Two-Dimensional Geometry :-

(Co-ordinate Geometry)

Introduction :-

- Co-ordinate Geometry is a link between the geometry and algebra, in which the geometrical problems are solved through algebra using curves and lines.
- It is a part of geometry, where the position of points is described using an ordered pair of numbers on the plane.

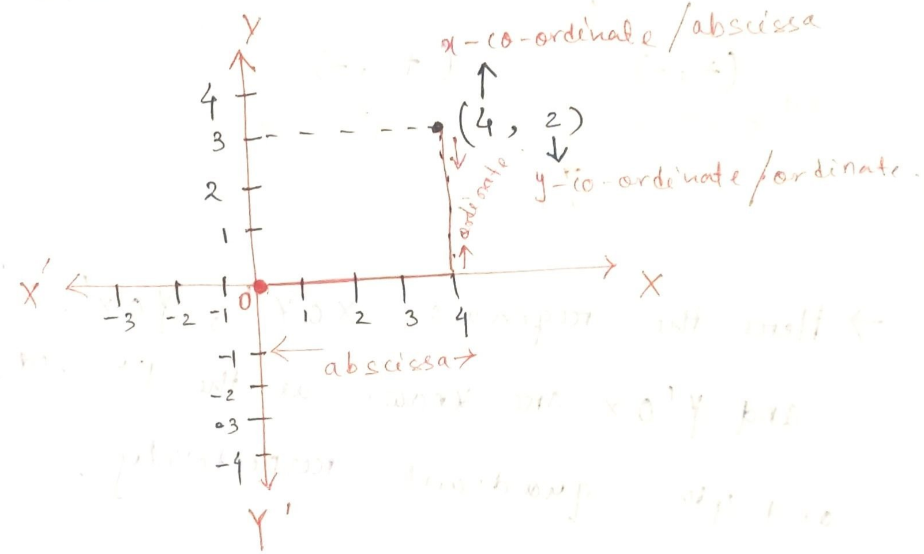
Application :- using co-ordinate geometry, it is possible to find the distance between two pts, to calculate area of a triangle in co-ordinate plane.

Co-ordinate :-

Co-ordinates are a set of values which helps to show the exact position of a pt. in the co-ordinate plane.

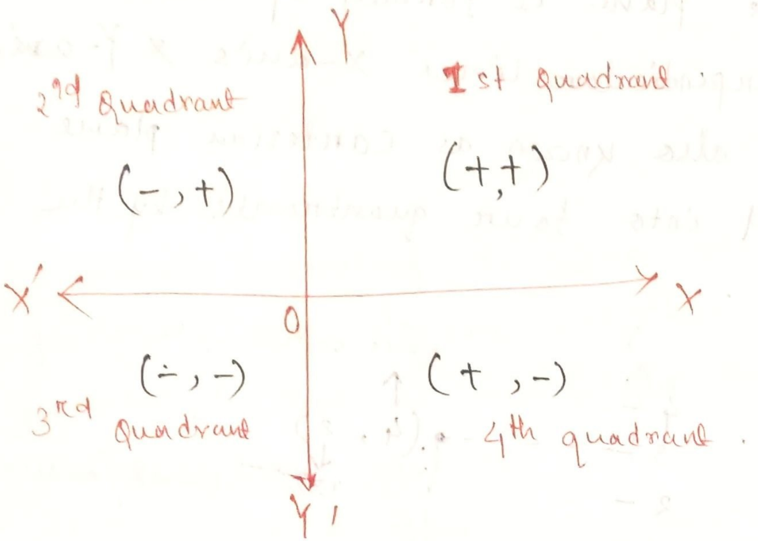
Co-ordinate plane :-

Co-ordinate plane is formed by intersection of two perpendicular lines X-axis & Y-axis, which is also known as cartesian plane is divided into four quadrants by the two axes.



* The point at which the axes intersect is known as the origin.
* The location of any pt. on a plane is expressed by a pair of values (x, y), known as the co-ordinates.
* The horizontal line (X-axis) & vertical line (Y-axis) are known as co-ordinate axes.

Quadrants :-



→ Here the regions XOY', YOX', X'OY' and Y'OX are known as the 1st, 2nd, 3rd and 4th quadrant respectively.

→ Signs for a point in different quadrants are given as follows:

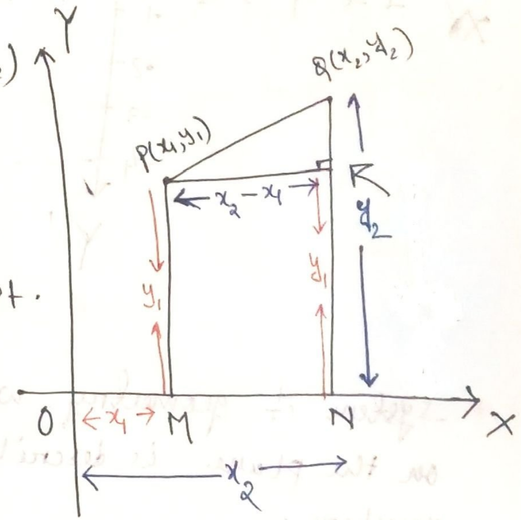
- 1st quadrant → $(+x, +y)$
- 2nd quadrant → $(-x, +y)$
- 3rd quadrant → $(-x, -y$
- 4th quadrant → $(+x, -y)$.

Co-ordinate Geometry Formulas & Theorems :-

Theorem 1 :- The distance between two pts $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

→ Here let $P(x_1, y_1)$ & $Q(x_2, y_2)$ be two pts.



→ let's draw perpendiculars PM and QN from the pt. P and Q on X-axis.

→ Also draw $PR \perp QN$.

Then $OM = x_1$ $ON = x_2$

$PM = y_1$ $QN = y_2$

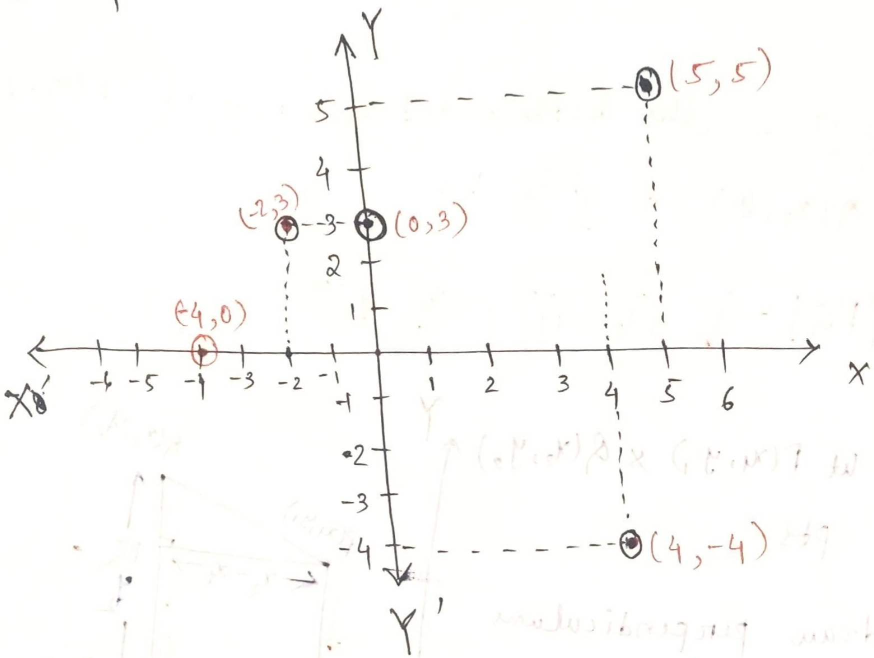
Then $PR = MN = ON - OM = x_2 - x_1$

Again $QR = QN - RN = y_2 - y_1$

Now consider the right angled triangle PQR.

plotting of pts on cartesian plane :-

Let's plot · (-2, 3), (4, -4), (5, 5), (0, 3), (-4, 0)



* System of geometry where the position of pts on the plane is described by using ordered pair of numbers.

* The plane where the pts are placed on, is known as co-ordinate plane.

It has two dimensions.

* A pt's location on a plane is given by two numbers, 1st tells where it is on X-axis & second where it is on Y-axis. & together they define a single & unique position on the plane.

By Pythagoreas theorem,

we have $(PQ)^2 = (PR)^2 + (QR)^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\Rightarrow \boxed{PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$

Q.1 :- find the distance between the pts (2, -4), (5, 6).

Solution :- let P(2, -4) and Q(5, 6) are two given pts.

