Probable Questions SUB-MATHEMATICS
for 2nd Semester.

CHAPTER – LIMIT & CONTINUITY

2 MARKS

Q.1 Evaluate $\lim_{n\to\infty} \frac{n!}{(n+1)! - n!}$

Q.2 Evaluate $\lim_{x\to 0} \frac{\sqrt{x+1} - 1}{x}$

Q.3 Evaluate $\lim_{x\to 0} \frac{\sin 5x}{\tan 3x}$

Q.4 Evaluate $\lim_{x\to 0} \frac{\sin x^0}{x}$

Q.5 Evaluate $\lim_{x\to 0} \frac{\sin 3x}{5x}$

Q.6 Evaluate $\lim_{x\to 5} \frac{\sqrt{x} - \sqrt{5}}{x-5}$

Q.7 find $\lim_{x\to 5/2} [x]$

Q.8 Evaluate $\lim_{x\to 0} \frac{x}{\tan^{-1} x}$

Q.9 Evaluate $\lim_{x\to 0} \frac{1-\cos x}{x}$

Q.10 Evaluate $\lim_{x\to 0} \frac{e^{\sin x} - 1}{x}$

Q.11 Evaluate $\lim_{x\to 0} \left(\frac{1-\cos x}{x^2}\right)$

Q.12 Evaluate $\lim_{x\to 4} \frac{x^2 - 16}{x-4}$

Q.13 Evaluate $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1} - 1}$

Q.14 Prove that $\lim_{x\to 0} \frac{\tan x}{x} = 1$.

Q.15 Evaluate the L.H.L of the function:

$$f(x) = \begin{cases} \frac{|x-4|}{x-4}, & x \neq 4 \\ 0, & x = 4 \end{cases} \quad \text{at } x = 4$$

5 MARKS

Q.1 Evaluate $\lim_{n\to\infty} \frac{1^3 + 2^3 + 3^3 + \cdots + n^3}{n^4}$

Q.2 Evaluate $\lim_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sin^{-1} x}$

Q.3 $\lim_{x\to 0} \frac{(x+9)^{3/2} - 27}{x}$

Q.4 $\lim_{x\to 0} \frac{\operatorname{cosec} x - \cot x}{x}$

Q.5 Evaluate $\lim_{x\to 1} \frac{\log_e(2x-1)}{x-1}$

Q.6 Examine the continuity of the function f defined by

$$f(x) = \begin{cases} 2x+1, & \text{if } x<1 \\ 0, & \text{if } x=0 \\ x^2-1, & \text{if } x>1 \end{cases}$$

Q.7 Discuss the continuity of the function.

$$f(x) = \begin{cases} x - \frac{|x|}{x}, & x \neq 0 \\ 2, & x=0 \end{cases} \quad \text{at } x=0$$

Q.8 Examine the continuity of the function.

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x=0 \end{cases} \quad \text{at } x=0$$

Q.9 If A function $f(x)$ is defined as $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ K, & x=3 \end{cases}$

is continuous at $x=3$

then find the value of K.

Q.10 Evaluate $\lim_{x\to 0} \left(\frac{\tan x - \sin x}{\sin^3 x} \right)$

10 Marks

Q.1 Examine the continuity of the function

$$f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x \neq 0 \\ 0, & \text{if } x=0 \end{cases} \quad \text{at } x=0$$

Q.2 If $f(x) = \begin{cases} ax^2+b, & \text{if } x<1 \\ 1, & \text{if } x=1 \\ 2ax-b, & \text{if } x>1 \end{cases}$ is continuous at $x=1$, then find a & b.

