1 = -5/2 = -2.5$ m/s

$v_2 = 0$

2) A ball is dropped from a height of 10m on a smooth floor and it rebound to a height of 5m. Determine the coefficient of restitution between the ball & the floor & also determine the expected height of the 2nd rebound.

u → velo before impact
v → " after "
h → hight before " 10m
h_1 → " after 1st rebound 5m
h_2 → " " 2nd " ?