P(2, -4), $x_1 = 2$, $y_1 = -4$

Q(5, 6), $x_2 = 5$, $y_2 = 6$

Then $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5-2)^2 + (6-(-4))^2}$

$= \sqrt{(3)^2 + (10)^2}$

$= \sqrt{9+100} = \sqrt{109}$

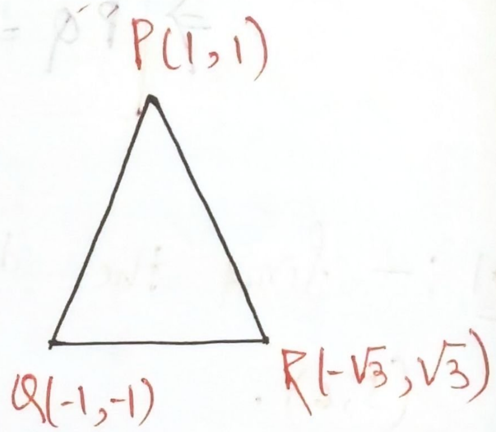
Q:-2 Show that the pts (1, 1), (-1, -1) and $(-\sqrt{3}, \sqrt{3})$ are the vertices of an equilateral triangle.

let $P(1,1) \Rightarrow x_1 = 1,\ y_1 = 1$

$Q(-1,-1) \Rightarrow x_2 = -1,\ y_2 = -1$

$R(-\sqrt{3}, \sqrt{3}) \Rightarrow x_3 = -\sqrt{3},\ y_3 = \sqrt{3}$

are three given pts.



$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-1-1)^2 + (-1-1)^2}$$

$$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$$

$$QR = \sqrt{(-\sqrt{3} - (-1))^2 + (\sqrt{3} - (-1))^2}$$

$$= \sqrt{(-\sqrt{3}+1)^2 + (\sqrt{3}+1)^2}$$

$$= \sqrt{(1)^2 + (-\sqrt{3})^2 + 2(-\sqrt{3})(1) + (\sqrt{3}) + (1)^2 + 2(\sqrt{3})(1)}$$

$$= \sqrt{1+3+3+1}$$

$$= \sqrt{8}$$

$$PR = \sqrt{(-\sqrt{3}-1)^2 + (\sqrt{3}-1)^2}$$

$$= \sqrt{(-\sqrt{3})^2 + (-1)^2 + 2(-\sqrt{3})(-1) + (\sqrt{3})^2 + (1)^2 + 2(\sqrt{3})(-1)}$$

$$= \sqrt{3+1+3+1}$$

$$= \sqrt{8}$$

Here $PQ = QR = PR$

$\Rightarrow PQR$ is an equilateral triangle.

Theorem-2 (section formula).

Section formula helps in finding the co-ordinate of a pt. which divides a line segement in some ratio let m : n.

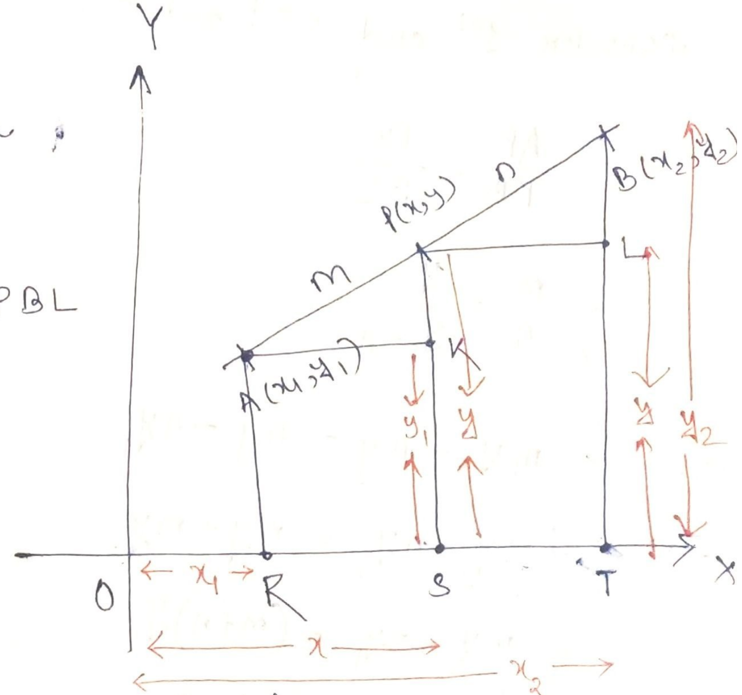
⇒ If m = n, then the pt. is the mid point.

Internal divison with Section formula

Let P(x, y) be a pt. which lies on a line-segment $\overline{AB}$ and satisfies AP : PB = m : n then we can say P divides the line $\overline{AB}$ ~~in the ro~~ internally in the ratio m : n.

Then co-ordinates of P will be

$$= \left(\frac{mx_2 + nx_1}{m+n} , \frac{my_2 + ny_1}{m+n} \right)$$



In the given figure

△APK & △PBL are simillare.

⇒ Sides are proportional.

i.e. $\frac{AP}{PB} = \frac{AK}{PL} = \frac{PK}{BL}$

consider first two ratios.

$$\frac{AP}{PB} = \frac{AK}{PL}$$

$$\Rightarrow \frac{m}{n} = \frac{x - x_1}{x_2 - x}$$

$$\Rightarrow m(x_2 - x) = n(x - x_1)$$

$$\Rightarrow mx_2 - mx = nx - nx_1$$

$$\Rightarrow mx_2 + nx_1 = mx + nx$$

$$\Rightarrow mx_2 + nx_1 = x(m+n)$$

$$\Rightarrow \boxed{x = \frac{mx_2 + nx_1}{m+n}}$$

Consider 1st and 3rd ratio.

$$\frac{AP}{PB} = \frac{PK}{BL}$$

$$\Rightarrow \frac{m}{n} = \frac{y - y_1}{y_2 - y}$$

$$\Rightarrow my_2 - my = ny - ny_1$$

$$\Rightarrow my_2 + ny_1 = ny + my$$

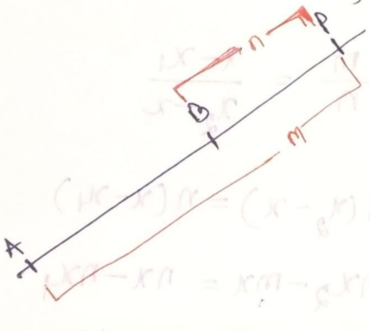
$$\Rightarrow my_2 + ny_1 = (m+n)y$$

$$\Rightarrow \boxed{y = \frac{my_2 + ny_1}{m+n}}$$

External Division with Section Formula

Let $P(x, y)$ be a pt. lies on the ~~exten~~ extention of the line segment AB and satisfies $AP : BP = m : n$

$\Rightarrow$ P divides the line segement externally in the ratio $m : n$

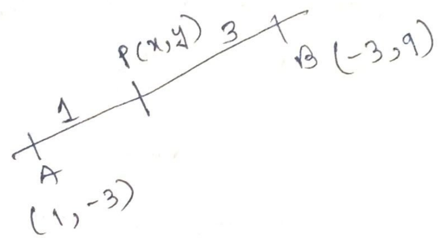


Then $x = \frac{mx_2 - nx_1}{m - n}$

$y = \frac{my_2 - ny_1}{m - n}$

Q:-1 :- find the co-ordinates of the pt. which divides the line segement $(1, -3)$ and $(-3, 9)$ in the ratio $1 : 3$

Sol^n



Let $A(1, -3) \;||\; x_1 = 1, \; y_1 = -3$

and $B(-3, 9) \;||\; x_2 = -3, \; y_2 = 9$

~~be~~ makes the line segement AB.

and $P(x, y)$ divides AB in the ratio $1 : 3$

so $m = 1$
$n = 3$

Then co-ordinates of P is obtained by

$$= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$= \left(\frac{1(-3) + 3(1)}{1+3}, \frac{1(9) + 3(-3)}{1+3} \right)$$

$$= \left(\frac{0}{4}, \frac{0}{4} \right) = (0, 0)$$

Q.2 find the co-ordinate of the pt. which divides the line segement joining (1, -3) & (-3, 9) in the ratio 1:3.

Solution :- Let P(x, y) be the pt which divides the line segement joining (1, -3) $\| \begin{array}{l} x_1 = 1 \\ y_1 = -3 \end{array}$

and (-3, 9) $\| \begin{array}{l} x_2 = -3 \\ y_2 = 9 \end{array}$

in the ration 1 : 3

$\Rightarrow m = 1$

$n = 3$

Then $x = \dfrac{mx_2 + nx_1}{m+n} = \dfrac{1(-3) + 3(1)}{1+3}$

$= \dfrac{-3+3}{4} = \dfrac{0}{4} = 0$

$y = \dfrac{my_2 + ny_1}{m+n} = \dfrac{1(9) + 3(-3)}{1+3} = \dfrac{9-9}{4} = \dfrac{0}{4}$

$= 0$

So the required co-ordinate is (0, 0).

Q.3 find the ratio in which the pt (3, -2) divide the line segment joining pts. (1, 4) & (-3, 16).

Solution :- Let P(3, -2) $\| \begin{array}{l} x = 3 \\ y = -2 \end{array}$

divides the line segement joining (1, 4) $\| \begin{array}{l} x_1 = 1 \\ y_1 = 4 \end{array}$

and (-3, 16) $\| \begin{array}{l} x_2 = -3 \\ y_2 = 16 \end{array}$

in the ratio m : n

Then $x = \dfrac{mx_2 + nx_1}{m+n}$

$\Rightarrow 3 = \dfrac{m(-3) + n(1)}{m+n}$

$\Rightarrow 3(m+n) = -3m + n$

$\Rightarrow 3m + 3n = -3m + n$

$\Rightarrow 6m = -2n$

$\Rightarrow \dfrac{m}{n} = \dfrac{-2}{6} = \dfrac{-1}{3}$

$\therefore$ P(3, -2) divides the line segment in the ratio 1 : 3 externally

② ~~Externally~~

Q.4 Find the co-ordinate of the pt. which divides the line segement joining (2, -1) & (-3, 4) in the ratio 2 : 3 externally.