Q.3 Discuss the continuity of the function at $x=\frac{1}{2}$,

$$f(x) = \begin{cases} x, & 0 \le x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ 1-x, & \frac{1}{2} < x \le 1 \end{cases}$$

Q.4 find the value of a if $\lim_{x\to 2} \frac{\log(2x-3)}{a(x-2)} = 1$

CHAPTER - DERIVATIVE

2 Marks find the derivative of the followings.

(i) $y = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$

(ii) $y = \log(\log(\log x))$

(iii) $y = \sqrt{e^{\sqrt{x}}}$

(iv) $y = \sqrt{\cot^{-1}\sqrt{x}}$

(v) $y = \frac{a^x - b^x}{x}$

(vi) find y_1 and y_2 if $y = \tan x$.

(vii) If $y = A\cos x + B\sin x$ then P.T. $\frac{d^2y}{dx^2} + y = 0$

(viii) If $z = x^2y + xy^2$. find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

(ix) State Euler's Theorem. 3.14

(x) Test whether the function $f(x,y) = \sin^{-1}\left(\frac{x}{y}\right)$ is homogeneous or not. 3.1

(xi) find the partial derivatives of $u = e^{xyz}$ * $u = x^y + y^x$

(xii) what is the slope of the curve $y = \sin x$ at $x = \frac{\pi}{6}$.

(xiii) Differentiate $\sin[\cos(\tan x)]$

(xiv) If $f(x) = \sin^2 x$, find $f'(x)$.

(xv) find the derivative of $\sin x^0$.

5 Marks

Q.1 Differentiate $\tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$

Q.2 Differentiate $\tan^{-1}\left(\sqrt{1+x^2} + x\right)$

Q.3 Differentiate $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

Q.4 find $\frac{dy}{dx}$ if $x = a(\theta + \sin\theta), y = a\cos\theta$

Q.5 find $\frac{dy}{dx}$, if $e^y \ln x + x \ln y = 0$

Q.6 Differentiate $\sin^2 x$ w.r.t. $(\ln x)^2$.

Q.7 find $\frac{dy}{dx}$ if $x^y y^x = 1$

Q.8 Differentiate (i) $(\ln x)^x$ (ii) $(\ln x)^{\tan x}$ (iii) $(\sin x)^{\cos^{-1} x}$.

Q.9 If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, find $\frac{d^2y}{dx^2}$.

Q.10 If $y = \log(x + \sqrt{1+x^2})$, P.T. $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

Q.11 If $y = \frac{1}{2}(\sin^{-1} x)^2$, then P.T. $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 1$

Q.12 If $z = f\left(\frac{y}{x}\right)$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$

Q.13 Differentiate $\tan^{-1}(\sec x + \tan x)$

3.14 Prove that if $z = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \tan z$

3.15 If $f(x,y,z) = \log(x^3+y^3+z^3-3xyz)$ P.T.

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 3$$

10 Marcks

Q.1 If $y = e^{m\cos^{-1}x}$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$

Q.2 If $x = \sin t$, $y = \sin(pt)$ then show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$

Q.3 If $\sqrt{1-x^4} + \sqrt{1-y^4} = k(x^2-y^2)$, then show that $\frac{dy}{dx} = \frac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$

Q.4 Differentiate $(\cos x)^{\ln x} + (\log x)^x$

Q.5 Differentiate $(\cos x)^y = (\sin y)^x$

Q.6 (i) Differentiate $\frac{e^{x^2}\tan^{-1}x}{\sqrt{1+x^2}}$

(ii) find $\frac{dy}{dx}$, when $x^y = y^x$

Q.7 If $u = (x^2+y^2+z^2)^{-1/2}$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

Chapter - Integration

2 Marks

(1) Evaluate $\int \sqrt{1-\sin 2x}\, dx$.

(2) Evaluate $\int \sin^{-1}(\cos x)\, dx$.

(3) Evaluate $\int (\tan x + \cot x)^2\, dx$.

(4) Evaluate $\int \frac{x^2}{x^2+1}\, dx$.

(5) Evaluate $\int \frac{\operatorname{cosec}^2 x}{1+\cot x}\, dx$.

(6) Evaluate $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}}\, dx$.

(7) Evaluate $\int 2x \operatorname{cosec}^2 x^2\, dx$.