Solution :- Let P(x, y) be the pt. which divides the line segment joining (2, -1) || $x_1 = 2$, $y_1 = -1$

and (-3, 4) || $x_2 = -3$, $y_2 = 4$

in the ratio 2:3 externally.

Then $x = \dfrac{mx_2 + nx_1}{m+n} = \cancel{\dfrac{m(-3)+n(2)}{m}}$

$= \cancel{\dfrac{2(-3)+3(2)}{2+3}} = \cancel{-6+}$

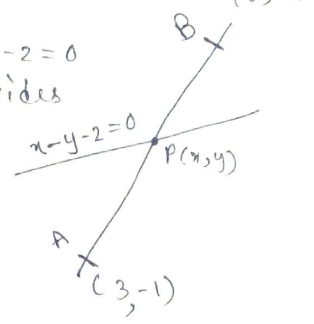
$x = \dfrac{mx_2 - nx_1}{m-n} = \dfrac{2(-3) - 3(2)}{2-3} = \dfrac{-6-6}{-1} = \dfrac{-12}{-1} = 12$

$y = \dfrac{my_2 - ny_1}{m-n} = \dfrac{2(4) - 3(-1)}{2-3} = \dfrac{8+3}{-1} = \dfrac{11}{-1} = -11$

So the required co-ordinate of pt. 'p' is (12, -11).

Q.5 Find the ratio in which the line $x-y-2=0$ cuts the line segment joining (3, -1) & (8, 9).

Solution



Let P(x, y) be a pt. on the line $x-y-2=0$ which divides the line segment joining the line A(3, -1) and B(8, 9) in the ratio m : n.

$x_1 = 3$, $y_1 = -1$ $\quad$ $x_2 = 8$, $y_2 = 9$

Then $x = \dfrac{mx_2 + nx_1}{m+n}$ & $y = \dfrac{my_2 + ny_1}{m+n}$

$= \dfrac{8m + 3n}{m+n}$ $\qquad$ $= \dfrac{9m - n}{m+n}$

As $P\left(\dfrac{8m+3n}{m+n}, \dfrac{9m-n}{m+n}\right)$ be a pt. on $x-y-2=$

$\Rightarrow \dfrac{8m+3n}{m+n} - \dfrac{9m-n}{m+n} - 2 = 0$

$\Rightarrow \dfrac{8m+3n-9m+n-2(m+n)}{m+n} = 0$

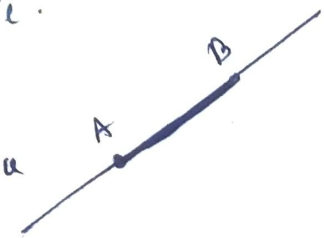
$\Rightarrow \dfrac{8m+3n-9m+n-2m-2n}{m+n} = 0$

$\Rightarrow -3m + 2n = 0$ $\qquad$ $\Rightarrow 3m = 2n$

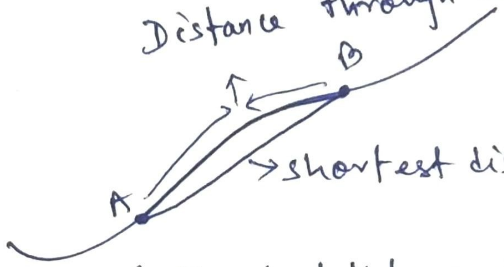
$\Rightarrow \dfrac{m}{n} = \dfrac{2}{3}$

STRAIGHT LINE

Definition:- In a straight line if we take any two points then the shortest distance between the points is equal to the distance between the pts through the curve.



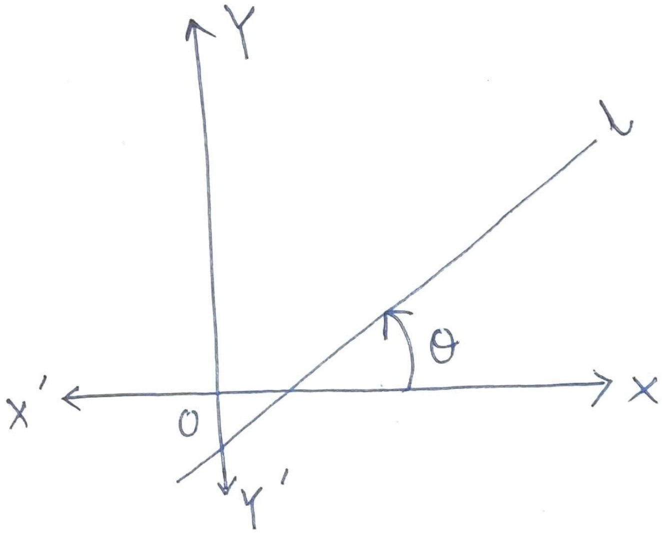
Here shortest distance = Distance through curve (Straight line)



(Shortest distance ≠ distance through curve)

Inclination of a straight line :-

Inclination of a line is the angle made by the line with the positive X-axis and measured in anti clockwise direction.



Here 'θ' is the inclination of the st. line 'l'.

* Inclination of X-axis or any line parallel to X-axis is 0°

* Inclination of Y-axis or line $\parallel^r$ to Y-axis is 90°.

Slope of a line :-

① Slope of a line which makes an angle θ with positive x-axis is $\tan\theta$ and denoted by the letter 'm'.

i.e. if inclination of a line is 'θ'

then its slope is $\boxed{m = \tan\theta}$

* Slope of a line $\parallel^r$ to x-axis or x-axis is

$m = \tan 0° = 0$ (or slope of Horizontal lines is 0)

* slope of a line $\parallel^r$ to Y-axis or Y-axis is

$m = \tan 90° =$ undefined.

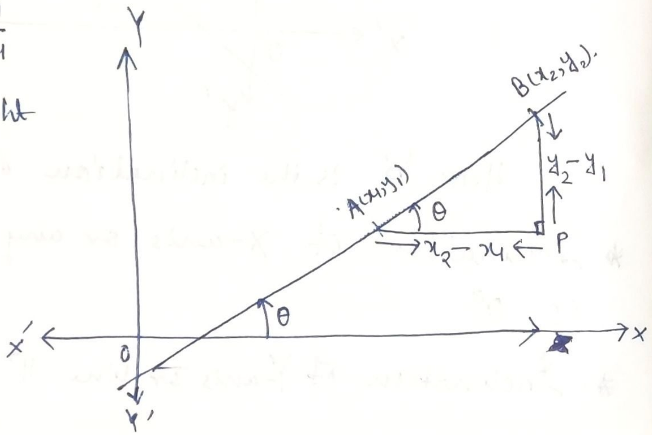
(or slope of Vertical lines is 0)

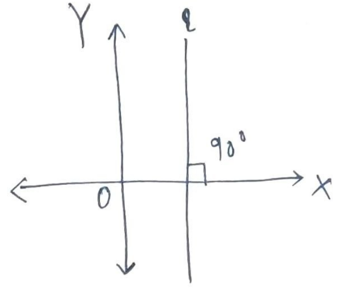
② Slope of line passing through two fixed pts $A(x_1, y_1)$ and $B(x_2, y_2)$

Slope $= \tan\theta = \frac{P}{B} = \frac{y_2 - y_1}{x_2 - x_1}$

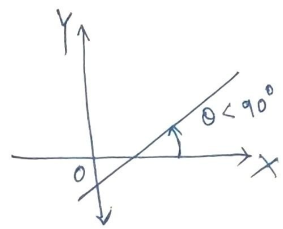
by considering the right angle Δ ABP

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

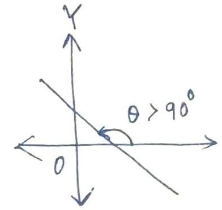




(slope = undefined.)



(slope is +ve)

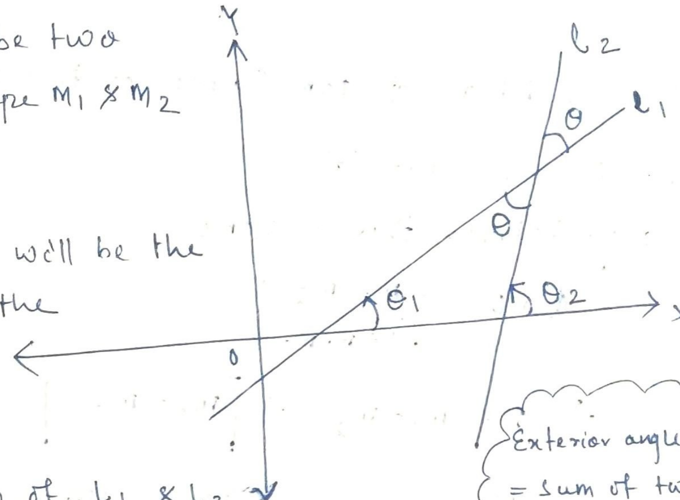


(slope -ve)

Angle between two lines :-

Let l_1 and l_2 be two lines with slope m_1 & m_2 respectively.

Then Here 'θ' will be the angle between the two lines.



Exterior angle = sum of two interior opposite angles.

→ Here inclination of L_1 & L_2 be θ_1 & θ_2 respectively (let).

Then $\theta_2 = \theta + \theta_1$

$\Rightarrow \theta = \theta_2 - \theta_1$

$\Rightarrow \tan\theta = \tan(\theta_2 - \theta_1)$

$= \dfrac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \cdot \tan\theta_1}$

$$\Rightarrow \tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1} \quad \left(\begin{array}{l} \text{as } m_1 = \tan\theta_1 \\ m_2 = \tan\theta_2 \end{array}\right)$$

$$\text{or } \tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right| \quad (\text{as } \theta \text{ is an acute angle}).$$

$$\text{or } \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\text{or } \boxed{\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|}$$

NOTE 1 condition of parallelism of two lines.

Two lines are parallel

$\Rightarrow$ angle between them is zero.

$$\Rightarrow \tan 0 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow 0 = m_1 - m_2 \Rightarrow \boxed{m_1 = m_2}$$

NOTE 2 condition of perpendicularity

Two lines are perpendicular.

$\Rightarrow$ angle between them is 90°

$$\Rightarrow \tan 90^\circ = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\Rightarrow \infty = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\Rightarrow 0 = \frac{1 + m_1 m_2}{m_1 - m_2} \qquad \Rightarrow m_1 m_2 + 1 = 0$$

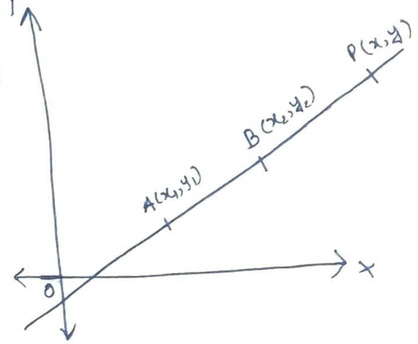
$$\Rightarrow \boxed{m_1 m_2 = -1}$$

EQUATION OF STRAIGHT LINE

① Equation of a line in two pt. form :-

Let L be a line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$

& let's take $P(x, y)$ be a general pt. on the line.



Then slope of AP = slope of AB

$$\Rightarrow \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow \boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$ required eq^n of the line

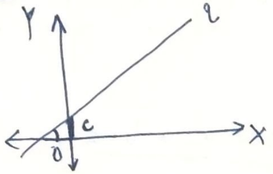
② Equation of line in point-slope form :-

Let m be the slope of the line and the line passing through (x_1, y_1)

then required eq^n is

$$\boxed{y - y_1 = m(x - x_1)}$$

(3) we have $y - y_1 = m(x - x_1)$ **Slope-intercept form**



$\Rightarrow y - y_1 = mx - mx_1$

$\Rightarrow y = mx - mx_1 + y_1$

$\Rightarrow \boxed{y = mx + c}$ also the eqⁿ of straight line

where coefficient of x is slope of line l.

$\Rightarrow$ So Eqⁿ of straight line is a linear eqⁿ in x & y.

i.e. $mx - y + c = 0$

(4) **General Eqⁿ of straight line.**

Any linear Eqⁿ in x and y is the eqⁿ of a straight line.

i.e. $\boxed{ax + by + c = 0}$ is the required eqⁿ.

NOTE 1 :-

$by = -ax - c$

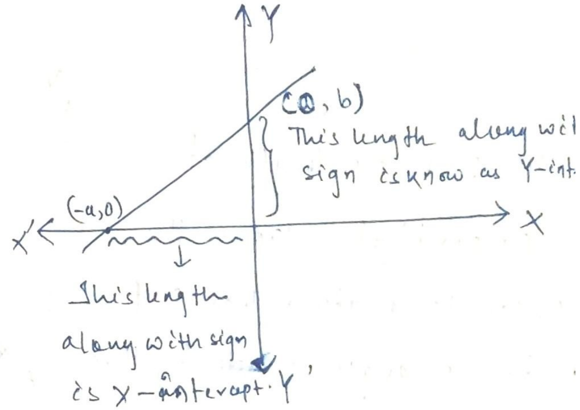
$y = \left(-\frac{a}{b}\right)x + \left(-c/b\right)$

which is in slope-intercept form.

Then $\boxed{\text{slope} = -a/b}$

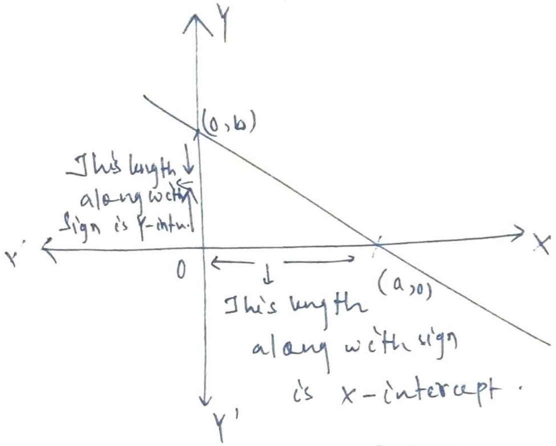
$\boxed{y\text{-intercept} = -c/b}$

(5) **Intercept form**



X-intercept = -a

Y-intercept = b

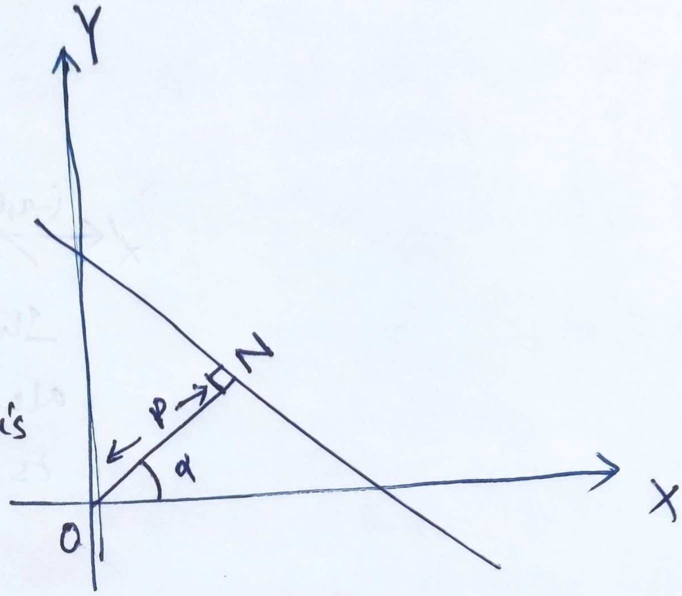


The eqⁿ of the line $\boxed{\frac{x}{a} + \frac{y}{b} = 1}$

(using two point form)

⑥ Normal / perpendicular form :-

Let p be the length of perpendicular from the origin to a given line and α be the angle made by this perpendicular line with the x-axis



then eqn of the line $x \cos\alpha + y \sin\alpha = p$

Reduction of General form to standard form

① Reduction of General form to slope intercept form.

General form $\Rightarrow ax + by + c = 0$

$$\Rightarrow by = -ax - c$$

$$\Rightarrow y = \left(-\frac{a}{b}\right)x + \left(-\frac{c}{b}\right)$$

So here

Slope $= -a/b$

Y-intercept $= -c/b$

② Intercept form

General form is

$$ax + by + c = 0$$

$$\Rightarrow ax + by = -c$$

$$\Rightarrow \frac{ax}{-c} + \frac{by}{-c} = 1$$

$$\Rightarrow \frac{x}{(-c/a)} + \frac{y}{(-c/b)} = 1$$

So here

X-intercept $= -c/a$

Y-intercept $= -c/b$

* Another condition of parallalism of two line in general form

Two lines are $\parallel^r$

then $m_1 = m_2$

$$\Rightarrow \frac{-a_1}{b_1} = \frac{-a_2}{b_2}$$

$$\Rightarrow \boxed{\frac{a_1}{b_1} = \frac{a_2}{b_2}}$$

* Another condition of perpendicularity

Let $a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$ are two perpendicular

lines then $m_1 m_2 = -1$

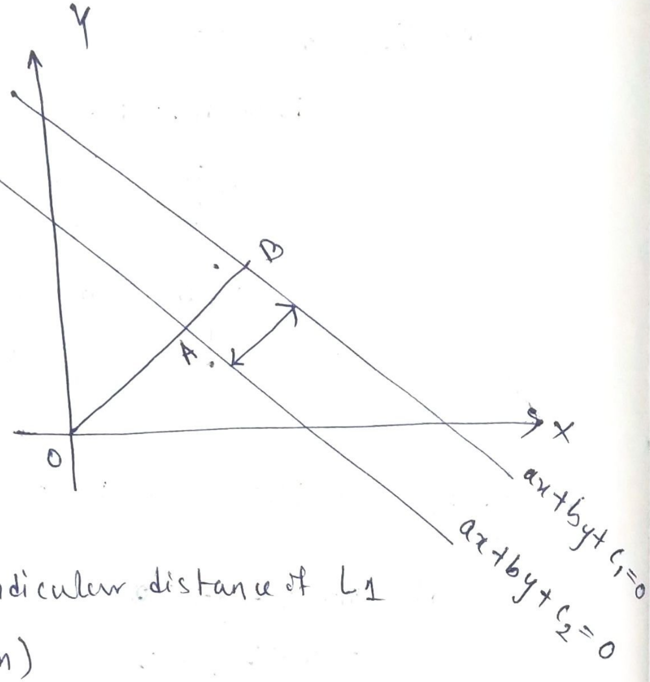
$$\Rightarrow \left(\frac{-a_1}{b_1}\right)\left(\frac{-a_2}{b_2}\right) = -1$$

$$\Rightarrow a_1 a_2 = -b_1 b_2$$

$$\Rightarrow \boxed{a_1 a_2 + b_1 b_2 = 0}$$

Distance between two parallel lines :-

Distance = shortest/ perpendicular distance between two lines.