(8) Evaluate $\int \frac{\operatorname{cosec}^2(\ln x)}{x}\, dx$.

(9) Evaluate $\int \frac{dx}{x\sqrt{25-(\ln x)^2}}$

(10) Evaluate $\int \frac{dx}{x \ln x \sqrt{(\ln x)^2-4}}$

(11) Evaluate $\int \frac{e^{5x}}{\sqrt{e^{10x}-4}}\, dx$.

(12) Evaluate $\int_0^1 \frac{dx}{1+x^2}$

(13) Evaluate $\int_2^3 2x\, e^{x^2}\, dx$.

(14) Evaluate $\int_1^4 [x]\, dx$.

(15) Evaluate $\int_{-3}^{4} |x|\, dx$.

5 Marks

(1) Prove that $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$

(2) Prove that $\int \frac{dx}{\sqrt{x^2+a^2}} = \log|x+\sqrt{x^2+a^2}| + c$

(3) Evaluate $\int \frac{dx}{\sqrt{2-4x+x^2}}$

(4) Evaluate $\int \frac{2x+1}{\sqrt{x^2+2x-1}}\,dx$ and $\int \frac{x+5}{\sqrt{x^2+6x-7}}\,dx$.

(5) Evaluate $\int \frac{\cos\theta\, d\theta}{\sin^2\theta\sqrt{\operatorname{cosec}^2\theta - 4}}$

(6) Evaluate $\int \frac{1}{\sin^2\theta\sqrt{\cot^2\theta+2}}\,d\theta$

(7) Evaluate $\int \tan^{-1}x\,dx$.

(8) Evaluate $\int \ln(x^2+x+2)\,dx$ and $\int (\log x)^2\,dx$.

(9) Evaluate $\int x\tan^{-1}x\,dx$.

(10) Evaluate $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$.

(11) Evaluate $\int e^x\left(\frac{1+x\log x}{x}\right)dx$.

(12) Evaluate $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$.

(13) Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx$.

(14) Evaluate $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$.

(15) Evaluate $\int_0^{\pi/2} \log\tan x\,dx$

10 Marks

1) Evaluate $\int \sqrt{a^2-x^2}\, dx$.

2) Evaluate $\int e^{3x} \cos 2x\, dx$.

3) Evaluate $\int \log(x+\sqrt{x^2+a^2})\, dx$.

4) Prove that $\int_0^{\pi/4} \log(1+\tan\theta)\, d\theta = \frac{\pi}{8}\log 2$

5) Prove that $\int_0^{\pi/2} \log(\sin x)\, dx = -\frac{\pi}{2}\log 2$

6) find the whole area of the circle $x^2+y^2=a^2$

-: Differential Equations :-

Q.1 find the order and degree.

(i) $\left(\frac{dy}{dx}\right)^2 + 3y^2 = 5x$ (ii) $\frac{d^2y}{dx^2} = \sqrt{3 + \frac{dy}{dx}}$

(iii) $\frac{dy}{dx} = \frac{3}{\frac{dy}{dx}}$ (iv) $\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$

(v) $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{5/2} = 3\left(\frac{d^2y}{dx^2}\right)$

Q.2 (i) find the diff. eq^n of the family of curves.

$y = A\cos x + B\sin x$.

(ii) find the diff eq^n of the family of curves.

$y = Ae^{2x} + Be^{-3x}$.

Q.3 Solve $(1+x^2)\,dy + (1+y^2)\,dx = 0$

Q.4 Solve $x(1+y^2)\,dx + y(1+x^2)\,dy = 0$

Q.5 Solve $e^x \tan y\,dx + (1+e^x)\sec^2 y\,dy = 0$

Q.6 Solve $\sec x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

Q.7 Solve $\frac{dy}{dx} + (\sec x)\,y = \tan x$.

Q.8 Solve $(1+x^2)\frac{dy}{dx} + 2xy - x^3 = 0$

Q.9 Solve $x\left(\frac{dy}{dx}\right) + 3y = x^2$