Here OB (Perpendicular distance of L_1 from origin)

$$OB = \left|\frac{c_1}{\sqrt{a^2+b^2}}\right|$$

OA = Perpendicular distance of L_2 from origin.

$$OA = \left|\frac{c_2}{\sqrt{a^2+b^2}}\right|$$

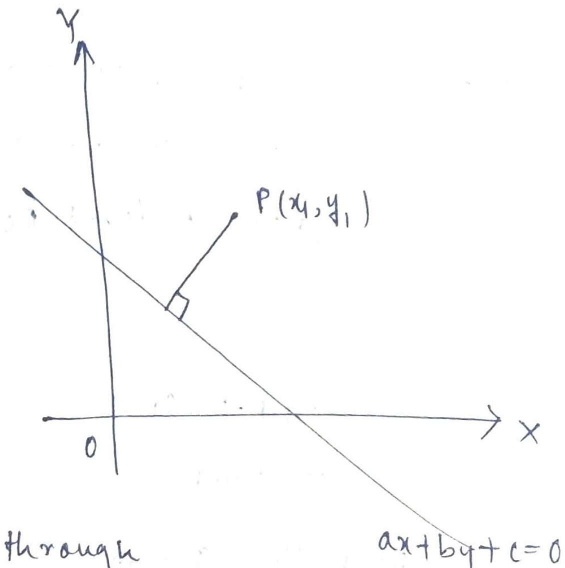
$$AB = OB - OA = \left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$$

So $\boxed{D = \dfrac{c_1 - c_2}{\sqrt{a^2+b^2}}}$

(~~Imp~~ When using this formula coefficient of x and y must be same in both lines.)

Distance between a pt. (x_1, y_1) and a line $ax+by+c=0$

(Perpendicular Distance)



Eq^n of the line passing through pt $P(x_1, y_1)$ and parallel to $ax+by+c=0$

Then slope will be same is $-a/b$

Then using slope-intercept form.

eq^n of the required line.

$$y - y_1 = -\frac{a}{b}(x - x_1)$$

$$\Rightarrow ax + by - ax_1 - by_1 = 0$$

$$\Rightarrow ax + by - (ax_1 + by_1) = 0$$

Then Distance between the lines.

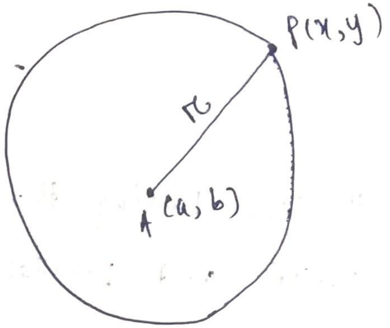
$$D = \left|\frac{c - \{-(ax_1 + by_1)\}}{\sqrt{a^2+b^2}}\right| = \boxed{\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|}$$

CIRCLE

Definition:— circle is a locus of a point 'P' which moves in such a way that it's distance from a fixed point is always constant.

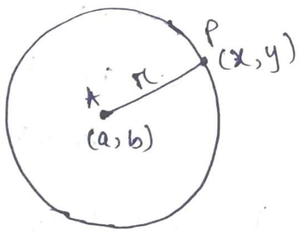
where fixed pt $= (a,b)$ is centre.

r = radius = Distance AP



Standard Equation of a circle:—

Eq^n of the circle whose centre (a,b) and radius is 'r'



Distance of AP

$$r = \sqrt{(x-a)^2+(y-b)^2}$$

Squaring both sides.

$$\Rightarrow \boxed{r^2 = (x-a)^2 + (y-b)^2}$$

which is the required eq^n of the circle.

General Eq^n of the circle:—

Standard form.

$$(x-a)^2+(y-b)^2 = r^2.$$

$$\Rightarrow x^2+2(-a)x+a^2+y^2-2by+b^2 = r^2$$

$$\Rightarrow x^2+y^2-2ax-2by+a^2+b^2-r^2 = 0$$

let $-a = g$

$-b = f$

$a^2+b^2-r^2 = c$

$$\Rightarrow \boxed{x^2+y^2+2gx+2fy+c=0}$$

required eq^n of circle.

Then centre $= (a,b) = \boxed{(-g,-f)}$ = centre

Radius $= r$

We have $a^2+b^2-r^2 = c.$

$$\Rightarrow r^2 = a^2+b^2-c$$

$$= (-g)^2+(-f)^2-c$$

$$\Rightarrow \boxed{r = \sqrt{g^2+f^2-c}}$$

NOTE :- $r = \sqrt{g^2+f^2-c}$

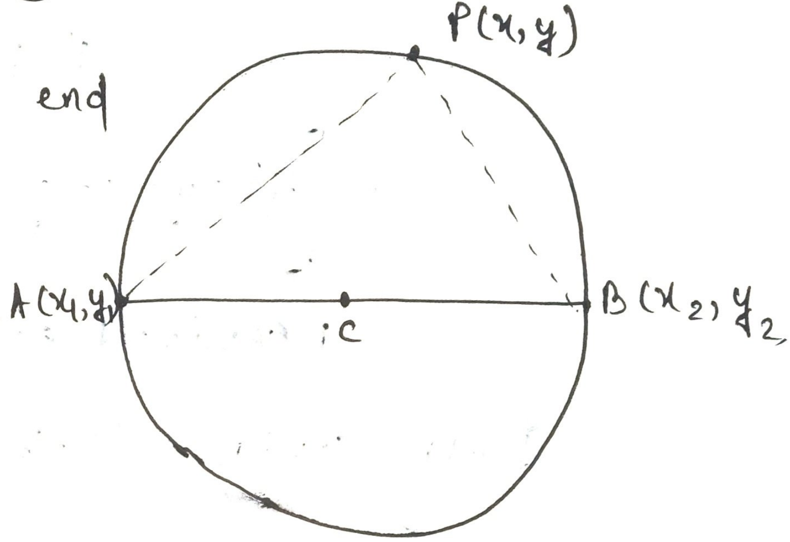
(i) if $g^2+f^2-c > 0 \Rightarrow$ real circle.

(ii) if $g^2+f^2-c < 0 \Rightarrow$ imaginary circle.

(iii) $g^2+f^2-c = 0 \Rightarrow$ Point circle. of radius '0'.

Diametrical Eqⁿ of a circle :-

Eqⁿ of a circle whose end pts of diameter are (x_1, y_1) and (x_2, y_2)



Here $AP \perp BP$

$\Rightarrow$ slope of AP · slope of BP $= -1$

$\Rightarrow \dfrac{y-y_1}{x-x_1} \cdot \dfrac{y-y_2}{x-x_2} = -1$

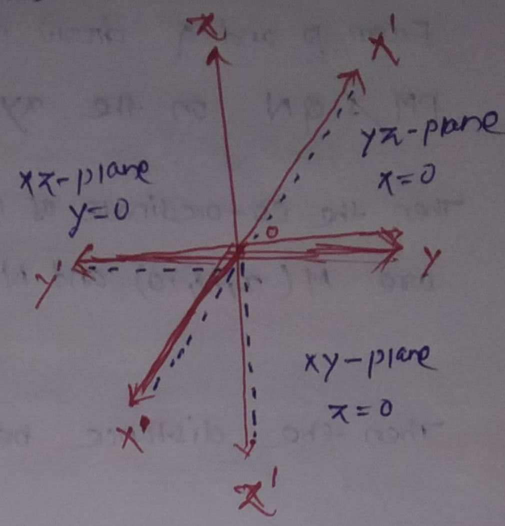
$\Rightarrow (y-y_1)(y-y_2) = -(x-x_1)(x-x_2)$

$\Rightarrow \boxed{(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0}$

Co-ordinate Geometry in Three Dimensions :

We take three perpendicular lines as axes.

O, the point of intersection is called origin.



$x'ox$ is called x-axis.

$y'oy$ is called y-axis.

$z'oz$ is called z-axis.

The three lines taken together are called rectangular co-ordinate axes.

x-axis written as $(x, 0, 0)$.

y-axis written as $(y, 0, 0)$.

z-axis written as $(z, 0, 0)$.

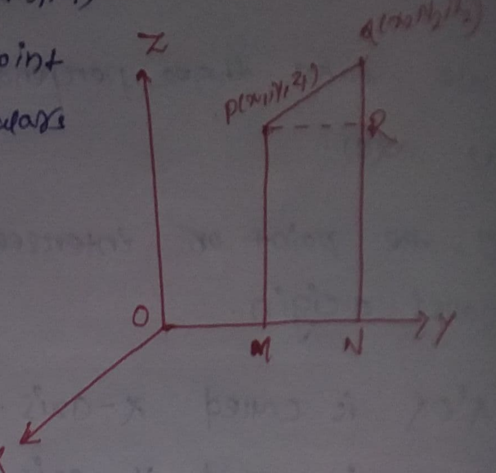
Distance Formula, Division Formula :

Theorem-1 (Distance Formula)

Prove that the distance bet^n the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by.

$$\boxed{PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

Proof Let O be origin and Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the given point. From P and Q draw perpendiculars PM & QN on the xy-plane.



Then the co-ordinate of M and N are $M(x_1, y_1, 0)$ and $N(x_2, y_2, 0)$

Then the distance betn $MN = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (0-0)^2}$

$$\Rightarrow MN^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

Now, from P draw $PR \perp QN$.

Then PR is parallel and equal to MN.

Now, in Right Angle triangle PRQ, we have,

$$PQ^2 = PR^2 + RQ^2$$
$$= MN^2 + (QN - RN)^2$$
$$= MN^2 + (QN - PM)^2$$

$$\therefore \boxed{PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}} \quad [\because PM = z_1 \ \& \ QN = z_2]$$

Corollary: The distance of the point $P(x, y, z)$ ~~and~~ from the origin $O(0, 0, 0)$ is

$$OP = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$
$$= \sqrt{x^2 + y^2 + z^2}$$

Ex-1 Find the distance bet^n two points (2,3,5) and (4,3,1).

Sol^n Distance bet^n two points (2,3,5) and (4,3,1) is

$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$

$= \sqrt{2^2 + 0^2 + (-4)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ Ans

Imp

Ex-2 Find the value of x if distances bet^n two points $(x,-8,4)$ and $(3,-5,4)$ is 5.

Sol^n Given points, $(x,-8,4)$ and $(3,-5,4)$

The distance bet^n these points = 5

$\sqrt{(x-3)^2 + (-8+5)^2 + (4-4)^2} = 5$

or, $(x-3)^2 + 9 + 0 = 25 \Rightarrow (x-3)^2 = 16$

or, $(x-3) = \pm 4$ or, $x = 7, -1$.

Ex-3 Show that the points A$(-2,-6,-7)$, B$(4,-4,-5)$, C$(7,-3,-4)$ are collinear.

Imp

Sol^n We have

$|AB| = \sqrt{(4+2)^2 + (-4+6)^2 + (-5+7)^2}$

$= \sqrt{6^2 + 2^2 + 2^2}$

$= \sqrt{36+4+4}$

$= \sqrt{44} = 2\sqrt{11}$

$|BC| = \sqrt{(7-4)^2 + (-3+4)^2 + (-4+5)^2}$

$= \sqrt{3^2 + 1^2 + 1^2}$

$= \sqrt{9+1+1}$

$= \sqrt{11}$

$$|CA| = \sqrt{(-2-7)^2 + (-6+3)^2 + (-7+4)^2}$$

$$= \sqrt{81 + 9 + 9} = \sqrt{99} = 3\sqrt{11}$$

$\therefore$ $|AB| + |BC| = |CA|$

Hence the points A, B & C are collinear.

Ex-4 imp

Show that the points $A(1,2,3)$, $B(-1,-2,-1)$, $C(2,3,2)$ and $D(4,7,6)$ are the vertices of a parallelogram ABCD, but it is not a rectangle.

Soln We have,

$$AB = \sqrt{(-1-1)^2 + (-2-2)^2 + (-1-3)^2} = \sqrt{4+16+16} = 6$$

$$BC = \sqrt{(2+1)^2 + (3+2)^2 + (2+1)^2} = \sqrt{9+25+9} = \sqrt{43}$$

$$CD = \sqrt{(4-2)^2 + (7-3)^2 + (6-2)^2} = \sqrt{4+16+16} = 6$$

$$DA = \sqrt{(1-4)^2 + (2-7)^2 + (3-6)^2} = \sqrt{9+25+9} = \sqrt{43}$$

$\therefore$ $AB = CD$ and $BC = DA$. Hence opposite sides are equal.

$\therefore$ ABCD is a parallelogram.

Again

$$AC = \sqrt{(2-1)^2 + (3-2)^2 + (2-3)^2}$$

$$= \sqrt{1+1+1} = \sqrt{3}$$

$$BD = \sqrt{(4+1)^2 + (7+2)^2 + (6+1)^2}$$

$$= \sqrt{25+81+49} = \sqrt{155}.$$

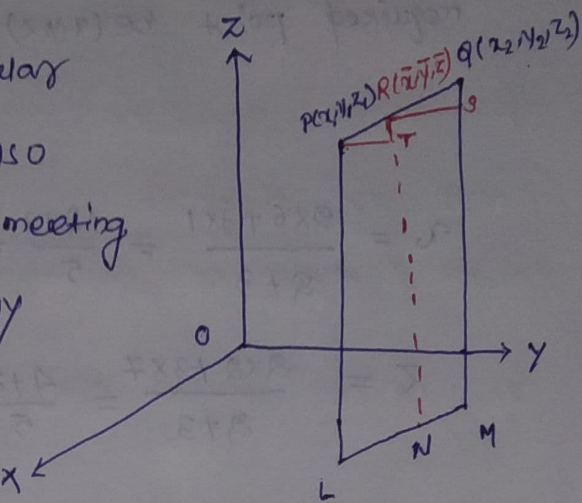
$\therefore$ $AC \neq BD$ i.e the diagonal are not equal,

Division Formula :- (Ratio Formula)

Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points and let $R(\bar{x}, \bar{y}, \bar{z})$ be a point on PQ dividing it in the ratio m : n prove that :

$$\bar{x} = \frac{mx_2 + nx_1}{m+n}, \quad \bar{y} = \frac{my_2 + ny_1}{m+n}, \quad \bar{z} = \frac{mz_2 + nz_1}{m+n}$$

Let's draw P, Q & R perpendicular PL QM and RN on xy-Plane. Also draw RS ⊥ QM and PT ⊥ RN meeting QM & RN at S and T respectivily



PRT and RQS are similar Triangles.

We have $\frac{RT}{QS} = \frac{PR}{RQ}$

$$\Rightarrow \frac{RN - TN}{QM - SM} = \frac{m}{n}$$

$$\Rightarrow \frac{\bar{z} - z_1}{z_2 - \bar{z}} = \frac{m}{n}$$

$$\Rightarrow n\bar{z} - nz_1 = mz_2 - m\bar{z}$$

$$\Rightarrow m\bar{z} + n\bar{z} = mz_2 + nz_2$$

$$\Rightarrow \bar{z}(m+n) = z_2(m+n)$$

$$\Rightarrow \bar{z} = \frac{mz_2 + nz_1}{\bar{z}}$$

Similarly $\bar{x} = \frac{mx_2 + nx_1}{m+n}$, $\bar{y} = \frac{my_2 + ny_1}{m+n}$

For internal division :-

Replacing n by -n.

$$\bar{x} = \frac{mx_2 - nx_1}{m-n}$$

$$\bar{y} = \frac{my_2 - ny_1}{m-n}$$

$$\bar{z} = \frac{mz_2 - nz_1}{m-n}.$$

Ex-1 Find the co-ordinates of a point which divides the points $(1,3,7)$, $(6,3,2)$ in the ratio $2:3$.

Sol^n

Let the co-ordinate of the required point be (x,y,z).

$(1,3,7) \longleftarrow \bullet \longrightarrow (6,3,2)$

(x,y,z)

$$x = \frac{2\times6+3\times1}{2+3} = \frac{12+3}{5} = 3, \quad y = \frac{2\times3+3\times3}{2+3} = \frac{6+9}{5} = 3$$

$$z = \frac{2\times2+3\times7}{2+3} = \frac{4+21}{5} = 5$$

$\therefore$ Required point is $(3,3,5)$.

Imp

Ex-2 Find the ratio in which the line Joining the points $(4,4,-10)$ and $(-2,2,4)$ is divided by

(a) the yz-plane (b) $x+y+z=3$.

Sol^n The given points are $(4,4,-10)$ and $(-2,2,4)$.

Let the ratio is $K:1$.

Then $$x = \frac{-2k+4}{k+1}$$

$$y = \frac{2k+4}{k+1}$$

$$z = \frac{4k-10}{k+1}$$

(a) If the plane lies in yz-plane then $x=0$

$\therefore \frac{-2k+4}{k+1} = 0$ or, $-2k+4=0$

$\Rightarrow -2k = -4$

$\Rightarrow \boxed{k=2}$

$\therefore$ The required Ratio is 2 : 1.

(b) $x + y + z = 3$.

$$\Rightarrow \frac{-2k+4}{k+1} + \frac{2k+4}{k+1} + \frac{4k-10}{k+1} = 3$$

$$\Rightarrow \frac{-2k+4+2k+4+4k-10}{k+1} = 3$$

$$\Rightarrow \frac{4k-2}{k+1} = 3$$

$\Rightarrow 4k - 2 = 3k + 3$

$\Rightarrow 4k - 3k = 3 + 2$

$\Rightarrow \boxed{k=5}$.

$\therefore$ The required ratio is 5 : 1.

EX-3 Find the ratio in which the line through (2,4,5), (3,5,−4) is divided by xy-plane.
IMP

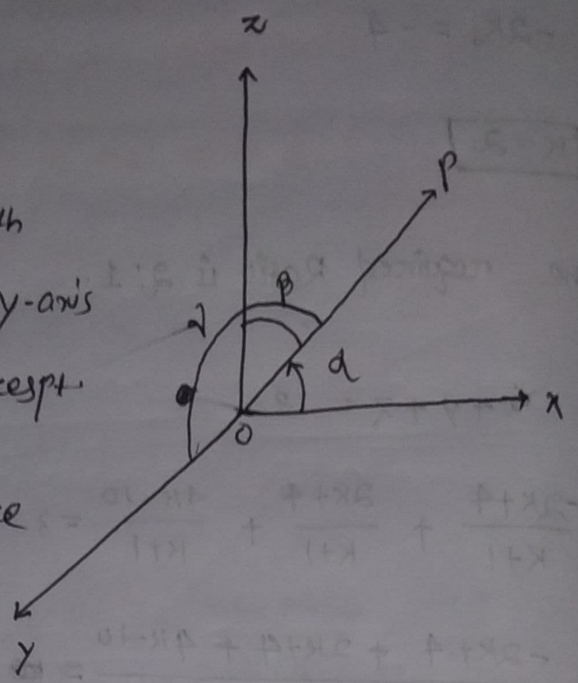
EX-4 Find the ratio in which the line Joining the points (2, −3, 1), (3, −4, −5) is divided by the laces $2x + y + z = 7$.
IMP

EX-5 Find the ratio in which the line segment Joining the points (4,3,2) & (1, 2, −3) is divided by xy-plane.
IMP

Direction cosine :

Let $\vec{op}$ be a st. line.

$\vec{op}$ makes angle α with ox-axis and β with oy-axis and γ with oz-axis respt.



then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the direction cosines (d.c's) of the line.

The (d.c's) of the line are denoted by l, m, n.

$\therefore l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma.$

In particular the d.c's of x-axis are 1, 0, 0.

Similarly the d.c's of y-axis and z-axis are 0, 1, 0 and 0, 0, 1 respectively.

Note: $\boxed{l^2 + m^2 + n^2 = 1}$

Direction Ratios : The number a, b, c are called d.r's. They are ~~the~~ written in the form $\langle a, b, c \rangle$ or $[a, b, c]$ or a, b, c.

$$\therefore \frac{a}{l} = \frac{b}{m} = \frac{c}{n} = K \text{ (say)}$$

$$\Rightarrow a = Kl,\ b = Km,\ c = Kn \quad \text{—— (1)}$$

where 'k' is the constant of proportionality. Squaring & adding these eqⁿs. we get

$$a^2+b^2+c^2 = k^2(l^2+m^2+n^2)$$

$$\therefore a^2+b^2+c^2 = k^2 \quad [\because l^2+m^2+n^2=1]$$

$$\Rightarrow k = \pm\sqrt{a^2+b^2+c^2}$$

$$l = \frac{a}{\pm\sqrt{a^2+b^2+c^2}}, \quad m = \frac{b}{\pm\sqrt{a^2+b^2+c^2}}, \quad n = \frac{c}{\pm\sqrt{a^2+b^2+c^2}}$$

Ex If a line has direction ratio $\langle -18, 12, -4\rangle$, then determine its d.c's.

Direction cosines & Direction ratios of the line segment Joining 2 pts.

Suppose $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$ are two points.

The direction ratios are $\langle x_2-x_1,\ y_2-y_1,\ z_2-z_1\rangle$

Angle betⁿ two lines :-

① The angle betⁿ two lines having d.c's l_1, m_1, n_1 & l_2, m_2, n_2
$\theta \to$ Angle.

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\Rightarrow \boxed{\theta = \cos^{-1}(l_1 l_2 + m_1 m_2 + n_1 n_2)}$$

② If $\langle a_1, b_1, c_1\rangle$ and $\langle a_2, b_2, c_2\rangle$ be d.r's of two lines then the d.c's are

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2+b_1^2+c_1^2} \times \sqrt{a_2^2+b_2^2+c_2^2}}\right)}$$

EX-15 Find the acute Angle betⁿ the lines whose d.r's are (1, 1, 2) and ($\sqrt{3}-1, -\sqrt{3}-1, 4$) resp.

$(\sqrt{3}+1)^2 = 3+1+2\sqrt{3}$

Solⁿ

Here $\langle a_1, b_1, c_1\rangle = \langle 1, 1, 2\rangle$ and

$\langle a_2, b_2, c_2\rangle = \langle \sqrt{3}-1, -\sqrt{3}-1, 4\rangle$

Then
$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\times\sqrt{a_2^2+b_2^2+c_2^2}} = \frac{\sqrt{3}-1+(-\sqrt{3}-1)+8}{\sqrt{6}\times\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+4^2}}$$

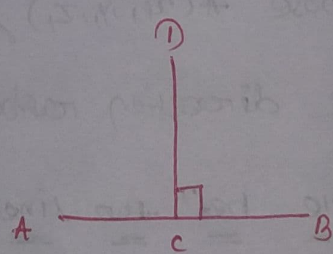
$$= \frac{6}{\sqrt{6}\times\sqrt{3+1-2\sqrt{3}+3+1+2\sqrt{3}+16}} = \frac{6}{\sqrt{6}\times\sqrt{24}}$$

$$= \frac{6}{\sqrt{144}} = \frac{6}{12} = \frac{1}{2}$$

$\therefore \cos\theta = 1/2 \Rightarrow \theta = \cos^{-1}\frac{1}{2} = 60^\circ$.

$\therefore$ Hence the reqd. Angle $\theta = 60^\circ$.

Case-1 condition For $\perp^r$:



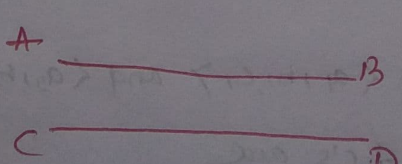
Here $\theta = 90^\circ$

$\cos\theta = \cos 90^\circ = 0$

$$\boxed{l_1l_2+m_1m_2+n_1n_2=0}$$

In term's of d.r's of two lines the $\perp^r$ condⁿ is

$$\boxed{a_1a_2+b_1b_2+c_1c_2=0}$$



AB $\parallel^r$ C

Case-2 Condⁿ for $\parallel^r$

Here $\theta = 0^\circ$

$\cos 0^\circ = 1$.

$$\boxed{\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2} = 1}$$

For d.r's

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = 1}$$

Unit-4 Plane

Plane :- A plane is defined as surface such that the st. line joining any two points on the surface lies on it.

Note $x=0$, is the eq^n of yz-plane

$y=0$, is the eq^n of zx-plane

$z=0$, is the eq^n of xy-plane.

Normal form :- The eq^n of plane is

$$\boxed{lx + my + nz = p} \quad \{\because l^2+m^2+n^2 = 1\}$$

Th^m-3 Equation of the plane ~~through~~ making off intercepts a, b, c on the co-ordinate axes is

$$\boxed{\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}.$$

Plane through a given point :-

The eq^n of any plane through (x_1, y_1, z_1) is

$$\boxed{a(x-x_1) + b(y-y_1) + c(z-z_1) = 0}.$$

Th^m-4 Find the eq^n of the plane passing through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

This is the reqd. eq^n of the plane

Thm-5 : To reduce the eq^n $ax+by+cz+d=0$ of the plane from general form to normal form.

Sol^n:

The given eq^n is

$$ax+by+cz+d=0$$

Hence, in general $\langle a,b,c\rangle$ are the drs of the normal to the plane. Let $\langle l,m,n\rangle$ be the direction cosine of the normal so, we have

$$l=\frac{a}{\pm\sqrt{a^2+b^2+c^2}}, \quad m=\frac{b}{\pm\sqrt{a^2+b^2+c^2}}$$

$$n=\frac{c}{\pm\sqrt{a^2+b^2+c^2}}.$$

Plane passing through the intersection of two points:-

Consider two intersecting planes given by the eq^s

$$a_1x+b_1y+c_1z+d_1=0 \quad \text{——— (1)}$$

$$a_2x+b_2y+c_2z+d_2=0 \quad \text{——— (2)}$$

We consider the eq^n

$$(a_1x+b_1y+c_1z+d_1)+K(a_2x+b_2y+c_2z+d_2)=0$$

where K is a parameter

i.e $(a_1+Ka_2)x+(b_1+Kb_2)y+(c_1+Kc_2)z+(d_1+Kd_2)=0$

which is also a plane.

Angle between two planes :

Angle between two planes is equal to the angle between their normals.

If the eq^n of two planes be

$a_1x + b_1y + c_1z + d_1 = 0$

and $a_2x + b_2y + c_2z + d_2 = 0$

If 'θ' be the angle bet^n the planes

$$\boxed{\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}}$$

(i) The plane will be parallel if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(ii) and perpendicular if $\cos\theta = 0$

i.e $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(iii) Two planes are identical if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}$$

Note-1: The distance of the point (x_0, y_0, z_0) from the plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_0 + by_0 + cz_0 + d}{\sqrt{a^2+b^2+c^2}}\right|$$

Note-2

Distance bet^n two parallel planes $ax+by+cz+d=0$ and $ax+by+cz+d_1=0$ is given by

$$D = \left|\frac{d - d_1}{\sqrt{a^2+b^2+c^2}}\right|.$$

Examples

Ex-1 Find the eqⁿ of the plane which passes through $(4,-2,1)$ and is perpendicular to the line whose direction ratios are $7, 2, -3$.

Solⁿ Equation of the plane through $(4,-2,1)$ is

$$a(x-4)+b(y+2)+c(z-1)=0 \quad \text{——(i)}$$

Since the drs of the normal to the plane are $7, 2, -3$.

$\therefore a=7, b=2, c=-3$.

Putting these values of a, b and c (i) we get

$$7(x-4)+2(y+2)-3(z-1)=0$$

or, $7x+2y-3z-21=0$.

is the reqd. eqⁿ of the plane.

Imp

Ex-2 Find the eqⁿ of the plane which passes through the point $(1,-1,4)$ and is parallel to the plane $2x-3y+7z=11$.

Solⁿ Any parallel to the plane $2x-3y+7z-11=0$ is of the form $2x-3y+7z+k=0$ ——(1)

Since it passes through $(1,-1,4)$

$\therefore 2(1)-3(-1)+7(4)+k=0$

or $2+3+28+k=0$

$\Rightarrow k=-33$

Put $k=-33$ in eqⁿ (1) we get

$$2x-3y+7z-33=0$$

Ex-3 Find the eqⁿ of the plane containing the line of intersection of the planes $x+y+z+1=0$, $2x-3y+5z-2=0$ and passing through the point $(-1,2,1)$.

Solⁿ

Eqⁿ of any plane passing through the line of intersection of the planes

$$x+y+z+1=0 \quad \text{and} \quad 2x-3y+5z-2=0$$

and passing through $(-1,2,1)$.

$$\Rightarrow (x+y+z+1)+\lambda(2x-3y+5z-2)=0 \quad \text{——— (i)}$$

Since (i) also passes through $(-1,2,1)$

$$\therefore (-1+2+1+1)+\lambda(-2-6+5-2)=0$$

or $3+\lambda(-5)=0$ or $\boxed{\lambda=\frac{3}{5}}$

Putting this values of $\frac{3}{5}$ in (i) we get

$$(x+y+z+1)+\frac{3}{5}(2x-3y+5z-2)=0$$

or $5x+5y+5z+5+6x-9y+15z-6=0$

or. $11x-4y+20z-1=0$ is the reqd. eqⁿ of the plane.

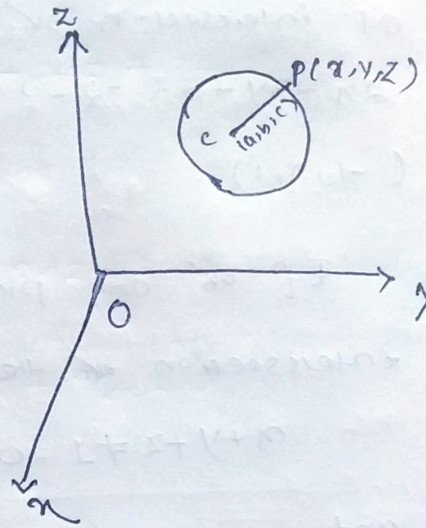
Imp

Ex-4 Find the eqⁿ of the plane passing through the point $(-1,-1,2)$ and $\perp^r$ to the planes

$$3x+2y-3z=1$$

and $5x-4y+z=5$

Sphere



— A sphere is the locus of a point in space which moves in such a way that it remains always at a constant distance from a fixed point.

The fixed point is called the centre and the constant distance is called radius of the sphere.

Thm-1 The eq^n of the sphere with centre at the point (a,b,c) and radius r is given by

$$\boxed{(x-a)^2+(y-b)^2+(z-c)^2=r^2}$$

General eq^n of a sphere:

General eq^n of a sphere expressed in the form

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

Hence the centre of the sphere is $(-u,-v,-w)$ and radius of the sphere is $\sqrt{u^2+v^2+w^2-d}$.

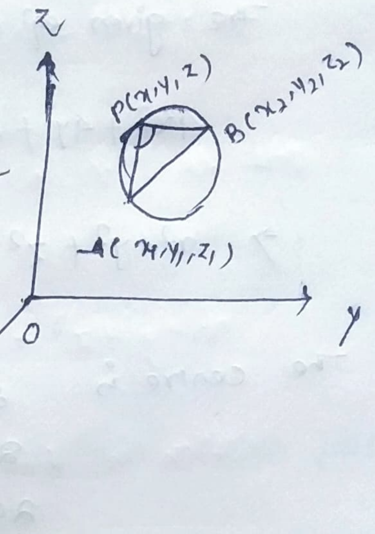
Note

If $u^2+v^2+w^2<d$ sphere may be called as imaginary sphere.

If $u^2+v^2+w^2>d$ sphere may be called as real sphere

Theⁿ - 2 The co-ordinates of end point of a diameter of a sphere are (x_1, y_1, z_1) and (x_2, y_2, z_2). find the eqⁿ of the sphere.

Solⁿ Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be two end points of diameter of a sphere.



using condition of perpendi-cularity

$$\boxed{(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + (z-z_1)(z-z_2) = 0}$$

Theⁿ-3 To find the eqⁿ of the sphere through four given points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4).

Solⁿ Let the eqⁿ of the sphere be

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

since it passes through (x_1, y_1, z_1) (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) is given by

$$x_1^2+y_1^2+z_1^2+2ux_1+2vy_1+2wz_1+d=0 \quad \text{——— (1)}$$

$$x_2^2+y_2^2+z_2^2+2ux_2+2vy_2+2wz_2+d=0 \quad \text{——— (2)}$$

$$x_3^2+y_3^2+z_3^2+2ux_3+2vy_3+2wz_3+d=0 \quad \text{——— (3)}$$

$$x_4^2+y_4^2+z_4^2+2ux_4+2vy_4+2wz_4+d=0 \quad \text{——— (4)}$$

Solving these eqⁿs we get the reqd. sphere.

Ex-1 Find the centre and radius of the sphere

$$4x^2 + 4y^2 + 4z^2 - 16x - 24z + 3 = 0$$

Soln The given eqn of sphere is

$$4x^2 + 4y^2 + 4z^2 - 16x - 24z + 3 = 0$$

$$\Rightarrow x^2 + y^2 + z^2 - 4x - 6z + 3/4 = 0$$

The centre is $2u = -4 \Rightarrow u = -2$

$2v = 0 \Rightarrow v = 0$

$2w = -6 \Rightarrow w = -3$

$$\boxed{(-u, -v, -w) = (2, 0, 3)}$$

Radius is $= \sqrt{u^2 + v^2 + w^2 - d} = \sqrt{4 + 0 + 9 - \frac{3}{4}}$

$= \sqrt{13 - \frac{3}{4}} = \sqrt{\frac{49}{4}} = 7/2.$

Hence the centre is $(2, 0, 3)$ and radius is $7/2$.

Ex-2 Find the eqn of the sphere on the Joint of $(2, 3, 5)$ and $(4, 9, -3)$ as diameter?

Soln We know that eqn of a sphere whose end points of the diameters are (x_1, y_1, z_1) and (x_2, y_2, z_2) is.

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + (z - z_1)(z - z_2) = 0$$

$\therefore$ Eq^n of the sphere whose end points of the diameter are $(2,3,5)$ and $(4,9,-3)$ is

$$(x-2)(x-4)+(y-3)(y-9)+(z-5)(z+3)=0$$

$$\Rightarrow x^2+y^2+z^2-6x-12y-2z+8+27-15=0$$

$$\Rightarrow x^2+y^2+z^2-6x-12y-2z+20=0$$

is the reqd. eq^n of the sphere.

Ex-3 Find the eq^n of the sphere which passes through the points $(0,0,0)$, $(0,1,0)$, $(1,0,0)$ & $(0,0,1)$.

Sol^n Let the eq^n of sphere be

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \quad \text{—— (1)}$$

Since $(0,0,0)$ lies on (i)

$$\therefore d=0 \quad \text{—— (2)}$$

Again $(0,1,0)$ lies on (i)

$$\therefore 1+2v+d=0$$

$$\text{or } 1+2v=0 \Rightarrow v=-\frac{1}{2} \quad (\because d=0)$$

Also $(1,0,0)$ lies on (i)

$$\therefore 1+2u+d=0$$

$$\text{or } 1+2u=0$$

$$\Rightarrow u=-\frac{1}{2}.$$

Again $(0,0,1)$ lies on (i)

$\therefore \quad 1 + 2w + d = 0$

$\Rightarrow 1+2w=0 \Rightarrow w = -\frac{1}{2} \quad (\because d=0)$

Put $u = v = w = -\frac{1}{2}$ and $d=0$ in (i) we get

$$x^2+y^2+z^2-x-y-z=0$$

which is the reqd. eqn of the sphere

EX-4 Find the eqn of the sphere with centre $(3,-2,5)$ and radius 4.

Soln The reqd. eqn of the sphere is

$$(x-3)^2+(y+2)^2+(z-5)^2 = 4^2 = 16$$

$$\Rightarrow x^2+y^2+z^2-6x+4y-10z+22=0$$

$$\Rightarrow x^2+y^2+z^2-6x+4y-10z+22=0$$

Imp

EX-5 Find the eqn of the sphere which passes through the points $(0,0,0)$, $(-a,b,c)$, $(a,-b,c)$ and $(a,b,-c)$.

Ans